19.

$$f'(x) = (x-1)^2(x+2)(x-5)$$

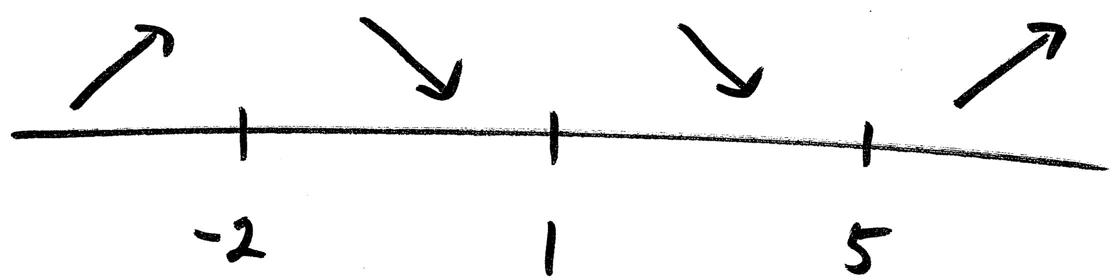
where are the relative max/min

find critical numbers : $f'(x) = 0$

$f'(x) = 0$ $\quad x-1=0, \quad x+2=0, \quad x-5=0$

$x=-2, \quad x=1, \quad x=5$

f' $\quad + \quad 0 \quad - \quad 0 \quad - \quad 0 \quad +$



rel. max
at $x=-2$

rel. min
at $x=5$

20 Find $\frac{d}{dx}\int_1^{2x} \sqrt{t^2+1}\,dt$ at $x=\sqrt{2}$

FTC 1:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

let $y = \int_1^{2x} \sqrt{t^2+1}\,dt$ want $\frac{dy}{dx}$

let $u = 2x$ $y = \int_1^{u} \sqrt{t^2+1}\,dt$

can find $\frac{dy}{du}$ from FTC 1

$$\frac{d}{du}\int_1^{u} \sqrt{t^2+1}\,dt = \sqrt{u^2+1}$$

from chain rule $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

$$\frac{dy}{dx} = \underbrace{\sqrt{u^2+1}}_{\frac{dy}{du}} \cdot \underbrace{2}_{\frac{du}{dx}} \qquad u = 2x$$

$$u^2 = (2x)^2 = 4x^2$$

$$\frac{dy}{dx} = 2\sqrt{4x^2+1}$$

at $x = \sqrt{2}$ $\quad \frac{dy}{dx} = 2\sqrt{4(\sqrt{2})^2+1} = 6$

21. $\int_{3}^{4} x\sqrt{25-x^2}\,dx = \int_3^4 x\,(25-x^2)^{1/2}\,dx$

compare: x vs $25-x^2$

let $u = 25-x^2$

$\dfrac{du}{dx} = -2x$

$du = -2x\,dx \longrightarrow x\,dx = \dfrac{du}{-2}$

new upper limit: $u = 25-4^2 = 9$

new lower limit: $u = 25-3^2 = 16$

$$\int_{16}^{9} \underset{\uparrow \atop (25-x^2)^{1/2}}{u^{1/2}} \cdot \underset{\nwarrow \, x\,dx}{\frac{du}{-2}} = \int_{16}^{9} -\frac{1}{2}u^{1/2}\,du = \frac{1}{2}\int_9^{16} u^{1/2}\,du$$

$$= \int_9^{16} \frac{1}{2} u^{1/2}\,du$$

$$= -\frac{1}{2}\ \frac{u^{3/2}}{3/2}\Bigg|_{16}^{9}$$

$$= -\frac{1}{2}\cdot\frac{2}{3}\,u^{3/2}\Big|_{16}^{9} = -\frac{1}{3}u^{3/2}\Big|_{16}^{9}$$

$$= \left[-\frac{1}{3}(9)^{3/2}\right] - \left[-\frac{1}{3}(16)^{3/2}\right]$$

$$= (-9) - \left(-\frac{64}{3}\right) = -9 + \frac{64}{3} = -\frac{27}{3} + \frac{64}{3}$$

$$= \frac{37}{3}$$

22. $\lim_{x\to\infty} \frac{1x^2+2x}{3x^2+4}$ $\quad \frac{\infty}{\infty}$ l'Hospital's Rule <u>OK</u>

$$\overset{L}{=} \lim_{x\to\infty} \frac{2x+2}{6x} \rightarrow \frac{\infty}{\infty}$$

$$\overset{L}{=} \lim_{x\to\infty} \frac{2}{6} = \frac{1}{3}$$ ↗ horizontal asymptote

23. $\displaystyle\lim_{x\to 0} \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$

$\displaystyle\lim_{x\to 0} \sin^{-1} x = 0$

$\displaystyle\lim_{x\to 0} \tan^{-1} x = 0$

$x = \sin y \longleftrightarrow \sin^{-1} x = y$

what does y have to do for $x \to 0$?

