Non-Homogeneous Equations

Method of Undetermined Coefficients:

If the suggested form for y_p is similar to another solution, multiply by t until it works

Variation of Parameters

for $ay'' + by' + cy = 0$, $y_p = \frac{1}{a}\left(-y_1 \int \frac{y_2 \cdot g}{W(y_1, y_2)} dt + y_2 \int \frac{y_1 \cdot g}{W(y_1, y_2)} dt \right)$

where W is the Wronskian: $W(y_1, y_2) = y_1 \cdot y_2' - y_1' \cdot y_2$

Be sure to normalize the equation such that a = 1.

Higher Order Differential Equations

-Solve using characteristic equations (factoring/complete the square may be needed)

-Make sure repeated roots are written out explicitly.

-Guessing roots: guess p/q where q is the leading (first) coefficient.

-Non-homogeneous situations are exactly the same (use undetermined coefficients)

-Wronskian of more than 2 functions (0 if dependent, nonzero if independent)

Laplace Transform: $L\{f(t)\}(s) = \int_0^\infty e^{-st} f(t)\,dt$

-Basic properties are the same as for integrals.

-Can apply to both sides of an IVP (initial value problem) to simplify solving. Then rearrange in terms of $L\{y\}$ and find the inverse Laplaces to find y

Unit Step Functions: $u_c(t)=\begin{cases}0 & t<c\\ 1 & t>c\end{cases}$

-Know how to break up discontinuous forcing functions into unit step functions + a constant.

$$L\{u_c f(t-c)\}(s)=e^{-cs}L\{f(t)\}(s) \qquad L\{u_c f(t)\}(s)=e^{-cs}L\{f(t+c)\}(s)$$

Impulse Forcing Functions: $\delta(t)=\begin{cases}0 & t\neq 0\\ \infty & t=0\end{cases}$

-Dirac Delta function (instantaneous): $\delta(t-c)=u_c'(t)$ $\quad \int_{-\infty}^{\infty}\delta(t-c)f(t)dt=f(c)$

$$L\{\delta(t-c)\}(s)=e^{-cs} \qquad L\{\delta(t)\}(s)=1$$

Some Laplace Transforms

$$L\{0\}(s)=0 \qquad L\{t\}(s)=\frac{1}{s^2} \qquad L\{t^2\}(s)=\frac{2}{s^3} \qquad L\{t^n\}(s)=\frac{n}{s^{n+1}}$$

$$L\{e^{at}\}(s)=\frac{1}{s-a} \qquad L\{e^{ct}f(t)\}(s)=L\{f(t)\}(s-c))$$

$$L\{\sin(at)\}(s)=\frac{a}{s^2-a^2} \qquad L\{\cos(at)\}(s)=\frac{s}{s^2+a^2}$$

$$L\{y'\}(s)=sL\{y\}-y(0) \qquad L\{y''\}(s)=s^2L\{y\}-sy(0)-y'(0)$$

Review HW from 3.5 - 3.8, 4.1 - 4.3, 6.1 - 6.4