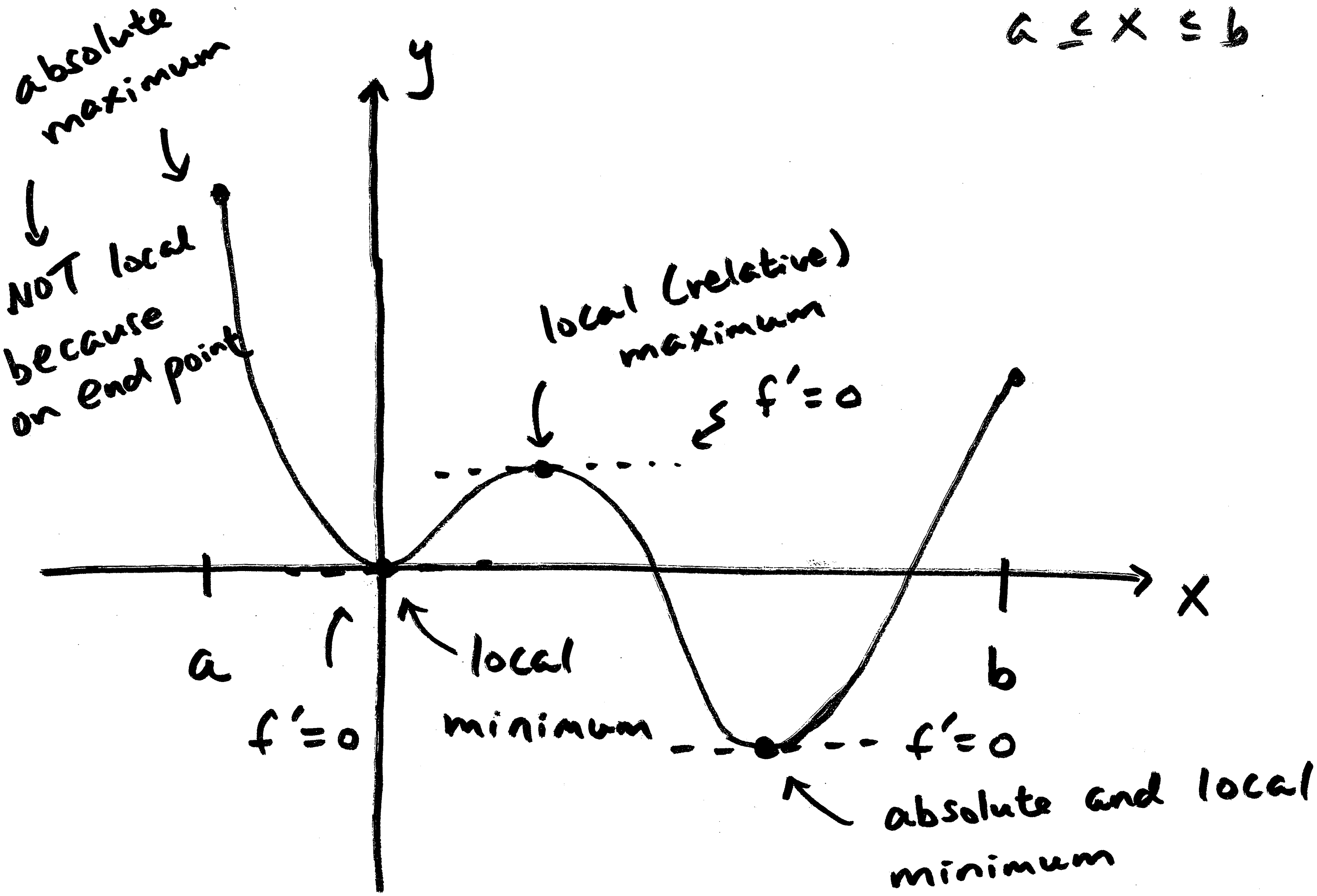
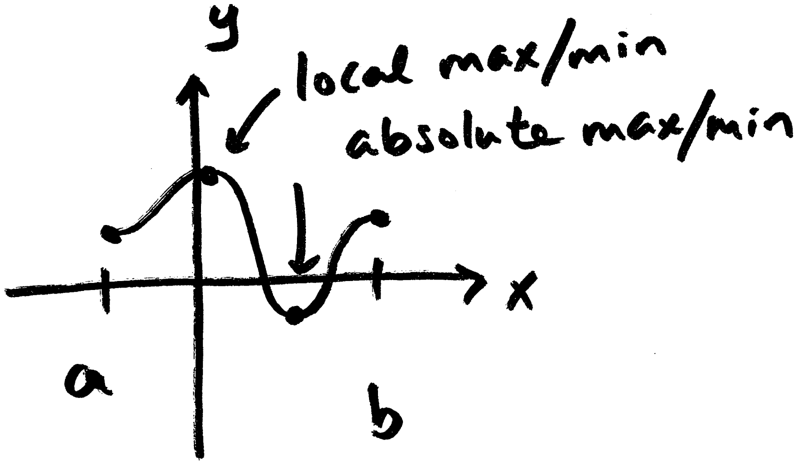
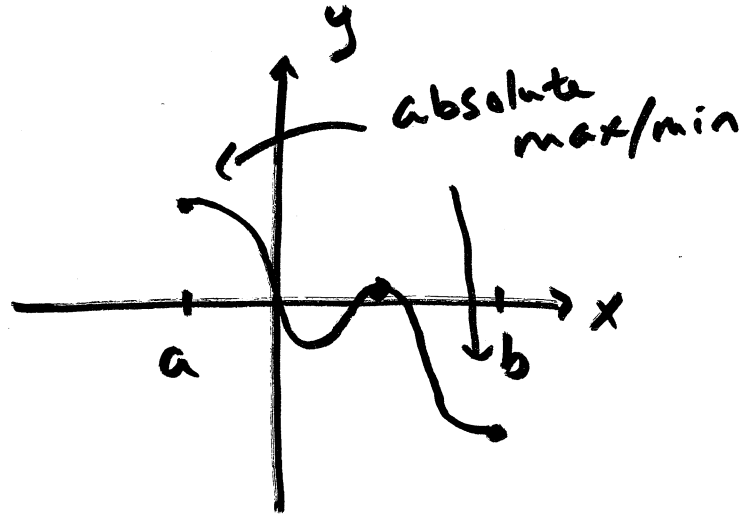
4.1 Maximum and Minimum Values

