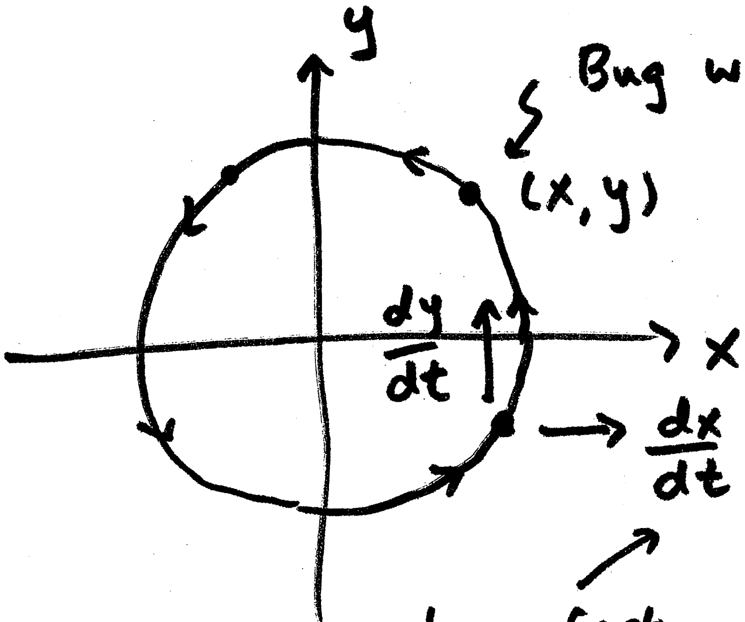
3.9 Related Rates (part 1)

Circular track with radius of 5 cm

Bug walks in track



how fast in x-direction

note: x and y are related by

$$x^2 + y^2 = 25$$

$\frac{dx}{dt}$ and $\frac{dy}{dt}$ are also related in some way.

<u>example</u>: At a particular instant, $\frac{dy}{dt} = 4$ and $y = 3$. What is $\frac{dx}{dt}$?

$$x^2 + y^2 = 25$$

given: $\frac{dy}{dt} = 4$ when $y = 3$

$x = 4$ or $\cancel{-4}$

because $\frac{dy}{dt} > 0$ (right half of track)

find: $\frac{dx}{dt}$

keep in mind $x = x(t)$

$y = y(t)$

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(25)$$

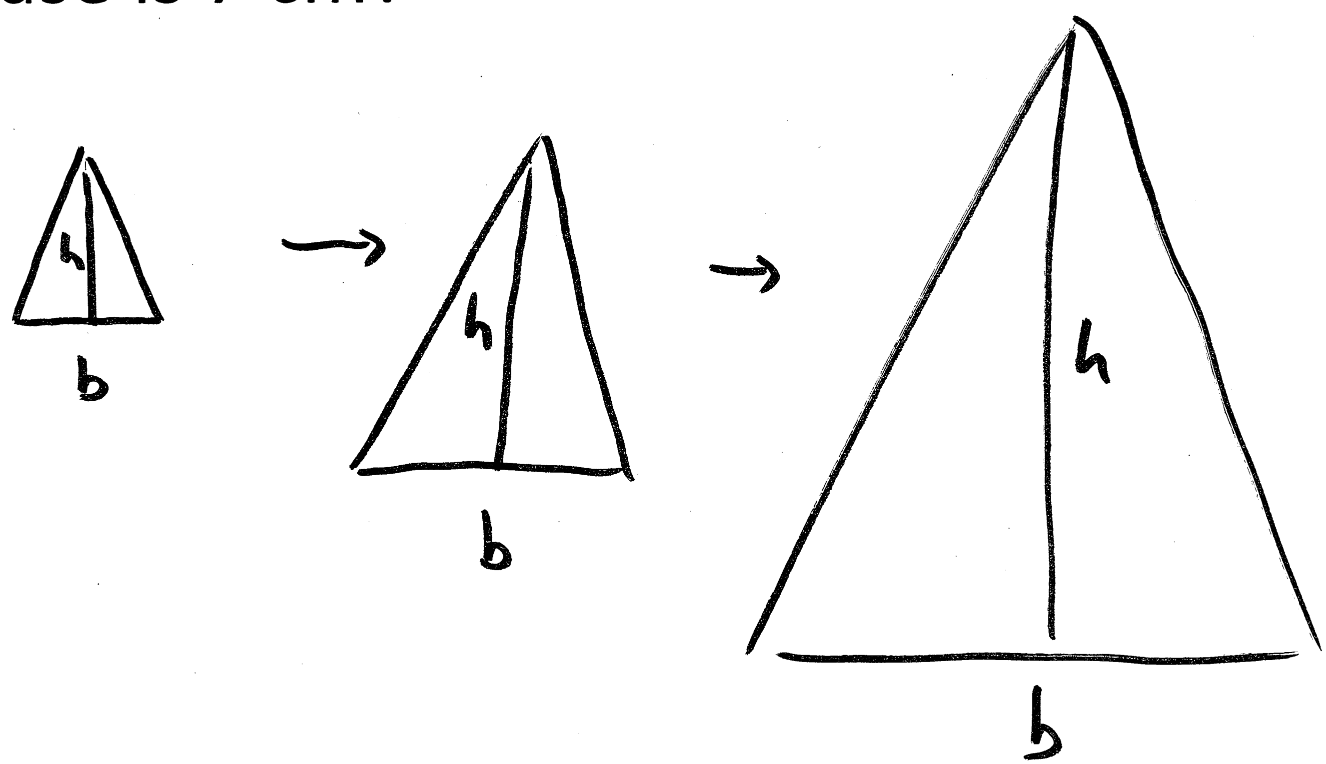
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \longrightarrow \frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt}$$

