Appendix D and part of 1.6   Trig Review

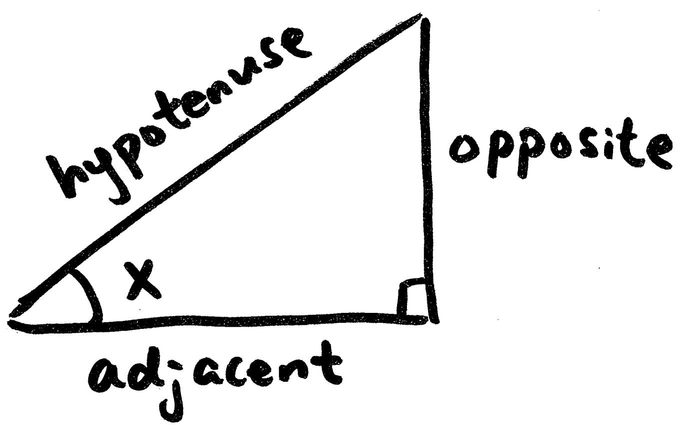
angle meesurement : degrees / radians

conversion : deg → rad multiply by $\frac{\pi}{180}$

$(360° = 360 \cdot \frac{\pi}{180} = 2\pi \text{ rad})$

rad → deg multiply by $\frac{180}{\pi}$

Trig Functions



sin   csc

cos   sec

tan   cot

$$\boxed{\sin x = \frac{\text{opp}}{\text{hyp}} \qquad \cos x = \frac{\text{adj}}{\text{hyp}}}$$

$$\tan x = \frac{\text{opp}}{\text{adj}}$$

SOH
CAH
TOA

$$\tan x = \frac{\sin x}{\cos x}$$

$$\csc x = \frac{1}{\sin x} = \frac{\text{hyp}}{\text{opp}}$$

$$\sec x = \frac{1}{\cos x} = \frac{\text{hyp}}{\text{adj}}$$

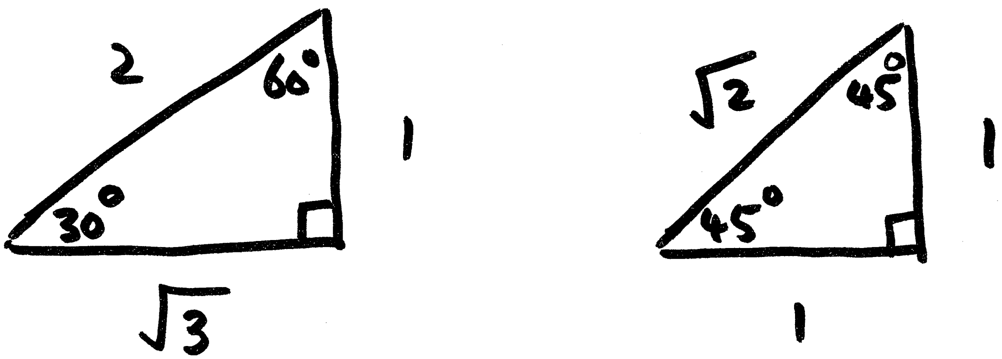
$$\cot x = \frac{1}{\tan x} = \frac{\text{adj}}{\text{opp}}$$

