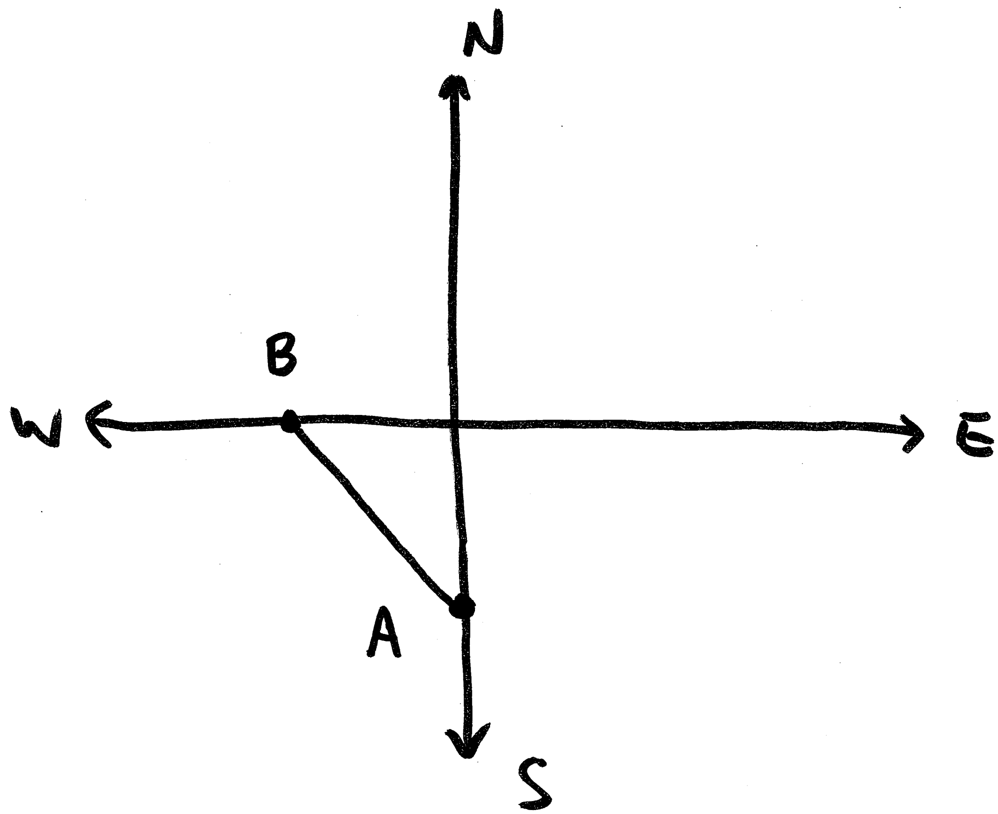
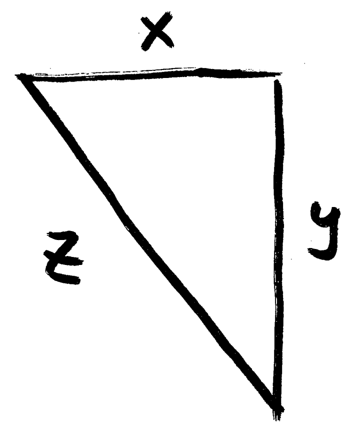
- Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

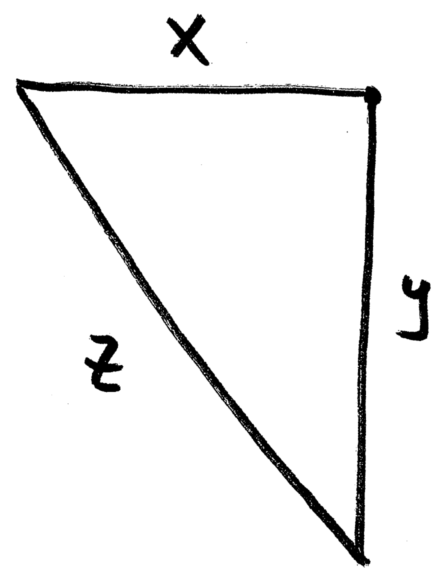
car A (One) — car B (other)