chain rule

plug in given #s

$$\frac{dx}{dt} = -\frac{3}{4}(4) = \boxed{-3}$$

- The height of an isosceles triangle is increasing at a rate of 4 cm/s and the base of the triangle is increasing at a rate of 5 cm/s. How fast is the area of the triangle increasing at the moment when the height is 10 cm and the base is 7 cm?



$A = \frac{1}{2} b h$ again, $b = b(t)$ $h = h(t)$

given: $\frac{dh}{dt} = 4$ $\frac{db}{dt} = 5$

find: $\frac{dA}{dt}$ when $h = 10$ and $b = 7$

$$\frac{d}{dt}(A) = \frac{d}{dt}\left(\frac{1}{2} b h\right)$$

product rule

$\frac{d}{dt}\left(b(t) \cdot h(t)\right)$

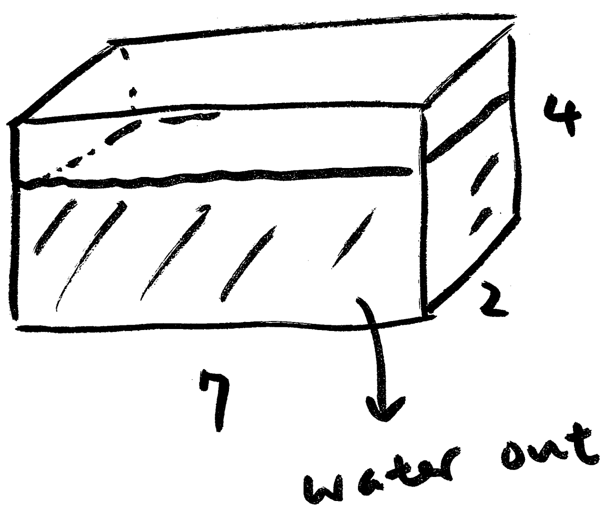
$$\frac{dA}{dt} = \frac{1}{2} \frac{d}{dt}(b h)$$

$$= \frac{1}{2}\left(b \frac{dh}{dt} + h \frac{db}{dt}\right)$$

plug in #s **AFTER** derivative

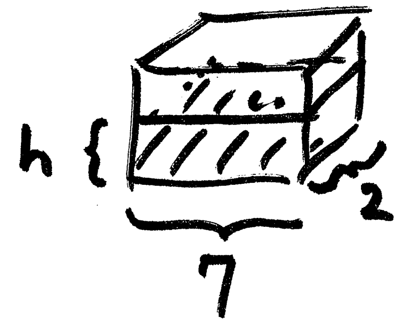
$$= \frac{1}{2}(7 \cdot 4 + 10 \cdot 5) = \boxed{39} \ \mathrm{cm}^2/\mathrm{s}$$

- A fish tank with a height of 4 ft, a length of 7 ft, and a width of 2 ft is currently 80% full of water. Water is being let out of the tank at a rate of 3 cubic feet per minute. How fast is the water level decreasing?