$y \to 0$

$x = \tan y \longleftrightarrow \tan^{-1} x = y$

$\downarrow \qquad \downarrow$

$x \to 0 \qquad y \to 0$

$y = \sin^{-1} x$ $\qquad \dfrac{dy}{dx} = \dfrac{d}{dx}(\sin^{-1} x) = ?$

$\sin y = x$

implicit diff

$\cos y \cdot \dfrac{dy}{dx} = 1$

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$$

since $\sin y = x$

$\sin^2 y = x^2 = 1 - \cos^2 y$

so $\cos^2 y = 1 - x^2$

$\cos y = \sqrt{1-x^2}$

$$\lim_{x\to 0} \frac{2x - \sin^{-1}x}{2x + \tan^{-1}x} \rightarrow \frac{0}{0}$$

l'Hopital's Rule again

$$\stackrel{L}{=} \lim_{x\to 0} \frac{2 - \dfrac{1}{\sqrt{-x^2+1}}}{2 + \dfrac{1}{x^2+1}} = \frac{2-1}{2+1} = \frac{1}{3}$$

24. $f''(x) > 0 \quad x < c$ $\longrightarrow$ concave up $x < c$

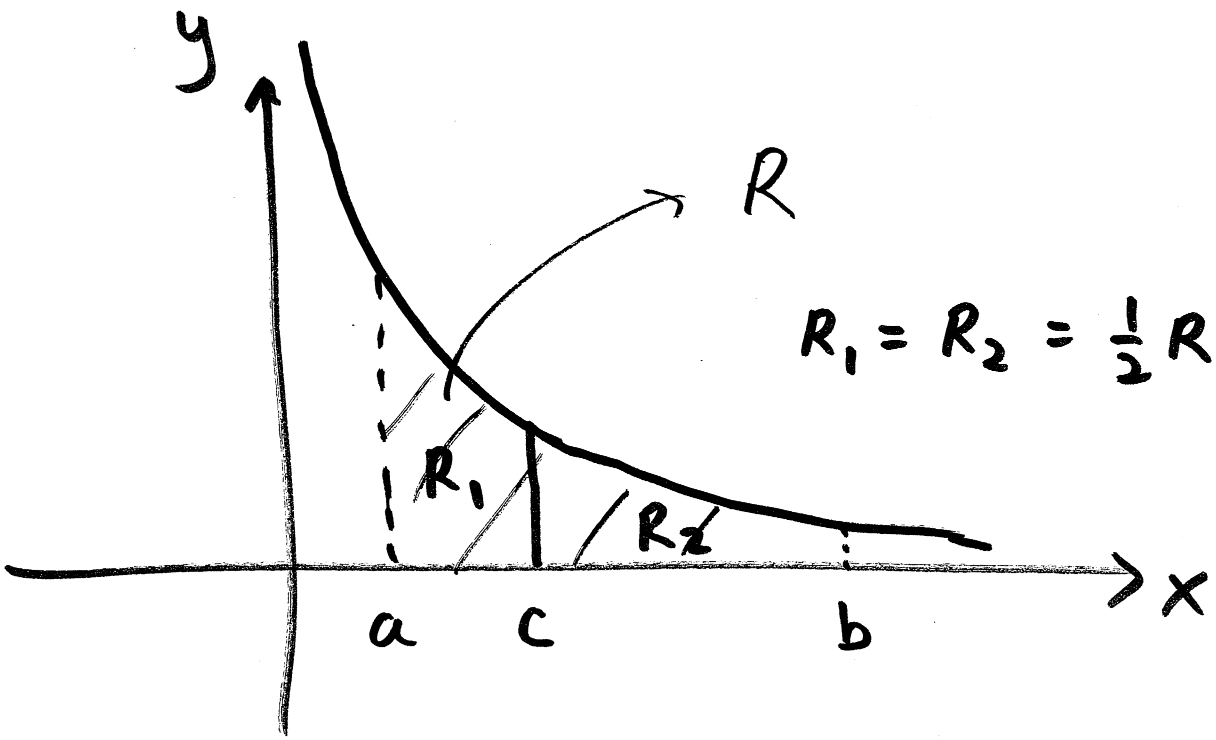
$f'(c) = 0$ $\longrightarrow$ zero tangent line slope at $x = c$

$f'(x) < 0 \quad x > c$ $\longrightarrow$ decreasing $x > c$

only B satisfies all three

25. R is region between $y = \frac{1}{x}$ and x-axis from $x = a$ to $x = b$

If the line $x = c$ cuts R into two equal halves, then $c = ?$



area of R: $\displaystyle\int_a^b \frac{1}{x}\,dx = \ln x \Big|_a^b$

$= \ln b - \ln a$

area of R_1: $\displaystyle\int_a^c \frac{1}{x}\,dx = \ln x \Big|_a^c$

$= \ln c - \ln a$

$= \frac{1}{2}(\ln b - \ln a)$

Solve for c: $\ln c - \ln a = \frac{1}{2}(\ln b - \ln a)$

$\ln c = \ln a + \frac{1}{2}\ln b - \frac{1}{2}\ln a$

$= \frac{1}{2}\ln a + \frac{1}{2}\ln b$

$= \frac{1}{2}(\ln a + \ln b) = \frac{1}{2}\ln ab = \ln\sqrt{ab}$

$\ln c = \ln\sqrt{ab}$

$c = \sqrt{ab}$

26. area of region between $y = \frac{1}{1+x^2}$

and x-axis from $x = -\sqrt{3}$ to $x = 1$

$$A = \int_{-\sqrt{3}}^{1} \frac{1}{1+x^2}\,dx$$

$y = \tan^{-1} 1 \Leftrightarrow \tan y = 1$

$\tan y = -\frac{\sqrt{3}/2}{1/2}$

$$= \tan^{-1} x \Big|_{-\sqrt{3}}^{1} = \tan^{-1} 1 - \tan^{-1}(-\sqrt{3})$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{3}\right)$$

$$= \frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$$