3.9 Related Rates (part 2)

example 1



$\frac{dx}{dt} = 25$
$\frac{dy}{dt} = 60$ } given

Find: $\frac{dz}{dt}$ at two hours after cars started moving

↳ $x = 50$

$y = 120$

equation relating x, y, z

$z^2 = x^2 + y^2$

$$z^2 = x^2 + y^2$$

$$\frac{d}{dt} z^2 = \frac{d}{dt}(x^2 + y^2)$$

$$2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{z}$$

$$= \frac{(25)(50) + (60)(120)}{130}$$

$$= \boxed{65} \text{ mi/h}$$

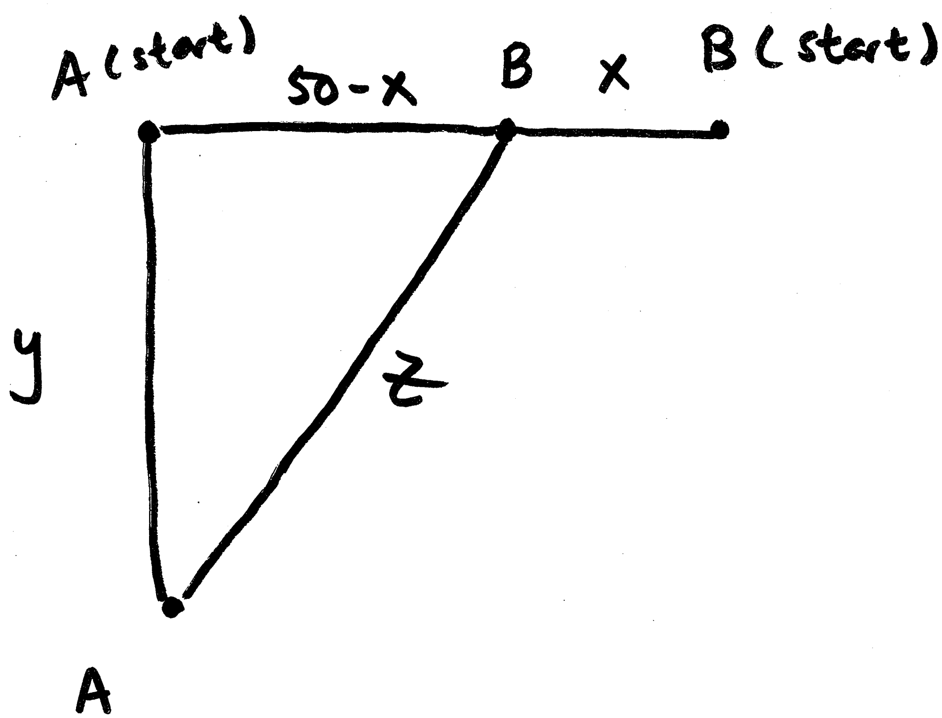
after derivative, plug in #s

$\frac{dx}{dt} = 25$ $\frac{dy}{dt} = 60$

$x = 50$ $y = 120$

$z = 130$

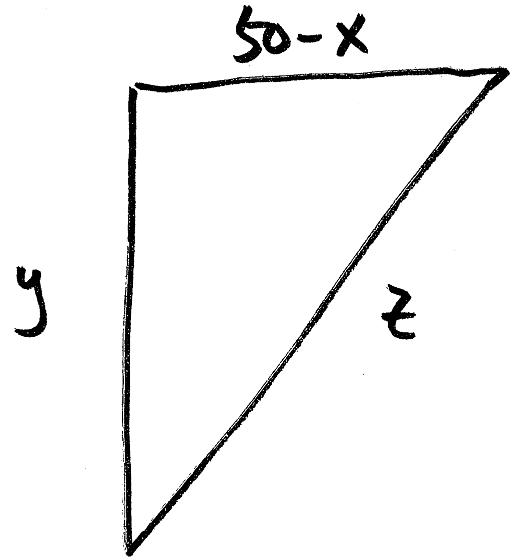
- Car A starts at a point 50 miles to the west of car B. Car A travels south at 60 mi/h and car B travels west at 25 mi/h. How fast are they moving away from each other 2 hours later?