<u>special angles</u>

$0°, 30°, 45°, 60°, 90°$, etc

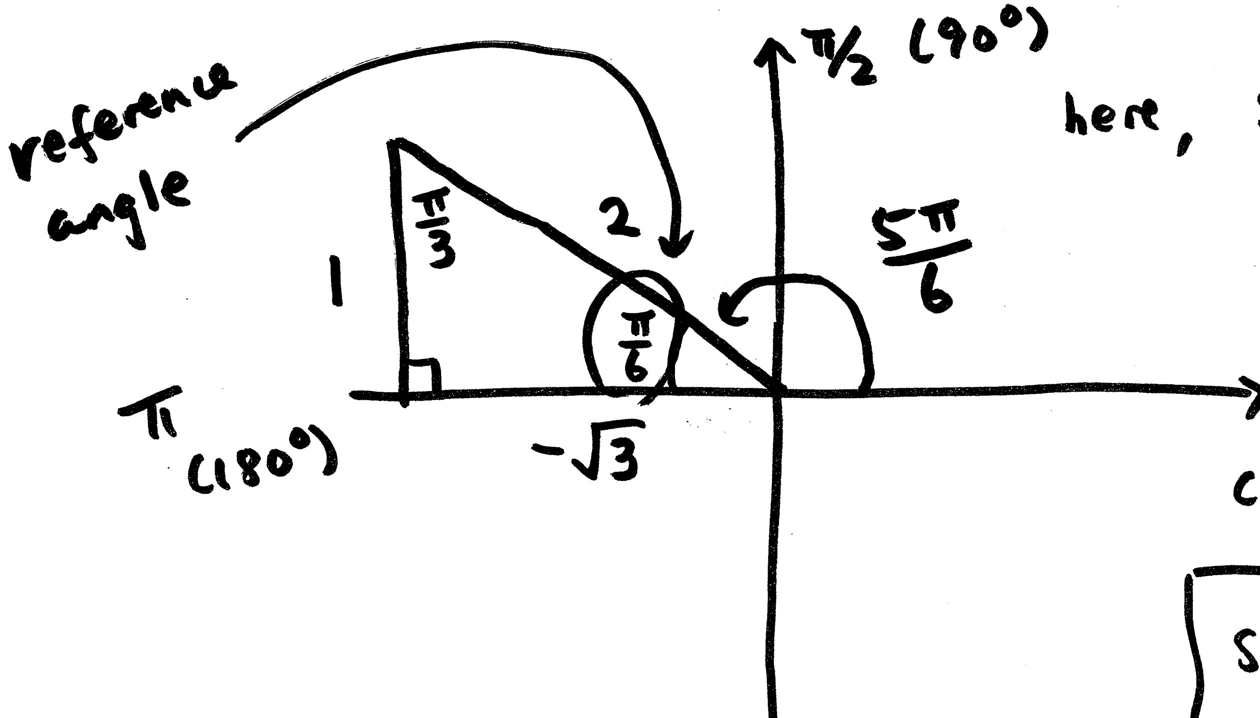
need to know sin and cos of these

From the book: special triangles



<u>example</u>: Find $\sec \frac{5\pi}{6}$

$\frac{5\pi}{6}$: quadrant II



here, $\sec \frac{5\pi}{6} = \sec \frac{\pi}{6}$

ref. angle in II

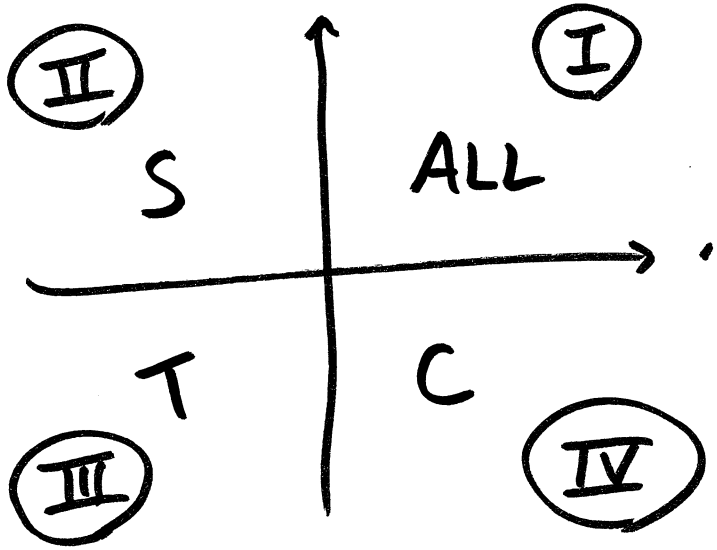
$\cos \frac{5\pi}{6} = \frac{-\sqrt{3}}{2}$