relate volume and height (depth)

$$V = 14h$$



given: $\frac{dV}{dt} = -3$ decreasing

find: $\frac{dh}{dt}$

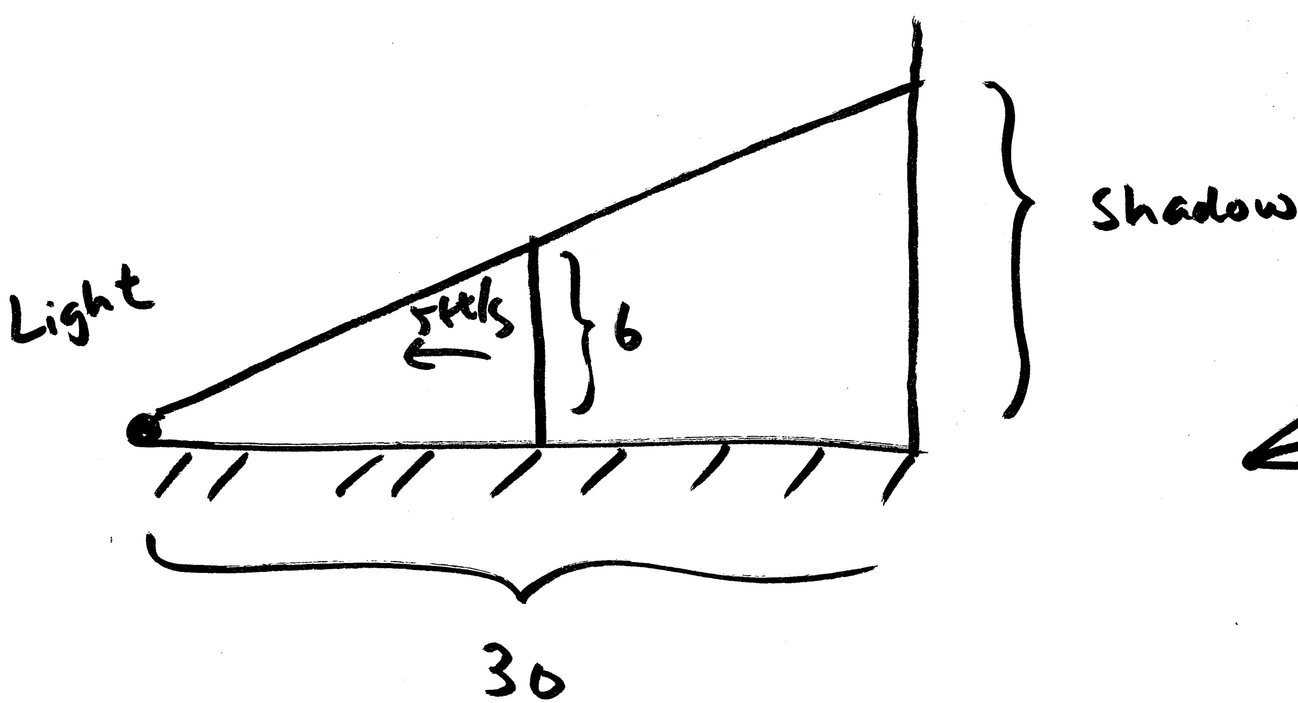
$$\frac{d}{dt}V = 14\frac{d}{dt}h$$

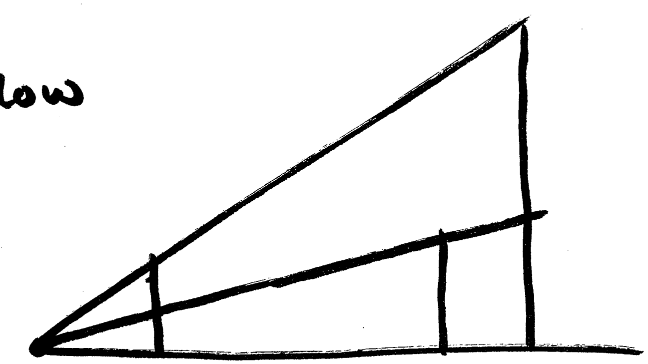
$$\frac{dV}{dt} = 14\frac{dh}{dt}$$

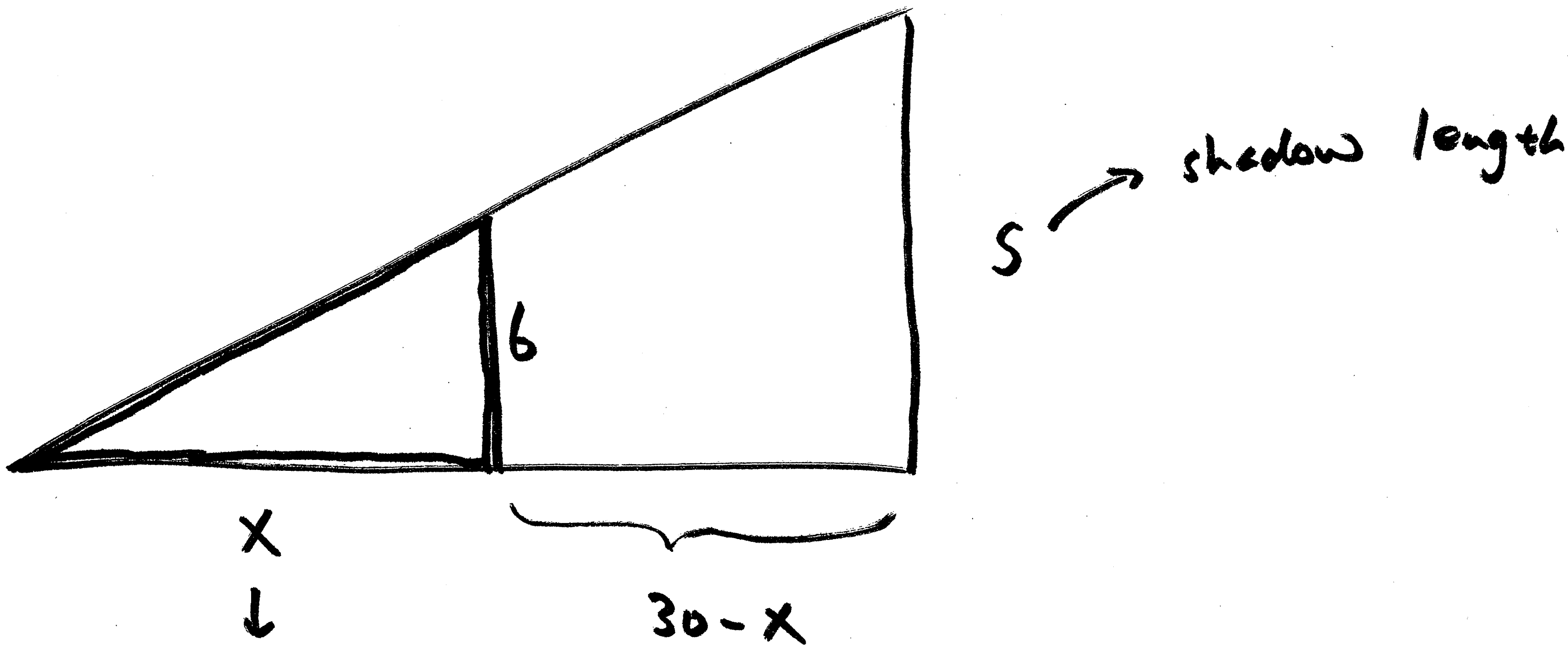
$$\frac{dh}{dt} = \frac{1}{14}\frac{dV}{dt}$$

$$= \boxed{-\frac{3}{14}} \text{ ft/min}$$

- A spotlight on the ground shines on a wall 30-ft away. If a 6-ft-tall man walks toward the light at a speed of 5 ft/s, how fast is the length of his shadow on the wall changing when he is 18-ft from the light?



