Separable Equations
-Isolate variables on either side of the equation and integrate to solve.

Integrating Factor Method (for linear equations: $a_0(t)y'(t)+a_1(t)y(t)=b(t)$)
1) Normalization: divide equation by a_0 so that there is a factor of 1 on $y'(t)$
the function should now look like: $y'(t)+\frac{a_1(t)}{a_0(t)}y(t)=g(t)=y'(t)+p(t)y(t)$
2) Form new function $\mu = e^{\int p(t)dt}$, which is the integrating factor.
3) Multiply the normalized equation by the integrating factor and solve.

Homogeneous Equations
-Equations are homogeneous of the powers of each term are the same
-Can be solved using substition.
-To solve, introduce new function $v(x)=y/x$ or $y=v(x)\cdot x$ and solve for v, then use use that to solve for y.

Modeling With First Order Diff. Eq's
-Chemicals in water tanks, gravity and velocity... just do practice problems from 2.3

$$\frac{dQ}{dt}=rate_{in}-rate_{out}=flow_{in}\cdot concentration_{in}-flow_{out}\cdot concentration_{out}$$

Autonomous Equations & Population Dynamics
-Equilibrium solutions mean that y(t)=c, thus y'(t)=0, plug in these values and solve
-Any 2 solutions of a First-Order D.E. cannot intersect each other, useful for finding whether a solution of a D.E. is stable, unstable, or semi-stable (convergence or divergence for different starting values for the equation

Exact Equations
-A diff. eq. in the form $M(x,y)+N(x,y)\frac{dy}{dx}=0$ is exact if $\frac{\partial}{\partial y}M(x,y)=\frac{\partial}{\partial x}N(x,y)$

-To solve, integrate one M or N, then derive in terms of the other variable. Then set that equal to the other term (M or N) and solve.
-Often, an equation must be rearranged must be rearranged to get in "exact" form

Existence & Uniqueness Theorem
-If we cannot solve a D.E. analytically,
1) use a numerical method
2) guess
3) use mathematical techniques to study properties of the equation

Existence & Uniqueness Theorem (continued)

-For linear first-order diff. eq's $y'(t)+p(t)y(t)=g(t)$ with an initial condition y[t] = C, if p[t] and g[t] are continuous functions on given domains, then the initial value problem has exactly 1 solution on that domain.

-Find the largest interval (a,b) on which one solution is guaranteed to exist for an eqn.

Linear Second Order Differential Equations: $ay''+by'+cy=0$

-Superposition theorem: if y_1 and y_2 are solutions, then so is $c_1y_1+c_2y_2$

-Characteristic equation for linear second order: $ar^2+br+c=0$, solve using quadratic equation/factoring to obtain r values such that $y=e^{rt}$ is a solution

-If $r_1 \neq r_2$ and r values are real, then $y_1=e^{r_1t}$, $y_2=e^{r_2t}$, $y=c_1e^{r_1t}+c_2e^{r_2t}$

-If $r_1 \neq r_2$ and r values are complex, then $r_{1,2}=\alpha \pm \beta i$, $y=c_1e^{\alpha t}\cos(\beta t)+c_2e^{\alpha t}\sin(\beta t)$

-If $r_1=r_2$, then $y_1=e^{r_1t}$, $y_2=ty_1=te^{r_1t}$, $y=c_1e^{r_1t}+c_2te^{r_1t}$ -reduction of order method

Linear Independence

-The Wronskian is defined as $W(y_1,y_2)=y_1\cdot y_2'-y_1'\cdot y_2$

-Linear dependence occurs when the Wronskian is zero or $y_1=cy_2$

-Otherwise, the two equations are linearly independent

Reduction of Order Method

-When one solution of a Diff. Eq. y_1 is known but another is needed, let $v=\dfrac{y}{y_1}$, substitute $y=vy_1$ and solve for v, then solve for y

Calculus Review Stuff

$$\int\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x+c \qquad \int xe^x\,dx=e^x(x-1) \qquad \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\int\frac{dx}{1+x^2}=\tan^{-1}x+c \qquad \int xe^{ax}\,dx=e^{ax}(ax-1)/a^2$$

Review HW from 1.2 - 1.3, 2.2 - 2.6, 3.1 - 3.4