$$\boxed{\sec \frac{5\pi}{6} = -\frac{2}{\sqrt{3}}}$$

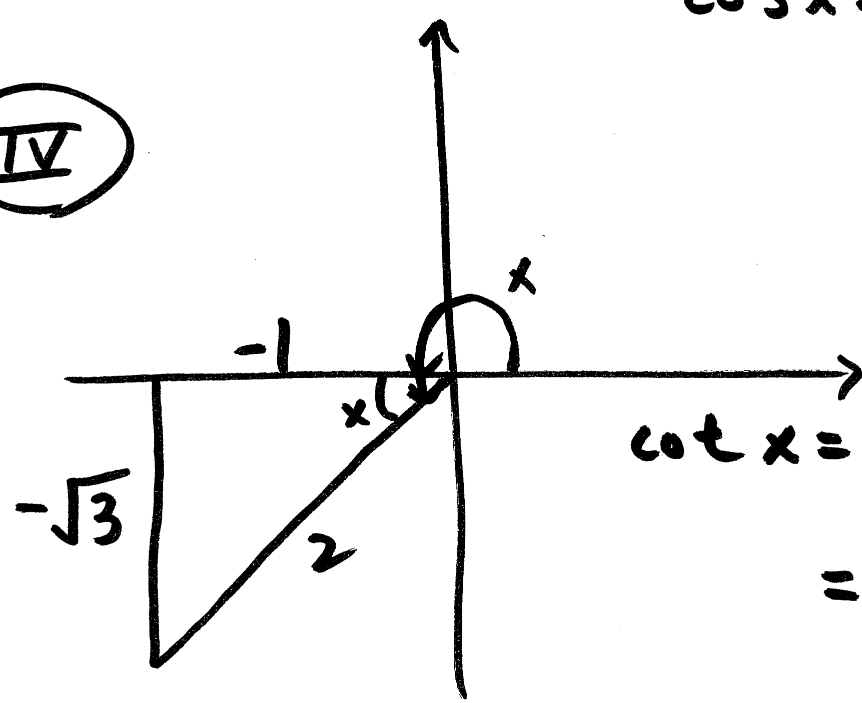
example: If $\cos x = -\frac{1}{2}$ and $\underbrace{\pi < x < \frac{3\pi}{2}}_{\text{III}}$

find $\cot x$

what trig function is positive in



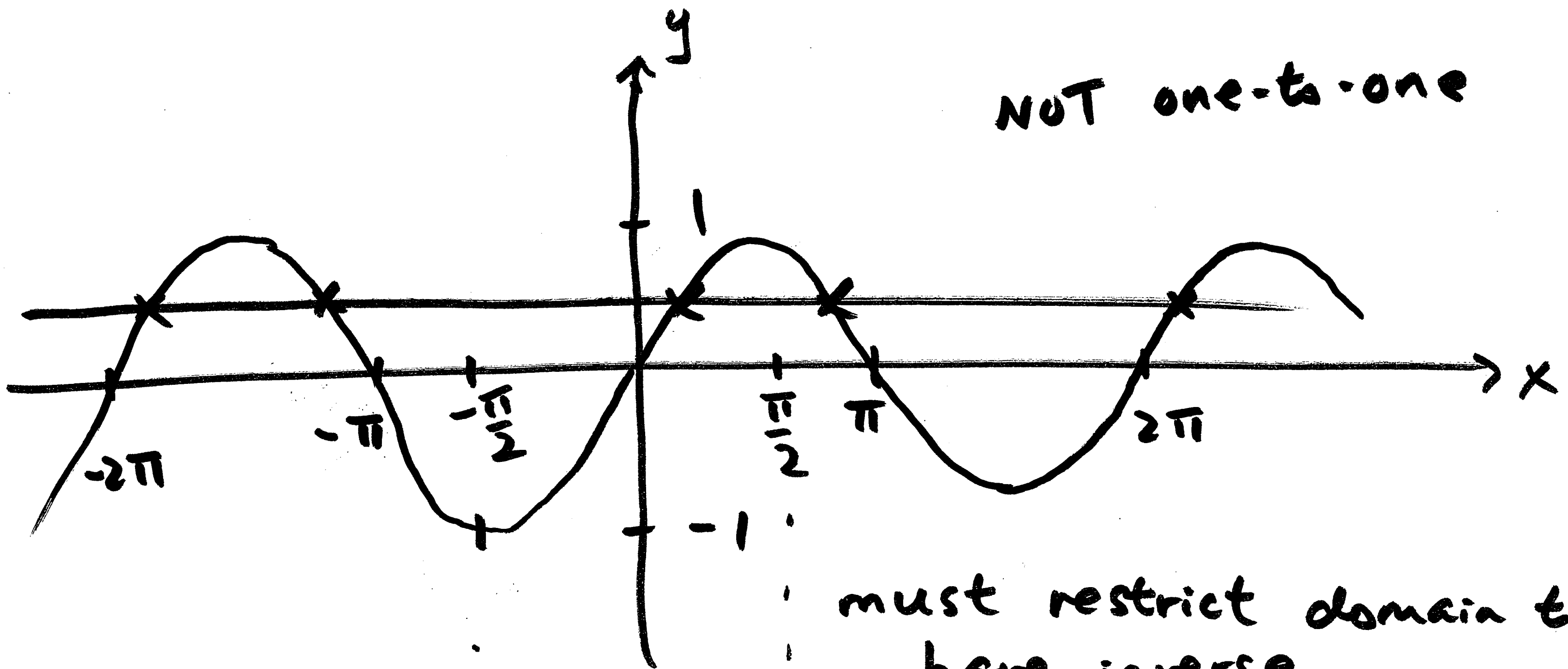
$$\cos x = -\frac{1}{2}$$
$$= \frac{\text{adj}}{\text{hyp}}$$



