EXAM 2

12 problems, 8 points each, 4 points for your name.

rules of differentiation (chain, product, quotient, exponential, log, trig, hyperbolic functions, inverse trig, etc)

implicit diff

related rates

special limit $\lim_{x \to 0} \frac{\sin x}{x} = 1$

definition of $\sinh(x)$ and $\cosh(x)$

exponential growth / decay

S12 # 3

If $f(x) = (\tan^2 x - 4)^4 = ((\tan x)^2 - 4)^4$

find $f'(\frac{\pi}{3})$ ↑ 2

$$f'(x) = 4((\tan x)^2 - 4)^3 \cdot \frac{d}{dx}((\tan x)^2 - 4)$$

$$= 4((\tan x)^2 - 4)^3 \cdot 2(\tan x) \cdot \sec^2 x$$

$$f'(\tfrac{\pi}{3}) = 8\left((\tan\tfrac{\pi}{3})^2 - 4\right)^3 (\tan\tfrac{\pi}{3}) \cdot \frac{1}{(\cos(\frac{\pi}{3}))^2}$$

$\tan\frac{\pi}{3} = \frac{\sin\pi/3}{\cos\pi/3}$

$$= 8\left(\frac{(\sqrt{3}/2)}{1/2} - 4\right)^3 \cdot \sqrt{3} \cdot \frac{1}{(1/2)^2} = -8 \cdot \sqrt{3} \cdot 4$$

$\frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$$= -32\sqrt{3}$$

S12 #6

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta + \tan\theta}$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

$$= \lim_{\theta \to 0} \frac{\frac{\sin\theta}{\theta}}{\frac{\theta + \tan\theta}{\theta}} = \lim_{\theta \to 0} \frac{\frac{\sin\theta}{\theta}}{1 + \frac{\tan\theta}{\theta}}$$

$$= \lim_{\theta \to 0} \frac{\frac{\sin\theta}{\theta}}{1 + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\theta}} = \lim_{\theta \to 0} \frac{\frac{\sin\theta}{\theta}}{1 + \frac{\sin\theta}{\theta} \cdot \frac{1}{\cos\theta}}$$

$\frac{\sin\theta}{\theta} \to 1$, $\frac{\sin\theta}{\theta} \to 1$, $\frac{1}{\cos\theta} \to 1$

$$\lim_{x \to 0} \frac{\sin(2x)}{(2x)} = 1$$

$$= \boxed{\frac{1}{2}}$$

$$\lim_{x \to 0} \frac{\sin^2 x}{x^2} = \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2 = 1$$

S12 #5

$$f(x) = \sqrt{x + \sqrt{x}} \qquad \text{find } f'(1)$$

$$f(x) = \left(x + x^{1/2}\right)^{1/2}$$

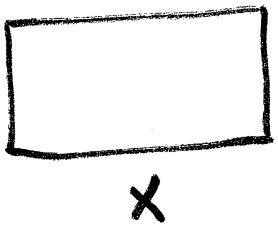
$$f'(x) = \frac{1}{2}\left(x + x^{1/2}\right)^{-1/2}\left(1 + \frac{1}{2}x^{-1/2}\right)$$

$$f'(1) = \frac{1}{2}(2)^{-1/2}\left(\frac{3}{2}\right) = \frac{3}{4\sqrt{2}}$$

$$= \frac{3}{2^2 2^{1/2}} = \frac{3}{2^{5/2}}$$

S10 #7

Base of rectangle increases at 2 cm/s and height increases at 3 cm/s, what is the rate of increase of area when base is 5 cm and height is 7 cm.



$A = xy$ derivative FIRST numbers later

$\frac{d}{dt} A = \frac{d}{dt}(xy)$ chain rule!

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} = 5(3) + 7(2) = 29$$

513 #15

Half-life 7 days

How long to lose two-thirds?

