

# Labor Mobility: Second Lecture

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LABOR ECONOMICS (ECON 385)

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# Introduction

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- Whether you're talking about an employed worker who is searching for a new job ("on the job" search) or an unemployed worker is searching for a job, the searcher has to choose between a known offer (current wage or the monetary value of leisure) and the probabilistic offer they expect when they search.
- The following basic model shows how search can be modeled for unemployed workers.

# Offer distribution

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- An unemployed individual faces a known distribution of offers. However he does not know—nor can he control—what kind of offer is drawn from the distribution when he searches. Formally consider the distribution characterized by:

$$f(w) \text{ and } F(w)$$

- These are the probability,  $f(w)$ , of getting an offered wage equal to  $w$  and the probability,  $F(w)$ , of getting an offered wage less than or equal to  $w$ .

# Microeconomics of search

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If the searcher accepts the offer in time period  $t$  (“today”), he receives the offered wage perpetually.

- This is obviously a simplification that he does not search anymore on the job. It is inconsequential with respect to the main implications of the search model.
- If the searcher rejects the offer today, he receives some “consolation” level of utility,  $c$ , and then he repeats the search next period (“tomorrow”). Searching tomorrow has the expected value (in present discounted terms) of:

$$\beta * E(V_{t+1})$$

where  $\beta$  is the rate of temporal preference (“patience”) and  $0 \leq \beta \leq 1$ .

- This problem creates a trade-off for the individual: the longer he searches, the more likely he can “hold out” for a better wage offer, but the longer he searches, the more time he spends unemployed and loses wages.
- The searcher will reject offers that are “too low” and accept them if the wage is sufficiently high. So there will be a cut-off of offers he accepts from those he rejects.

# The asking wage

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- Call the cut-off the asking wage and use notation for it,  $\bar{w}$ .
  - The probability that a searcher receives an offer less than or equal to  $\bar{w}$  is  $F(\bar{w})$ ; this is the probability the searcher will reject his wage offer this period.
  - He accepts an offer with probability,  $1 - F(\bar{w})$ . The searcher chooses  $\bar{w}$  by comparing two values ( $V_t$ ) of the present discounted expected utility—the probability-weighted average of all possible outcomes from the search:

$$V_t = \begin{cases} \sum_{\tau=t}^T \beta^{\tau} * w, & \&w > \bar{w} \\ c + \beta * E(V_{t+1}), & \&w \leq \bar{w} \end{cases}$$

# Microeconomics of search, continued

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- If the searcher accepts an offer, the present discounted value of his future wages is a geometric series that converges to the finite value,

$$\sum_{\tau=t=0}^T \beta^{\tau} * w \equiv s \rightarrow s\beta = \sum_{\tau=t+1=1}^T \beta^{\tau} * w$$
$$s - s\beta = s(1 - \beta) = \beta^0 w - \beta^T w$$

- If  $T$  is large, the second term goes to zero, and:

$$s = \frac{w}{1 - \beta}$$

- This is the present discounted sum of future wages if the searcher accepts a wage offer of,  $w$ , today.  
So,

$$V_t = \begin{cases} \frac{w}{1 - \beta}, & \&w > \bar{w} \\ c + \beta * E(V_{t+1}), & \&w \leq \bar{w} \end{cases}$$