$\frac{dy}{dt} = 60$

$\frac{dx}{dt} = 25$

stop at 2 hours later so distances are positive



$\frac{dx}{dt} = 25$

$\frac{dy}{dt} = 60$

2 hours later, $x = 50$

$y = 120$

$\frac{dz}{dt} = ?$ $z = 120$

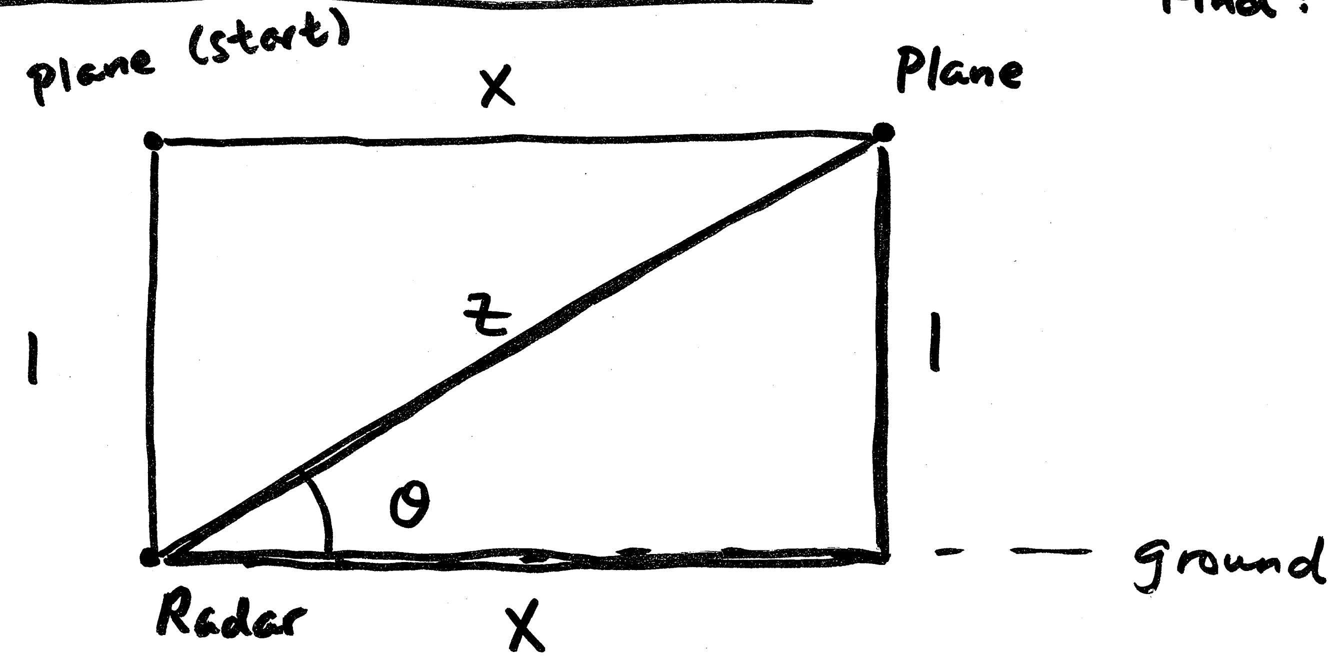
$$z^2 = (50-x)^2 + y^2$$

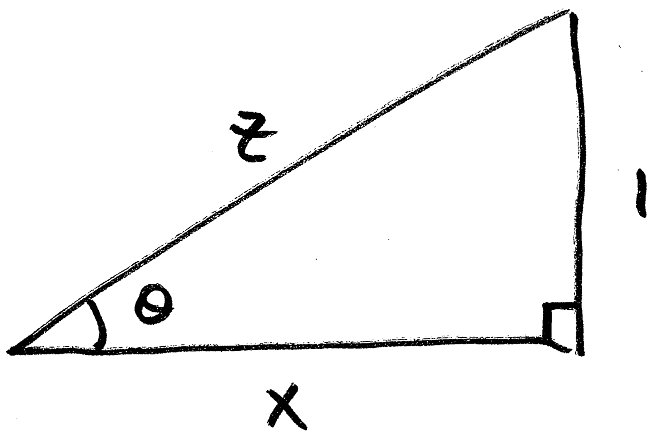
$$2z\frac{dz}{dt} = 2(50-x)\left(-\frac{dx}{dt}\right) + 2y\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{-(50-x)\frac{dx}{dt} + y\frac{dy}{dt}}{z} = \boxed{60}$$

- A plane flying horizontally at an altitude of 1 mi and a speed of 510 mi/h passes directly over a radar station. Find the rate at which the angle of elevation from the radar station to the plane is changing when the plane is 5 mi away from the station.

Find: $\frac{d\theta}{dt}$





given: $\frac{dx}{dt} = 510$

find: $\frac{d\theta}{dt}$ when $z = 5$

angles in <u>radians</u>

relate θ, x, z

$$\cos\theta = \frac{x}{z}$$

$$-\sin\theta \frac{d\theta}{dt} = \frac{z \cdot \frac{dx}{dt} - x\frac{dz}{dt}}{z^2}$$

need: θ, z, x, $\frac{dx}{dt}$, $\frac{dz}{dt}$ ← hard to find!

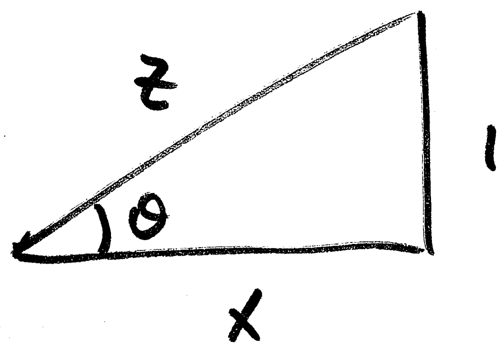
something simpler

$$\tan\theta = \frac{1}{x} \longrightarrow \cot\theta = x$$

$$-\csc^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$$

$$\frac{1}{\csc\theta} = \sin\theta$$

$$\frac{d\theta}{dt} = \frac{\frac{dx}{dt}}{-\csc^2\theta} = -\sin^2\theta \, \frac{dx}{dt}$$



now $z = 5$

$\sin\theta = \frac{1}{5}$ $\quad \sin^2\theta = \frac{1}{25}$

$$\frac{d\theta}{dt} = -\frac{1}{25}(510) = \boxed{-20.4 \text{ rad/hour}}$$