$P(t) = P(0)\, e^{kt}$ find k

7 days later

$\frac{1}{2} P(0) = P(0)\, e^{7k}$ $P(0)$ cancels out

$\frac{1}{2} = e^{7k}$

$\ln \frac{1}{2} = 7k$ $k = \frac{1}{7} \ln \frac{1}{2}$

$P(t) = P(0)\, e^{\frac{1}{7} \ln \frac{1}{2} t}$

$$P(t) = P(0)\, e^{\frac{1}{7}\ln\frac{1}{2}t}$$

$$\frac{1}{3}P(0) = P(0)\, e^{\frac{1}{7}\ln\frac{1}{2}t}$$

$$\frac{1}{3} = e^{\frac{1}{7}\ln\frac{1}{2}t}$$

$$\ln\frac{1}{3} = \frac{1}{7}\ln\frac{1}{2}t$$

$$\ln\frac{1}{3} = \ln 3^{-1} = -\ln 3$$

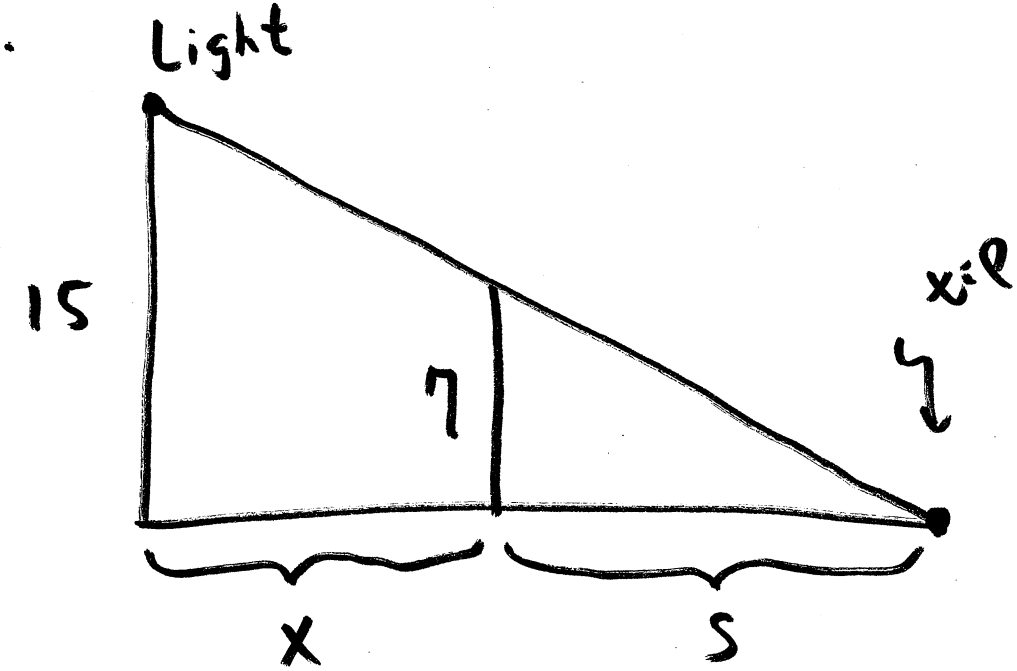
$$7 \cdot \frac{\ln\frac{1}{3}}{7 \cdot \frac{1}{7}\ln\frac{1}{2}} = t$$

$$\frac{7 \cdot -\ln 3}{-\ln 2} = t = \boxed{\frac{7\ln 3}{\ln 2}}$$

Man's shadow problem

Light on 15-ft pole. Man 7ft-ft walks away from light at 3 ft/s.

How fast is the tip of his shadow moving when he is 10-ft away from the light.



known: $\frac{dx}{dt} = 3$

find: $\frac{d}{dt}(x+s) = \frac{dx}{dt} + \frac{ds}{dt}$ → $\frac{ds}{dt} = \frac{d(x+s)}{dt} - \frac{dx}{dt}$

$\frac{ds}{dt}$ → how fast tip of shadow is moving from the man's point of view

Similar triangles $\frac{7}{s} = \frac{15}{x+s}$

$7(x+s) = 15s$

$7x + 7s = 15s$

$7x = 8s$

$x = \frac{8}{7}s$ → $\frac{dx}{dt} = \frac{8}{7}\frac{ds}{dt}$

$$\frac{dx}{dt} = \frac{8}{7}\frac{ds}{dt}$$

$$3 = \frac{8}{7}\frac{ds}{dt}$$

$$\frac{ds}{dt} = \frac{21}{8}$$

$$\frac{d}{dt}(x+s) = \frac{dx}{dt} + \frac{ds}{dt} = 3 + \frac{21}{8} = \frac{45}{8}$$

Sec #8

$$\tan(xy) = e^x + e^y \qquad \text{find } \frac{dy}{dx} \qquad y = y(x)$$

$$\sec^2(xy) \cdot \left(x \cdot \frac{dy}{dx} + y\right) = e^x + e^y \cdot \frac{dy}{dx}$$

$$x \cdot \sec^2(xy) \frac{dy}{dx} + y \cdot \sec^2(xy) = e^x + e^y \cdot \frac{dy}{dx}$$

$$\left(x \cdot \sec^2(xy) - e^y\right) \frac{dy}{dx} = e^x - y \sec^2(xy)$$

$$\frac{dy}{dx} = \frac{e^x - y \sec^2(xy)}{x \sec^2(xy) - e^y}$$