$$\cot x = \frac{1}{\tan x}$$
$$= \frac{\text{adj}}{\text{opp}} = \frac{-1}{-\sqrt{3}}$$
$$= \frac{1}{\sqrt{3}}$$

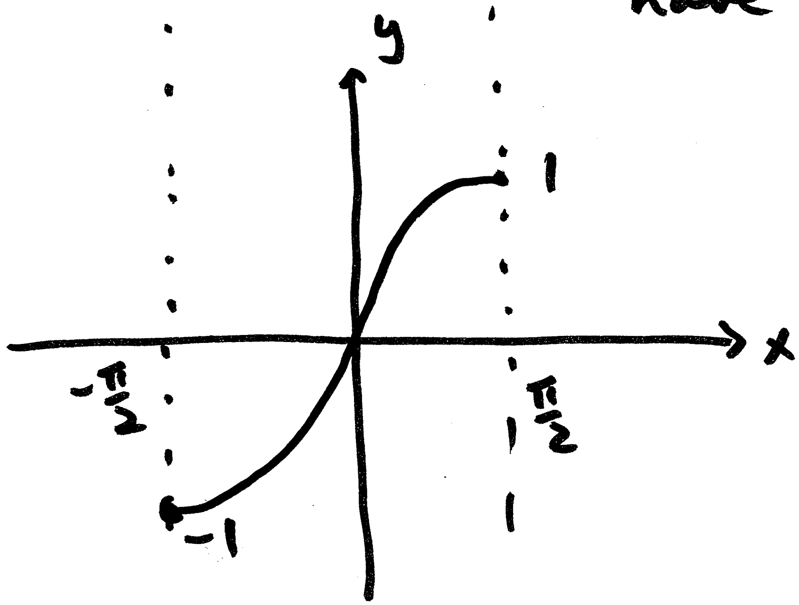
Inverse of $\sin x$

- function must be <u>one-to-one</u> to have inverse



NOT one-to-one

must restrict domain to have inverse



now is one-to-one

inverse sin : $\sin^{-1}(x) = \arcsin(x)$

has range $[-\frac{\pi}{2}, \frac{\pi}{2}]$

domain $[-1, 1]$

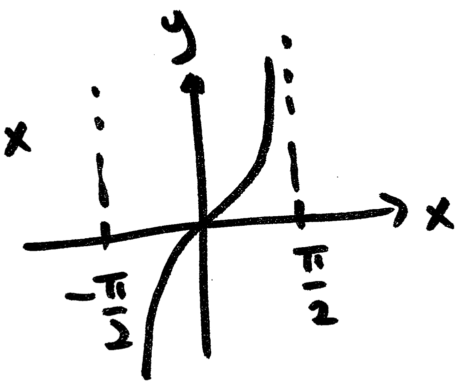
$\cos^{-1}(x) = \arccos(x)$

has range $[0, \pi]$

domain $[-1, 1]$

$\tan^{-1}(x) = \arctan(x)$

tan x



has range $(-\frac{\pi}{2}, \frac{\pi}{2})$

domain $(-\infty, \infty)$

$$\sin^{-1}(x) = y \longleftrightarrow \sin(y) = x$$

<u>example</u> : "simplify" $\sin(\tan^{-1} x)$

work inside-out

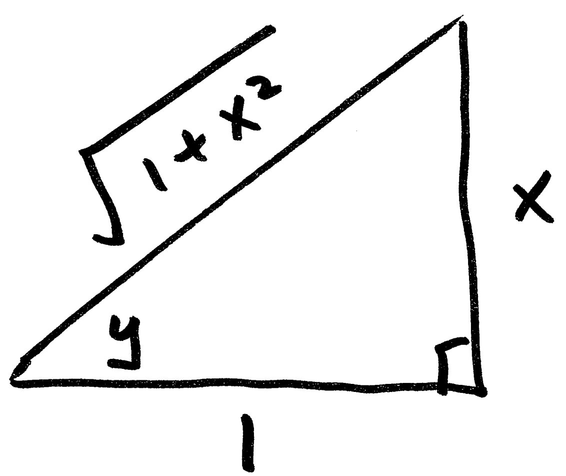
let $y = \tan^{-1} x$

then $\cancel{\tan y = y\ x}$