# Microeconomics of search, concluded

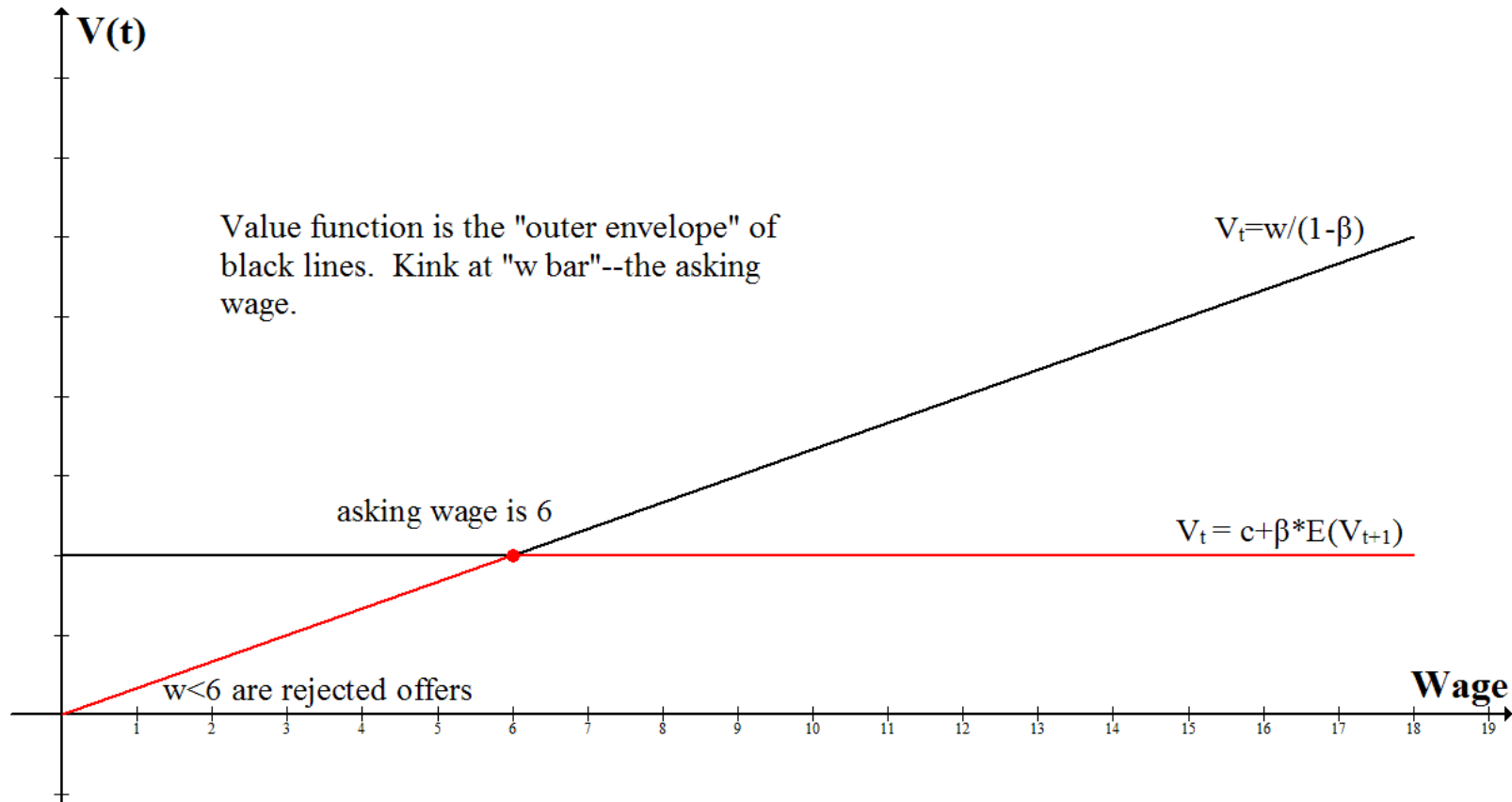
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- Pinning down the asking wage simply means defining the wage level where the searcher is indifferent to accepting and rejecting the offer, i.e., where  $V_t(\text{accept}) = V_t(\text{reject})$ . At  $\bar{w}$ ,

$$\frac{\bar{w}}{1 - \beta} = c + \beta * E(V_{t+1}) \Leftrightarrow \bar{w} = (1 - \beta)c + \beta(1 - \beta)E(V_{t+1})$$

- This is easy to illustrate graphically. The value function of accepted offers (wage) is an upward sloping line with a slope,  $1/(1-\beta)$ , and the value function of rejected offers is a constant. The searcher will always choose the line that is higher for a given  $w$ .
- In the illustration on the next slide, the asking wage is \$6.

