Labor Demand: First Lecture

LABOR ECONOMICS (ECON 385)

BEN VAN KAMMEN, PHD

Firms

- Firms turn inputs into output.
- The inputs at a real life firm are numerous and precisely differentiated. In addition to all the many different occupations (types of labor) represented at a moderate-sized company, there are at least as many types of capital (tools and machines), energy to power them, the land on which the building is located and the entrepreneurship (risk taking) that creates the firm in the first place.
- When a firm combines all of these, the result is a good or a service that is sold to consumers in a market. This is the firm's output—the good (or goods) it supplies to consumers.
  - Consumers pay for the output; there is a price for it, and this is the source of the firm's revenue.
  - Since the goods market is the focus for the majority of microeconomics courses, we will not dwell on it much here.

Production functions

In economic theory, firms turn inputs into output according to a production function.

- It is possible to analyze production functions with a realistic number of inputs ("n" inputs), but this makes the model exceedingly hard to solve. For the present purpose, we use a simplified model consisting of two inputs: capital and labor.
  - According to this simplification, labor represents all human inputs, and capital represents all non-human inputs. This is an important assumption about labor: it is homogenous in this model, i.e., it is not differentiated by skill or any other non-productive dimension.
- Modeling production using a mathematical function enables the economist to perform operations and derive quantifiable predictions from the model. For example, the downward slope of the labor demand is one prediction, and the elasticity of demand is a measurable indication of this prediction.
  - Any time we write down a mathematical model of economic behavior, however, we are making assumptions. When doing this, the theorist faces a trade-off between more restrictive assumptions and a more difficult-to-solve model. The "art" of economics is deciding which assumptions to make.

Production functions (continued)

- Notation for the production function typically looks like the following:

$$q = f(L, K)$$

where,

q is the level of output,

L is the number of hours of labor, and

K is the number of units of capital.

- The function, “f”, is the set of mathematical ”rules” that determine the level of output produced, given the values of L and K. To use the jargon, L and K are arguments in the function, and that function maps their values to the functional value, q. In English this means: “you tell me how much labor and capital you have, and the function will tell you how much output that will produce”.

Properties of production functions

So far we have made no assumptions about specifically how those inputs relate to output. Some of these will be necessary to get interesting solutions from the model, however. Among the most justifiable assumptions that can be made are the following.

- “More of a given input—if we leave the other one unchanged—will result in more output”.
- “More capital—leaving labor unchanged—increases the output per unit of labor”.
- “Doubling the levels of both labor and capital will result in no more than a doubling of output”.
- “If you continue increasing one input—without increasing the other—the increased output that results will get smaller and smaller”.
- The preceding statements can be translated into economics terminology as: a) positive marginal product, b) complementarity of inputs, c) non-increasing returns to scale, and d) diminishing returns to inputs.

Properties of production functions (continued)

- In mathematical notation, the properties are espoused as follows.

Positive Marginal Product $\rightarrow \frac{\partial f}{\partial L} \equiv MP_L > 0$ and $\frac{\partial f}{\partial K} \equiv MP_K > 0$

Complementarity $\rightarrow \frac{\partial MP_K}{\partial L} > 0$ and $\frac{\partial MP_L}{\partial K} > 0$

Non $-$ increasing Returns to Scale $\rightarrow tf(L, K) \geq f(tL, tK)$,

where t is any positive constant,

Diminishing Marginal Product $\rightarrow \frac{\partial MP_L}{\partial L} < 0$ and $\frac{\partial MP_K}{\partial K} < 0$.

Profit under perfect competition

The production function is a key component of the firm's profit function. In order to know how much profit the firm makes, it is compulsory to know how much it produces. There are two main parts to the profit function—revenue and costs. The revenue is all the money the firm makes from selling its output. Cost is the total money paid to all of the inputs in exchange for their productive services.

- As stated earlier, output is sold in a market at a price per unit. For now assume that the relationship between output and revenue is linear, i.e., that the price on all units sold is constant (Price=*P*). According to this assumption, revenue would be P*q.

$$\text{Total Revenue} = P * q = PK^{\alpha}L^{\beta}$$

  - Think back to microeconomics about this. When the price facing an individual firm is constant, this is a condition that arises under perfect competition. So this assumption translates into: "the firm sells its output in a perfectly competitive market."

Input costs

- The two inputs also each have prices. Capital has a rental rate, denoted by “r”, and labor is paid a wage, denoted by “w”. For the sake of simplicity, assume for now that the labor and capital markets are perfectly competitive.
  - The upshot of this is that the firm hires all units of capital and labor at the identical price. According to this assumption,

$$\text{Wage Bill} = wL \text{ and Capital Expenditure} = rK$$

- The total cost including both inputs, then, is $wL + rK$.
- See the last page of these notes for an example using Cobb-Douglas functional form.

