11. $f(x) = 3x^2 + 6x - 10 \qquad -2 \leq x \leq 2$

find absolute max/min

find critical numbers : $f' = 0$ or f' DNE

$f'(x) = 6x + 6 = 0 \quad \rightarrow \quad x = -1$

if outside given interval → discard.

compare :

$f(-2) = -10$

$f(-1) = -13$ abs. min

$f(2) = 14$ abs. max

12. $f(3)=5 \qquad f'(3)=-2$

use linear approx. to estimate $f(3.02)$

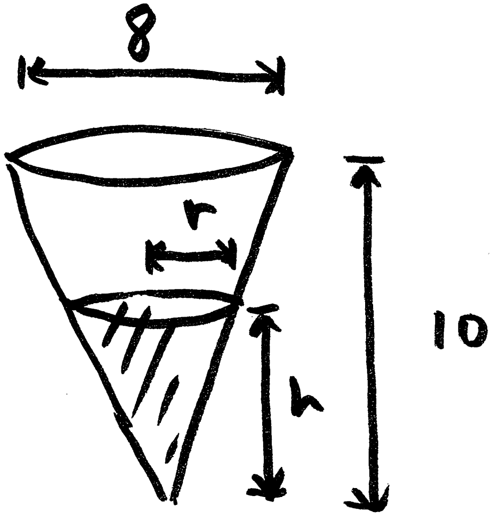
$$L(x) = f(a) + f'(a)(x-a)$$

here, $a = 3 \qquad x = 3.02$

$$f(3.02) \approx 5 + (-2)(3.02-3)$$

$$\approx 5 - 0.04 \approx 4.96$$

13.



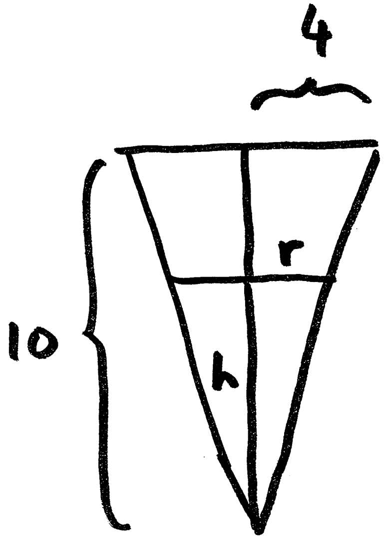
water is withdrawn at 5 ft^3/min → negative rate

How fast is water level falling when depth is 5? → want $\frac{dh}{dt}$

$$V = \frac{1}{3}\pi r^2 h \qquad\qquad \frac{dV}{dt} = -5$$

deriv. on BOTH sides

but has two variables on right side



similar Δs : $\frac{4}{10} = \frac{r}{h}$

so $r = \frac{2}{5}h$ or

$h = \frac{5}{2}r$

$V = \frac{1}{3}\pi r^2 h$ want $\frac{dh}{dt}$, so eliminate r

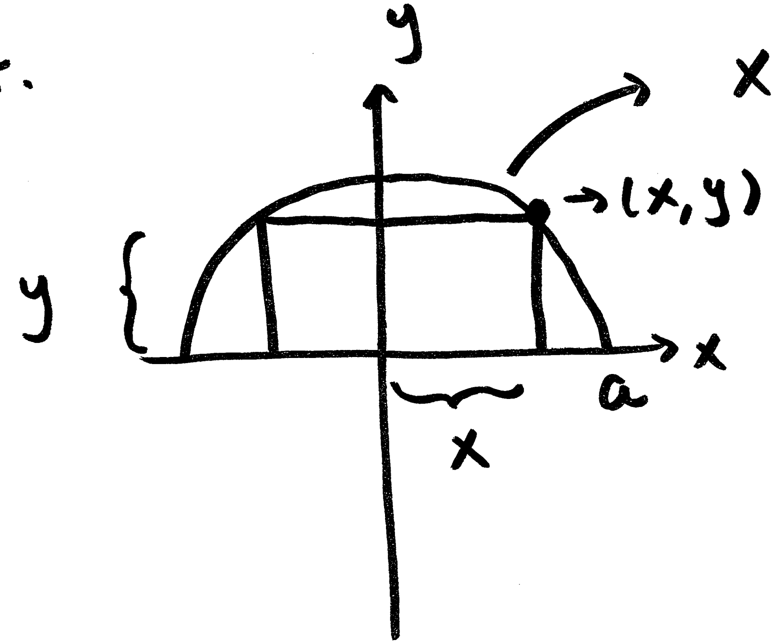
$V = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h$

$V = \frac{1}{3}\pi \cdot \frac{4}{25}h^2 \cdot h = \frac{4}{75}\pi h^3$

$\frac{dv}{dt} = \frac{4}{25}\pi h^2 \frac{dh}{dt}$ $\frac{dh}{dt} = \frac{\frac{dv}{dt}}{\frac{4}{25}\pi h^2} = \frac{-5}{\frac{4}{25}\pi(5)^2}$

$= -\frac{5}{4\pi}$

14.