# The value function, graphically





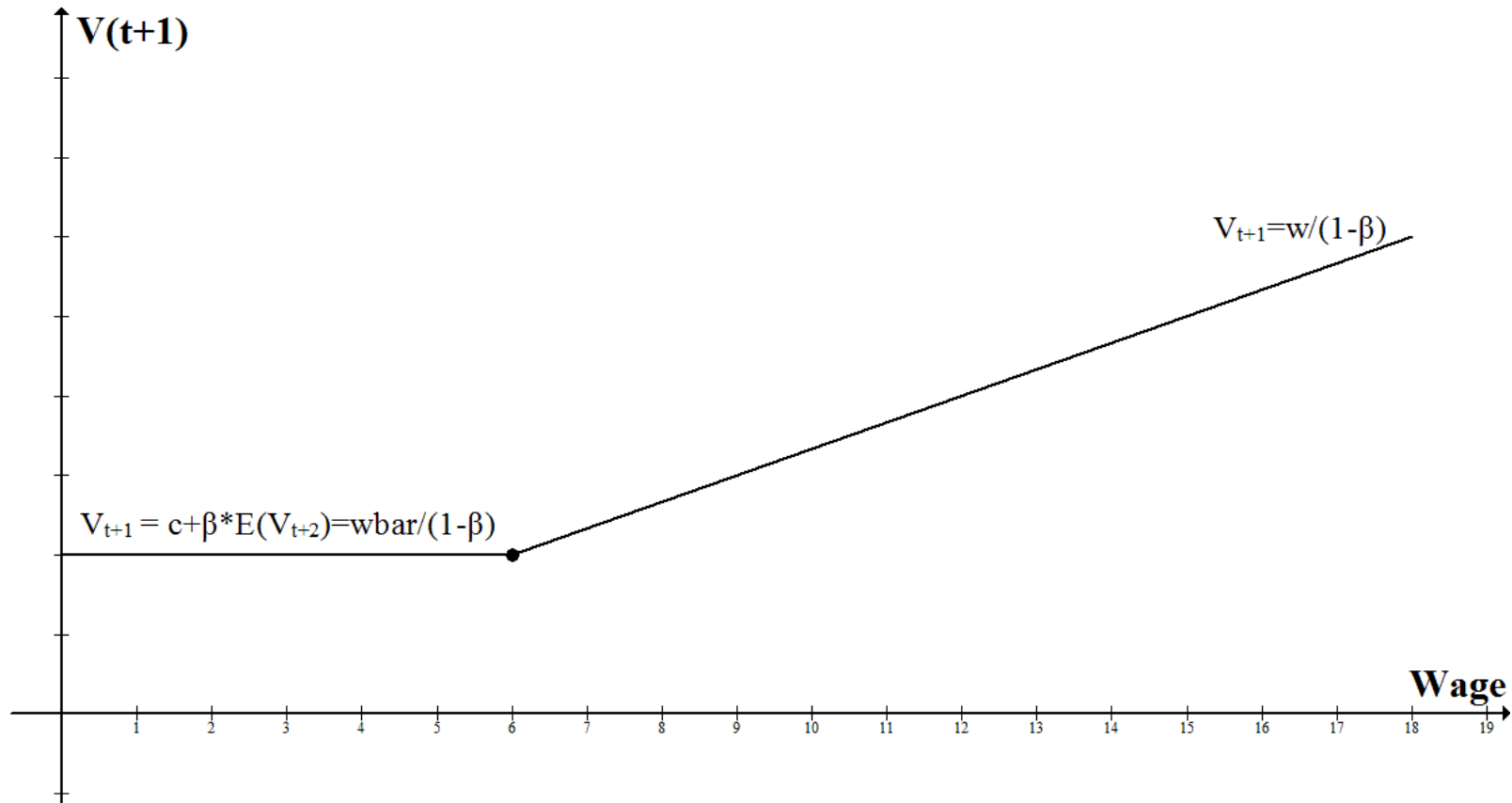
# Observations about the asking wage

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- The asking wage depends (positively) on both  $c$  and  $\beta$ . This means that:
  - Making unemployment relatively more attractive ( $\Delta c > 0$ ) will increase the asking wage and make individuals search longer on average.
  - More patient individuals will search longer.
- Formally the effect of raising  $c$  can be [shown](#) by modeling the expected value of search explicitly.

# The value function optimum

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# Raising $c$ : raising the generosity of unemployment insurance

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- The policy application is obviously to unemployment insurance. Raising the UI benefits is tantamount to increasing  $c$ .
- As we have shown, this raises the asking wage by making unemployed individuals more willing to search—but it does not raise the asking wage 1 for 1.
  - Additional UI benefits raise the asking wage by only a fraction of the  $\Delta c$ .