$$\tan y = x = \frac{\text{opp}}{\text{adj}} = \frac{x}{1}$$

$$\sin(\tan^{-1} x) = \sin(y)$$

$$= \boxed{\frac{x}{\sqrt{1+x^2}}}$$

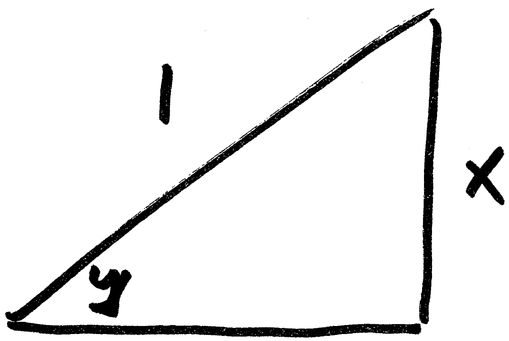


example :

$$\sin(\sin^{-1} x) \cancel{\neq x}$$

let $y = \sin^{-1} x$

$$x = \sin y = \frac{\text{opp}}{\text{hyp}} = \frac{x}{1}$$



can be found from Pythagorean Theorem

$\sin(\sin^{-1} x)$

$= \sin y = x$

so $\boxed{\sin(\sin^{-1} x) = x}$

also, $\sin^{-1}(\sin x) = x$

IF x is in the domain of $\sin x$ or $\sin^{-1} x$

example:

$$\sin^{-1}(\sin x)$$

$\sin^{-1}(\sin x) = y$ is equivalent to

$\sin x = \sin y$ ~~provided~~

so $x = y$

or $\sin^{-1}(\sin x) = x$

provided $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (range of $\sin^{-1}$)

or $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

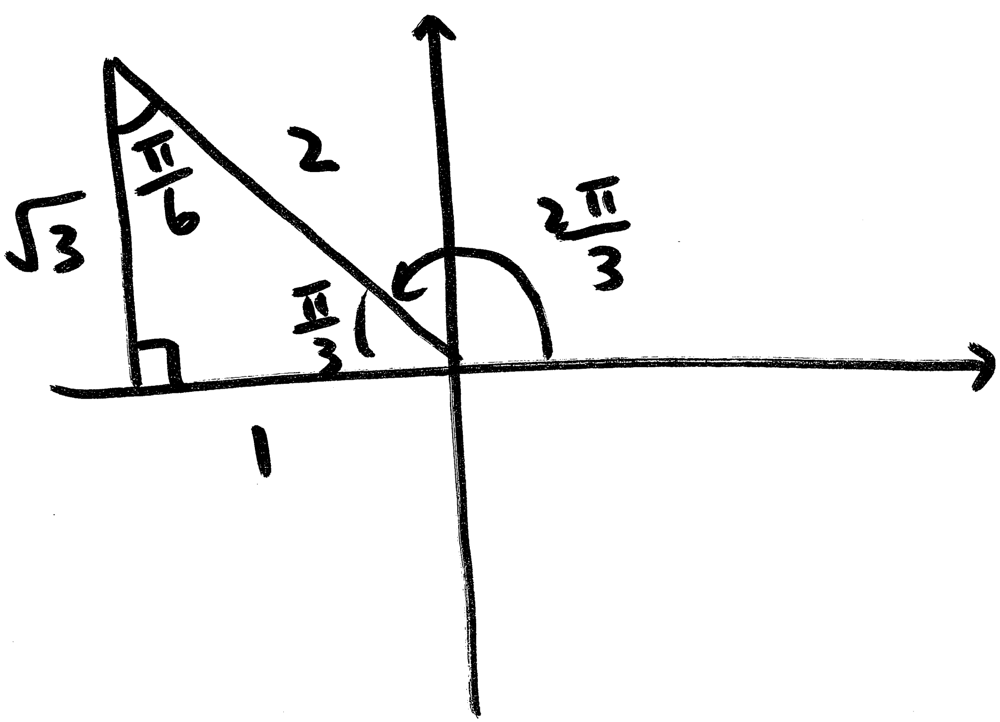
If not, then $\sin^{-1}(\sin x)$ may not be equal to x