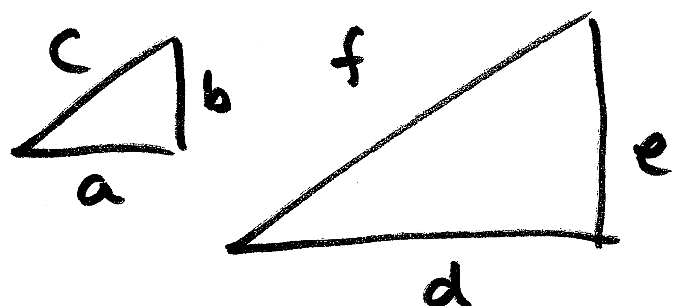




dist. to light

want to relate x, s $\left(\frac{dx}{dt}, \frac{ds}{dt}\right)$

Similar triangles



$$\frac{b}{a} = \frac{e}{d}$$

ratio is the same

$$\frac{6}{x} = \frac{s}{30}$$

$$\frac{dx}{dt} = -5$$

$$sx = 180$$

$$\underline{\underline{x = 18}}$$

$$s = 10$$

$$\frac{d}{dt}(xs) = \frac{d}{dt}(180)$$

$$x\frac{ds}{dt} + s\frac{dx}{dt} = 0$$

$$18 \cdot \frac{ds}{dt} + s \cdot (-5) = 0$$

want: $\frac{ds}{dt}$

$$18\frac{ds}{dt} - 50 = 0$$

$$\frac{ds}{dt} = \frac{50}{18} = \boxed{\frac{25}{9}} \text{ ft/s}$$