Profit function

Profit is merely the excess of revenue over costs. It is denoted in most books with the Greek letter, pi (Profit=Π).

$$\Pi = Revenue - Cost = P * f(L,K) - (wL + rK)$$

- It can be shown that under these assumptions the profit function is "concave" and, therefore, has a maximum point. This elaboration is omitted, however. Profit is a function, though, in the same sense that production is. And the arguments ("choice variables") for the firm are its levels of labor and capital,

$$\Pi\left(L,K\right) = P * f\left(L,K\right) - wL - rK.$$

Profit function (continued)

- Note also that by writing profit this way, one has made another assumption about firm behavior. The output price, P, is constant, i.e., $price = P \; \forall \; q$. This assumption states that the firm has no market power and sells its output in a competitive market.

- Another assumption that is implied by this profit expression is that the wage is constant, i.e., $wage = w \; \forall \; L$. So this is the counterpart of assuming perfect competition in the output market: the firm has no market power in the hiring of workers in the labor market, either.
  - Given that output is assumed to be competitive, this shouldn't be much of a stretch. Competition is achieved by having many firms to compete the price down to marginal cost. It stands to reason they would also compete the wage up to eliminate exploitation (paying below-market wages) by one firm.

SR profit maximization

Maximizing profit in the Short Run requires the firm to only select its labor.

- The short run is a conceptual interval of time in which one input (capital) is fixed. Intuitively think of capital as the "size of the factory"—which cannot be easily changed in a short period of time. Labor, on the other hand, can be changed with comparative ease by, say, contacting a temporary help service.
- Thinking about the short run is also a pedagogical tool for looking at one input decision at a time. This makes the problem of deriving input demand simpler.
- Mathematically the short run amounts to setting K equal to some constant level, and only considering the amount of labor to hire.

SR profit maximization (continued)

- Short run profit is:

$$\Pi(L;\bar{K}) = P * f(L; K = \bar{K}) - wL - r\bar{K}$$

where “K bar” is the level of short run capital and is taken as given (constant). The maximum profit in the short run occurs where

$$\frac{\delta\Pi|\bar{K}}{\delta L} = 0.$$

- If marginal profit is positive, the firm could obtain more profit by increasing L (and therefore output). If marginal profit is negative, the firm could obtain more profit by decreasing L. So the only place where profit is maximized is where marginal profit is strictly equal to zero.
- This is called the <u>first order condition</u> for a maximum.

SR profit maximization (concluded)

- The marginal profit is (using differential calculus notation):

$$\frac{\delta \Pi}{\delta L} = P * \frac{\delta f_{K=\bar{K}}}{\delta L} - w$$

  - The first term in the expression is called the value of the marginal product of labor (VMPL). It consists of the output price times the marginal product of labor, MPL (see above). This is the additional output produced by the marginal unit of labor times how much the output can bring in the goods market. Think of it as the "marginal benefit" of one extra hour of labor.
  - The second term (w) is the marginal cost of one extra unit (hour) of labor. So the first order condition implies: $VMP_L = w$.

- Setting this expression equal to zero to impose on it the first order condition yields:

$$\frac{\delta \Pi}{\delta L} = P * \frac{\delta f_{K=\bar{K}}}{\delta L} - w = 0$$

