3.1 Derivatives of Polynomials and Exponential

definition of derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The Power Rule

if $f(x) = x^n$ then $f'(x) = n x^{n-1}$

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

↳ "derivative of ___"

f $f(x) = x^{2}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x+h)$$

$$= 2x^1$$

$$f(x) = x^{3}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= \lim_{h \to 0} (3x^2 + 3xh + h^2)$$

$$= 3x^2$$

$$f(x) = x^{4}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^4 - x^4}{h} = \lim_{h\to 0} \frac{\cancel{x^4} + 4x^3h + 6x^2h^2 + 4xh^3 +}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(4x^3 + 6x^2h + 4xh^2 + h^3)}{\cancel{h}}$$

$$= \lim_{h\to 0} \left(4x^3 + 6x^2h + 4xh^2 + h^3\right)$$

$$= 4x^3$$

example

$$\frac{d}{dx}\left(x^{10,000}\right) = 10,000\,x^{9,999}$$

example

$f(x) = \sqrt{x} = x^{1/2}$ in x^n form!

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} = \frac{1}{2x^{1/2}}$$

example

$$f(x) = \frac{1}{\sqrt[3]{x}} = x^{-1/3}$$

$f'(x) = -\frac{1}{3}x^{-4/3}$ $\rightarrow$ $-\frac{1}{3} - \frac{3}{3} = -\frac{4}{3}$

$$\frac{d}{dx}(x^1) = 1 \cdot x^0 = 1$$

if c is a constant, then $\boxed{\frac{d}{dx}(c) = 0}$

because $\frac{d}{dx}(1) = \frac{d}{dx}(x^0)$

$$= 0 \cdot x^{-1} = 0$$

we know $\frac{d}{dx}(x^2) = 2x$

$$\frac{d}{dx}(3x^2) = 3\frac{d}{dx}(x^2) = 3 \cdot 2x = 6x$$

constant-multiple

<u>example</u>: $\frac{d}{dx}\left(\frac{2}{x^5}\right)$

$$= \frac{d}{dx}\left(2x^{-5}\right) = 2\frac{d}{dx}(x^{-5}) = 2 \cdot (-5x$$

$$= -10x^{-6} = \frac{-10}{x^6}$$

$$\frac{d}{dx}(3x^2) = 6x$$

$$\frac{d}{dx}(x^2 + x^2 + x^2)$$

separated by $+/-$
→ split

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(x^2) + \frac{d}{dx}(x^2)$$

$$= 2x + 2x + 2x = 6x$$

$$\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

$\boxed{\pm \text{ only!}}$

xample derivative of $\dfrac{9x^2+3x+5}{\sqrt{x}}$?

$$y = \frac{9x^2+3x+5}{x^{1/2}} = \frac{9x^2}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{5}{x^{1/2}}$$

$$y = 9x^{3/2} + 3x^{1/2} + 5x^{-1/2}$$

$$y' = 9\cdot\left(\tfrac{3}{2}x^{1/2}\right) + 3\cdot\left(\tfrac{1}{2}x^{-1/2}\right) + 5\cdot\left(-\tfrac{1}{2}x^{-3/2}\right)$$

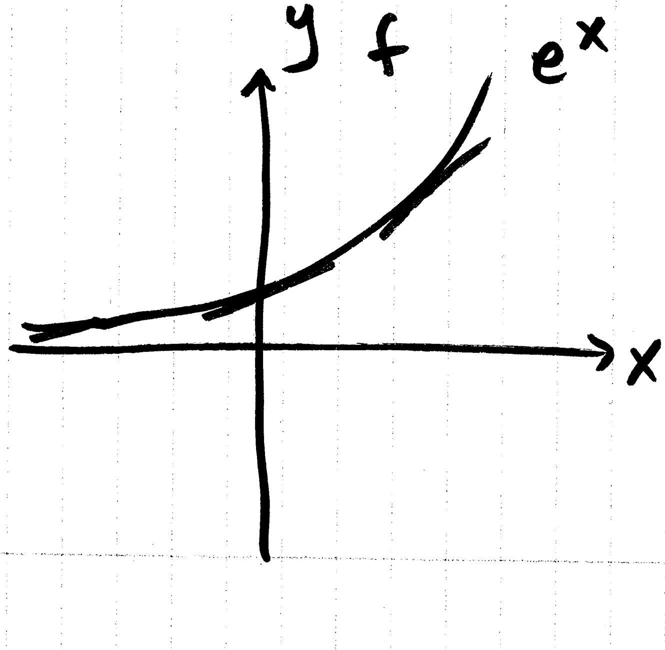
$$y' = \frac{27}{2}x^{1/2} + \frac{3}{2}x^{-1/2} - \frac{5}{2}x^{-3/2}$$

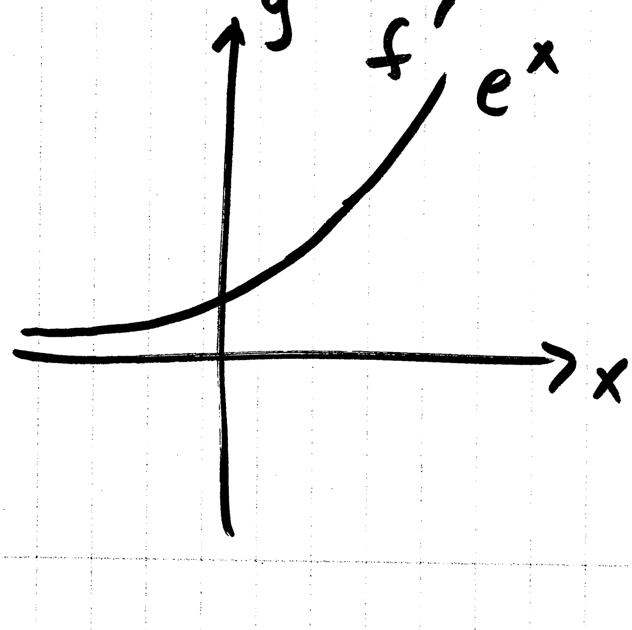
Exponential Function

if $f(x) = e^x$ then $f'(x) = e^x$

e^x is its own derivative

(NOT true for any other a^x if $a \neq$





Higher Derivatives

derivatives of derivatives

Leibniz notation

distance $y = f(x)$

velocity $y' = f'(x) = \frac{dy}{dx}$ first derivative

acceleration $y'' = f''(x) = \frac{d^2y}{dx^2}$ 2nd deriv.

jerk $y''' = f'''(x) = \frac{d^3y}{dx^3}$ 3rd deriv.

$y^{(4)} = f^{(4)}(x) = \frac{d^4y}{dx^4}$ 4^{th} deriv.