$x^2+y^2=a^2$

area of ~~maxi~~ largest rectangle.

$A = 2xy$ two variables

from $x^2+y^2=a^2$

$$y=\sqrt{a^2-x^2}$$

$$A(x) = 2x\,(a^2-x^2)^{1/2} \qquad 0 \le x \le a$$

(like #11)

$$A(x) = (4x^2)^{1/2}(a^2-x^2)^{1/2} = \left[(4x^2)(a^2-x^2)\right]^{1/2}$$

$$A = \left(4a^2x^2 - 4x^4\right)^{1/2}$$

$$A' = \frac{1}{2}\left(4a^2x^2 - 4x^4\right)^{-1/2}\left(8a^2x - 16x^3\right) = 0$$

$$\frac{8a^2x - 16x^3}{2\left(4a^2x^2 - 4x^4\right)^{1/2}} = 0$$

$$8x\left(a^2 - 2x^2\right) = 0$$

$$x = 0 \qquad x = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$$

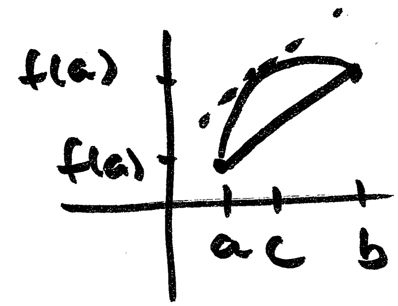
A is max at $x = \frac{a}{\sqrt{2}}$

15. f differentiable for all x

$f(2) = 4 \qquad f(7) = 10$

Mean Value Theorem states there is a number c such that:

Mean Value Theorem



f defined on $[a, b]$

f diff-able on (a, b)

then there is a c in (a, b) such that
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

here $f(2)=4, \quad f(7)=10$

so $a=2 \quad b=7$

$$f'(c)=\frac{10-4}{7-2}=\frac{6}{5} \qquad 2<c<7$$

16. radioactive substance decays from 18 gms to 2 gms in 2 days, how long for 12 gms to decay to 4 gms?

$$y = y(0)\, e^{kt}$$

$$2 = 18\, e^{2k}$$

$$\frac{2}{18} = e^{2k}$$

$$\ln\left(\frac{1}{9}\right) = 2k$$

$$k = \frac{1}{2}\ln\left(\frac{1}{9}\right)$$

$$k = \ln\left(\frac{1}{9}\right)^{1/2} = \ln\left(\frac{1}{3}\right)$$

$$y = y(0)\, e^{\ln\left(\frac{1}{3}\right)t}$$

$$\left[e^{\ln\left(\frac{1}{3}\right)}\right]^t$$

$$\left[\frac{1}{3}\right]^t$$

$$= y(0) \cdot \left(\frac{1}{3}\right)^t$$

$$4 = 12 \cdot \left(\frac{1}{3}\right)^t \rightarrow \frac{4}{12} = \left(\frac{1}{3}\right)^t \rightarrow \boxed{t = 1}$$

17. $g(x) = 4x^3 - 3x^4$, which is/are true?

(1) g is decreasing for $x > 1$

(2) g has a relative extreme value at $(0,0)$

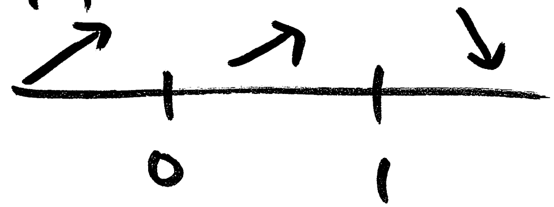
(3) g is concave up for all $x < 0$

from First Deriv. Test

$$g'(x) = 12x^2 - 12x^3 = 0$$

$$12x^2(1-x) = 0 \qquad x=0,\ x=1$$

g' $++$ 0 $++$ 0 $--$



(1) is true

(2) is false

$$g''(x) = 24x - 36x^2 = 0$$

$$12x(2 - 3x) = 0$$

$$x = 0, \quad x = \frac{2}{3}$$

g'' $- \; -$ $\quad 0 \qquad 0$

$\cap$

$0 \qquad 2/3$

(3) is false

18.

$$f(x) = \frac{2}{\sqrt{1+x^2}} \qquad \text{increasing?}$$

$$= 2\left(1+x^2\right)^{-1/2}$$

$$f'(x) = -\left(1+x^2\right)^{-3/2}(2x)$$

$$= \frac{-2x}{\left(1+x^2\right)^{3/2}}$$

$f' = 0 \rightarrow x = 0$

f' DNE $\rightarrow$ never

f' $+ \; +$ $\quad 0 \quad$ $- \; -$

0

inc: $(-\infty, 0)$

$x < 0$