$$\Leftrightarrow P * \frac{\delta f_{K=\bar{K}}}{\delta L} \equiv VMP_L = w.$$

Toward labor demand

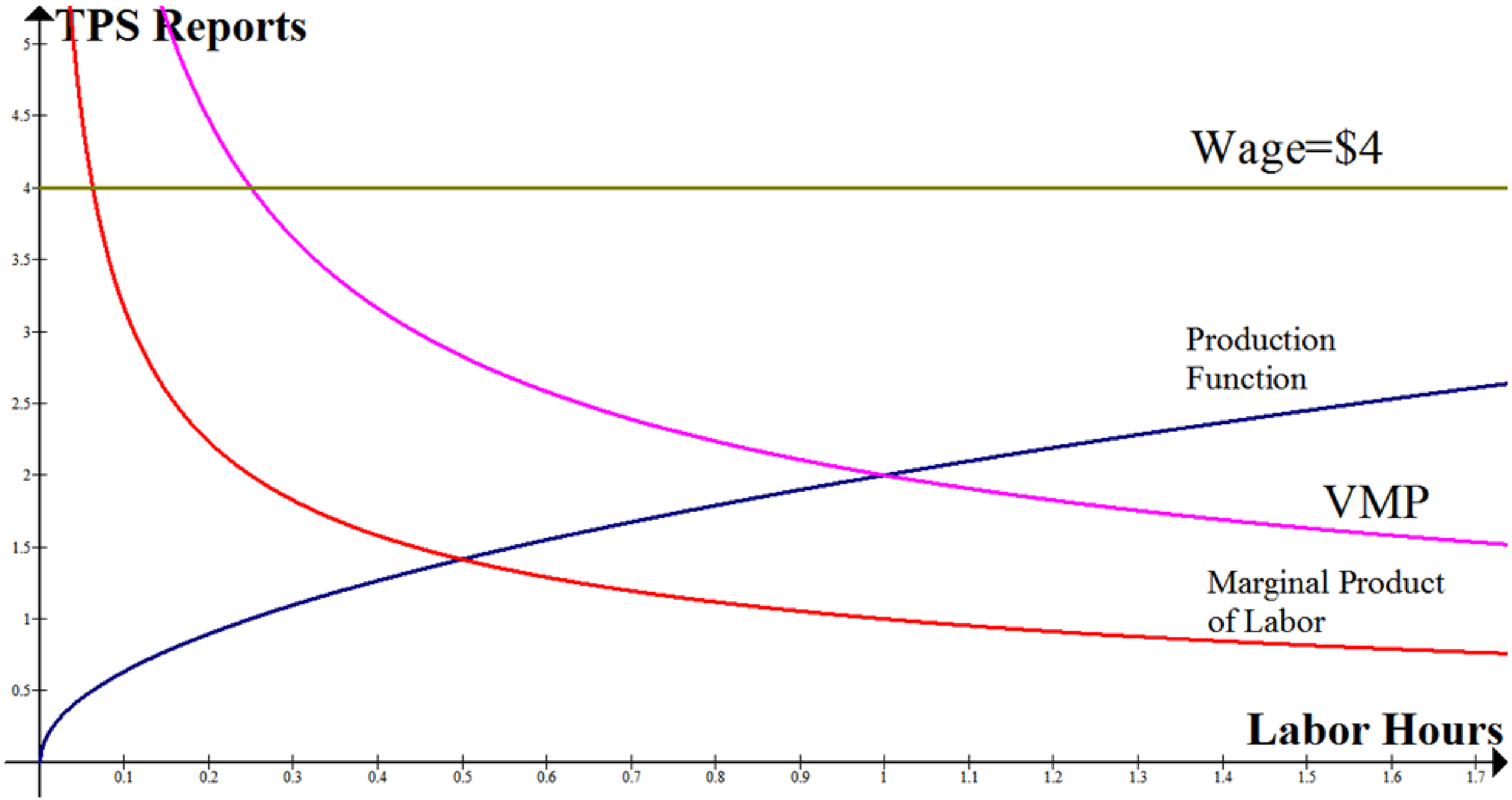
What does this relationship look like? Specifically one would like to know how much labor the firm will hire for a given wage. The wage is a non-negative concept, so we are only interested in output for which VMP_L is non-negative. One can imagine solving VMP_L for *L* to identify the profit-maximizing employment level, but how would that level change if *w* were to change?

- This is where the assumptions about $f(\cdot)$ come into play. How VMP_L changes as *L* changes depends on the production function. If labor has diminishing marginal product,

$$\frac{\partial MP_L}{\partial L} < 0, \text{and } \frac{\partial VMP_L}{\partial L} < 0, \text{because } \frac{\partial VMP_L}{\partial L} = P * \frac{\partial MP_L}{\partial L}.$$

- Then there is a downward-sloping (negative) relationship between *L* and VMP_L. Combining this assumption with perfect competition in the labor market (wage is a horizontal line) results in a graphical depiction like the one below.

VMP and labor input



SR labor demand

- Identifying the intersection of wage and VMP_L is the firm's short run labor demand. If the wage were to rise or fall, because of the dynamics of the overall labor market, this firm would move up (wage increase) or down (wage decrease) its VMP curve. So the VMP curve is the firm's short-run labor demand function:

$$L_{SR}^{*}(w) \equiv \text{firm's short−run labor demand.}$$

Labor demand and elasticity

Slope of the firm's labor demand and the Law of Demand. In order for labor demand to obey the (negative) relationship we expect, the slope should be negative.

- The slope of the labor demand is negative by the assumption of diminishing marginal product, i.e., $f''(L) < 0$.
- When the wage goes up, a firm demands fewer hours of labor.
- In order to find the actual quantity of labor the firm will employ, it just needs to know the wage (and output price). When this is plugged in to the function, the functional value of L tells you how much labor it employs.

Demand Elasticity. The responsiveness of labor demanded to wage changes is usually measured as a ratio of percentage changes:

$$Short\ Run\ Demand\ Elasticity \text{ is } \delta_{SR} = \frac{\%\Delta\ in\ L}{\%\Delta\ in\ w} = \frac{\Delta L_{SR}^*}{\Delta w}\frac{w}{L_{SR}^*}.$$