# Other extensions of the search model

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- The effect of raising the mean of the offer distribution.
  - This has the effect of raising the asking wage since the searcher expects a higher wage.
- The effect of raising the variance of the offer distribution.
  - This has the (perhaps surprising) effect of raising the asking wage because searchers always have the option of rejecting the really low wage offers. Meanwhile the offers on the high end are even better offers, and searchers will be more inclined to “hold out” for a really good offer.
- Search effort can be included, whereby the number of offers per period is increased at a price to the searcher. He then faces the additional decision of how hard to search.
- The effects of other searchers' efforts can be included, incorporating the willingness of other searchers to take offers that I would also take.
  - This extension is closely related to the work that won Peter Diamond, Dale Mortensen, and Christopher Pissarides the Nobel Prize in 2010.
  - One implication is that if searchers are too quick to take offers, they will deprive other searchers of that offer even if the other guy would have been a better match for the job. This suggests that policies to encourage longer search and longer spells of unemployment (!) can be beneficial to the quality of matches made.

# Optional material: the effect of raising the “consolation” utility, $c$

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- Next period the search has an expected value equal to the probability-weighted average of all search outcomes:

$$E(V_{t+1}) = \int_0^{\bar{w}} V_{t+1}(w) * f(w) dw + \int_{\bar{w}}^{\infty} V_{t+1}(w) * f(w) dw$$

where the integrals mean: “the sum of the products of each wage and the probability of receiving that wage.” Note that for wages less than or equal to the asking wage, the value function is constant. So the first integral simplifies to:

$$\int_0^{\bar{w}} V_{t+1}(w) * f(w) dw = \int_0^{\bar{w}} \frac{\bar{w}}{1 - \beta} * f(w) dw = \frac{\bar{w}}{1 - \beta} * F(\bar{w})$$

and the value function for accepted offers is  $w/(1-\beta)$ . This can be substituted in the second integral.

# Raising $c$ , continued

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$$\int_{\bar{w}}^{\infty} V_{t+1}(w) * f(w)dw = \int_{\bar{w}}^{\infty} \frac{w}{1-\beta} * f(w)dw = \frac{1}{1-\beta} \int_{\bar{w}}^{\infty} w * f(w)dw$$

These can be substituted into the asking wage function:

$$\begin{aligned}\bar{w} &= (1-\beta)c + \beta(1-\beta) \left[ \int_0^{\bar{w}} V_{t+1}(w) * f(w)dw + \int_{\bar{w}}^{\infty} V_{t+1}(w) * f(w)dw \right] \\ \Leftrightarrow \bar{w} &= (1-\beta)c + \beta(1-\beta) \left[ \frac{\bar{w}}{1-\beta} * F(\bar{w}) + \frac{1}{1-\beta} \int_{\bar{w}}^{\infty} w * f(w)dw \right] \\ (\text{simplifying}) \bar{w} &= (1-\beta)c + \beta \left[ \bar{w} * F(\bar{w}) + \int_{\bar{w}}^{\infty} w * f(w)dw \right]\end{aligned}$$

# Raising $c$ , concluded

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- Differentiating this with respect to  $c$  gives us:

$$\frac{\partial \bar{w}}{\partial c} = (1 - \beta) + \beta \left[ \frac{\partial \bar{w}}{\partial c} (F(\bar{w}) + \bar{w} * f(\bar{w})) - \frac{\partial \bar{w}}{\partial c} \bar{w} * f(\bar{w}) \right] = (1 - \beta) + \beta \left[ \frac{\partial \bar{w}}{\partial c} F(\bar{w}) \right]$$

Solving for the partial derivative gives the effect of raising  $c$  on the asking wage:

$$\frac{\partial \bar{w}}{\partial c} (1 - \beta F(\bar{w})) = (1 - \beta) \Leftrightarrow \frac{\partial \bar{w}}{\partial c} = \frac{(1 - \beta)}{(1 - \beta F(\bar{w}))}$$

- It can be shown that this effect is positive and less than 1.

[Back.](#)