EXAM 1

12 multiple-choice problems
(5 choices each)

topics covered:

- Domain
- Graph translation / reflection / stretch
- Log and exponentials
- Inverse
- Secant, tangent lines
- Limits
- Continuity
- Asymptotes

Graph of f and f'

6 from my review problems

$$\lim_{x\to\infty} \frac{3x^1 - 2}{9x^1 + 7}$$

same degrees : ratio of leading coefficients (# in front of x to highest)

divide top and bottom by x^n where n is the highest power in denominator

$$= \lim_{x\to\infty} \frac{\frac{3x-2}{x}}{\frac{9x+7}{x}} = \lim_{x\to\infty} \frac{3 - \frac{2}{x}}{9 + \frac{7}{x}} = \frac{3}{9} = \frac{1}{3}$$

limit at infinity

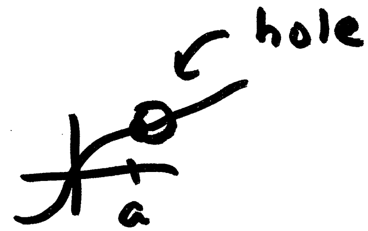
horizontal asymptote

top has lower degree : 0

top has higher degree : $\pm\infty$

#4 (continuity)

If $\lim_{x \to a} f(x) = L$ which of the following MUST be true?

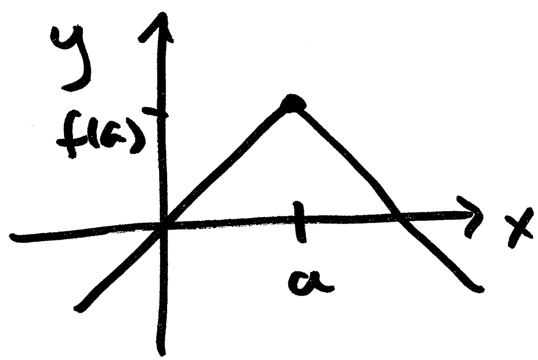


(~~i~~) f is defined at a

(~~ii~~) $f(a) = L$

(~~iii~~) f is continuous at a

(~~iv~~) f is differentiable at a

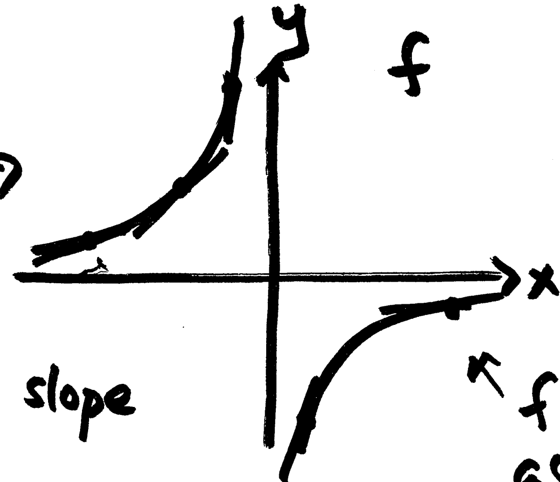


NOT differentiable

→ discontinuity (hole, jump, vert. asymptote)

→ sharp corner

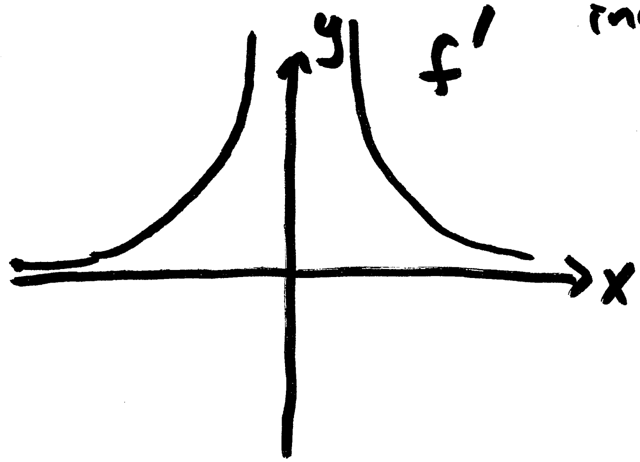
find where $\boxed{f' = 0}$
(if any)



tangent line slope
increases as
x increases

f' decreases
as x
increases

geometric meaning?
slope of tangent line
is zero
(flat)



Solve $\ln(x+1) - \ln x = 1$

$\ln(ab) = \ln a + \ln b$

$\ln \frac{a}{b} = \ln a - \ln b$

$\log_a x = y$

$\leftrightarrow x = a^y$

combine logs if possible

$$\ln\left(\frac{x+1}{x}\right) = 1$$

$\log_e$

$$\frac{x+1}{x} = e^1$$

$$x+1 = ex$$

$$x - ex = -1$$

$$x(1-e) = -1$$

$$x = \frac{-1}{1-e} = \frac{1}{e-1}$$

If $f(x) = 2x + \ln x$ find $\boxed{f^{-1}(x)}$ $f^{-1}(2)$

$y = 2x + \ln x$

$\ln e^{x} = x$

$y = \ln e^{2x} + \ln x$

$y = \ln(x e^{2x})$

$x = \ln(y e^{2y})$ $y = f^{-1}(x)$

$2 = \ln(f^{-1}(2)\, e^{2f^{-1}(2)})$

$e^{2} = f^{-1}(2)\, e^{2f^{-1}(2)}$

$f^{-1}(2) = 1$

$$p(t) = 6 + 8t - e^{2-t} \qquad \text{avg vel. } [0,2]$$

22 - 1

$$\frac{p(2) - p(0)}{2 - 0} = \frac{(6 + 16 - e^{0}) - (6 + 0 - e^{2})}{2}$$

$$= \frac{21 - 6 + e^{2}}{2} = \frac{15 + e^{2}}{2}$$

avg. velocity = secant line slope

inst. velocity = tangent " "

domain of $\dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt{x-1}}$

no division by zero

no negatives under even roots

no log of negative

first term: $x \neq 0$, $x > 0$

2nd term: $x \neq 1$, $x > 1$

BOTH must be satified

so $x > 1$ $\rightarrow$ $(1, \infty)$