example $\sin^{-1}\Big(\underbrace{\sin\frac{8\pi}{3}}_{y}\Big)$

$y = \sin\frac{8\pi}{3}$

$\frac{8\pi}{3}$ is in quadrant II

$\left(\frac{8\pi}{3} = \underbrace{\frac{6\pi}{3}}_{\text{one complete circle}} + \frac{2\pi}{3}\right)$



$y = \sin\frac{8\pi}{3} = \sin(\text{reference angle } \frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$\sin^{-1}(\sin\frac{8\pi}{3})$ is therefore

$= \sin^{-1}(\frac{\sqrt{3}}{2})$

this asks: what angle X gives $\sin(x) = \frac{\sqrt{3}}{2}$?

$\sin(x) = \frac{\sqrt{3}}{2}$ positive sin means quadrant I or II

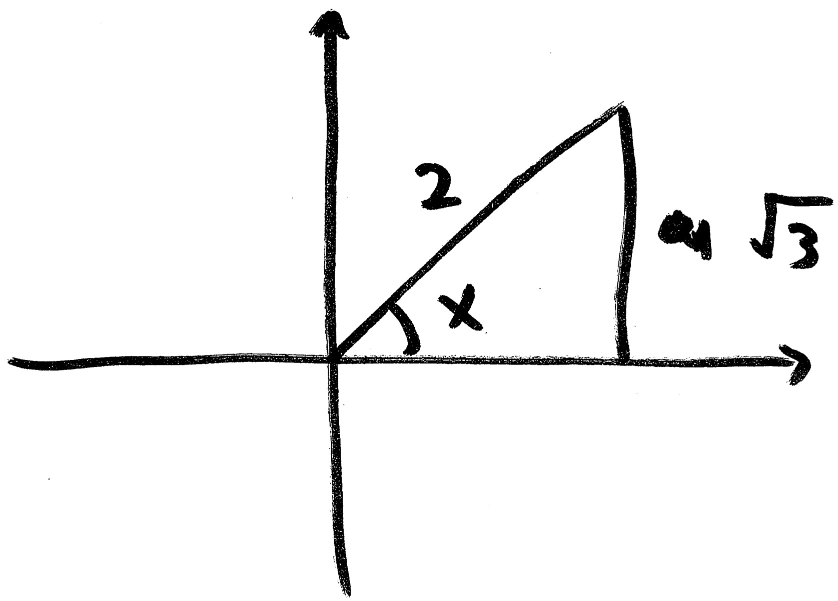
but range of $\sin^{-1}(\frac{\sqrt{3}}{2})$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Q IV QI

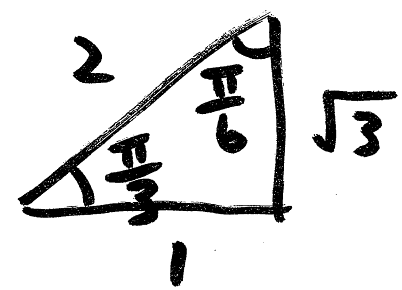
so X MUST be in QI

now we solve $\sin(x) = \frac{\sqrt{3}}{2}$ with x in QI

$$\frac{\sqrt{3}}{2} = \frac{\text{opp}}{\text{hyp}}$$



compare to special Δ



we know therefore $\boxed{x = \frac{\pi}{3}}$

so $\sin^{-1}\left(\sin\frac{8\pi}{3}\right) = \frac{\pi}{3}$ and NOT $\frac{8\pi}{3}$