The Chain Rule (part 2)

recall $\frac{d}{dx} u^n = \underbrace{n u^{n-1}}_{\text{power rule}} \frac{du}{dx}$

account for u not being just x

$$y' = f'(u)\, u'$$

<u>example</u> : $f(x) = \sqrt[4]{1 + \cot x} = (1 + \cot x)^{1/4}$

$$f'(x) = \frac{1}{4}(1 + \cot x)^{-3/4}(-\csc^2 x)$$

$$= \boxed{\frac{-\csc^2 x}{4(1 + \cot x)^{3/4}}}$$

example :

$$y = \left[(x^4-1)^3\right]\left[(x^3+1)^6\right]$$

deriv. deriv.

$$y' = (x^4-1)^3 \cdot \left[6\cdot(x^3+1)^5(3x^2)\right]$$

$$+ (x^3+1)^6 \cdot \left[3\cdot(x^4-1)^2(4x^3)\right]$$

$$= \underline{18}x^2(x^3+1)^5(x^4-1)^3 + \underline{12}x^3(x^3+1)^6(x^4-1)^2$$

$$= \underline{6}x^2(x^3+1)^5(x^4-1)^2\left[3(x^4-1)+2x(x^3+1)\right]$$

$$= 6x^2(x^3+1)^5(x^4-1)^2(3x^4-3+2x^4+2x)$$

$$= \boxed{6x^2(x^3+1)^5(x^4-1)^2(5x^4+2x-3)}$$

recall $(\cos x)^2 = \cos^2 x \neq \cos(x^2)$

<u>example</u>: $y = \cos^2 x + \cos x^2$

$$= (\cos x)^2 + \cos(x^2)$$

$$y' = 2(\cos x)^1(-\sin x) + -\sin(x^2)\cdot 2x$$

$$\boxed{= -2\left(+\cos x \cdot \sin x + x \sin x^2\right)}$$

example: $y = \sin(\sin(x))$

$= \sin(\square)$

recall $\frac{d}{dx} \sin x = \cos x$

$\frac{d}{dx} \sin \square = \cos \square \cdot \square'$

$y' = \cos(\sin(x)) \cdot \frac{d}{dx}(\sin(x))$

$= \boxed{\cos(\sin x) \cdot \cos x}$

example

$$y = \sin\left(\boxed{\cos(\sin(5x))}\right)$$

outside in

$$y' = \cos(\cos(\boxed{\sin(5x)})) \cdot -\sin(\boxed{\sin(5x)}) \cdot \cos 5x \cdot 5$$

deriv.

deriv.

deriv.

recall $\frac{d}{dx} e^x = e^x$ $\qquad$ $\frac{d}{dx} e^u = e^u \cdot u'$

<u>example</u> :

$$y = e^{x\cos 2x}$$

product rule

$$y' = e^{x\cos 2x} \cdot \frac{d}{dx}\left((x)(\cos 2x)\right)$$

$$y' = e^{x\cos 2x} \cdot \left[(x)(-\sin 2x)(2) + \cos(2x)\right]$$

$$= \boxed{e^{x\cos 2x}\left(-2x\sin 2x + \cos 2x\right)}$$

[illegible]

$$y = e^{\left(e^{e^x}\right)}$$

$$y' = e^{e^{e^x}} \cdot \frac{d}{dx}\left(e^{\left(e^x\right)}\right)$$

$$= e^{e^{e^x}} \cdot e^{e^x} \cdot \frac{d}{dx}\left(e^x\right)$$

$$= \boxed{e^{e^{e^x}} \cdot e^{e^x} \cdot e^x}$$

$$y = e^x$$

$$y' = e^x \cdot \underbrace{\frac{d}{dx}(x)}_{1}$$

when in doubt chain it out.