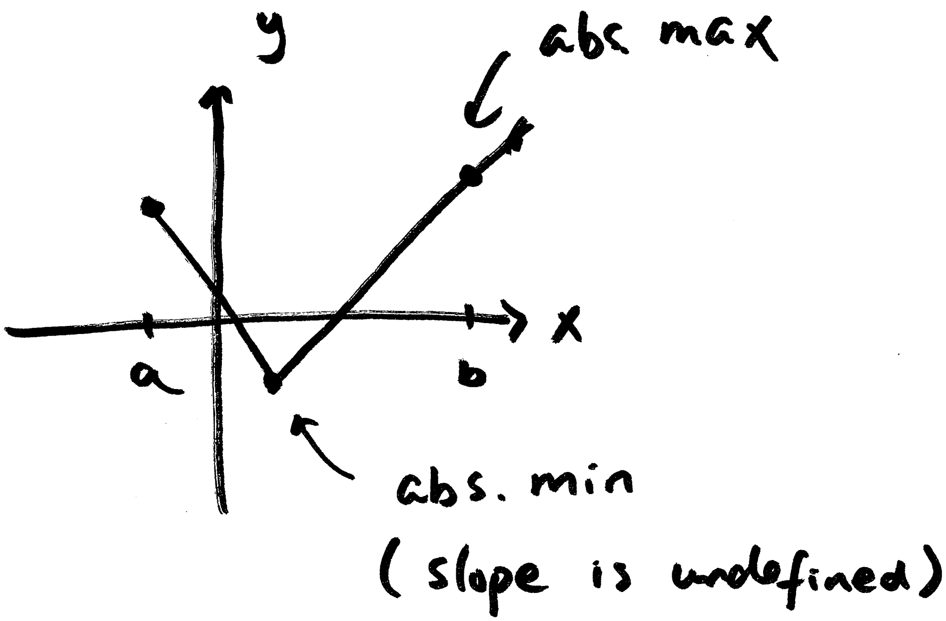
$f(x)$ on a closed interval $[a, b]$

$a \le x \le b$









Closed Interval Method

1. Find critical numbers (where $f' = 0$ or f' DNE)
2. Compare the local max/min to f end points.
3. Find absolute max/min

<u>example</u> Find the absolute max/min of $f(x) = 4x^3 - 12x^2 - 96x + 2$ on $[-3, 5]$

$f'(x) = 12x^2 - 24x - 96$

$f'(x) = 0 \longrightarrow 12x^2 - 24x - 96 = 0$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = -2, \quad x = 4$ critical numbers

possible locations of loc. max/min or abs. max/min

compare $f(-3)$, $f(-2)$, $f(4)$, $f(5)$

$f(-3) = 74$

$f(-2) = 150 \longrightarrow$ abs. max at $x = -2$

$f(4) = -318 \longrightarrow$ abs. min at $x = 4$

$f(5) = -278$

example : $\cancel{f(x) = \quad 1 +}$

$$f(x) = x + \frac{1}{x} \quad \text{on} \quad \boxed{\left[\tfrac{1}{2},\ 8\right]}$$

$$= x + x^{-1}$$

$$f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$$

$$f' = 0 \longrightarrow 1 - \frac{1}{x^2} = 0$$

$$x^2 - 1 = 0$$

$$\cancel{x = -1}, \quad x = 1$$

outside of $\left[\tfrac{1}{2}, 8\right]$

~~abs.~~

$$f' \text{ DNE} \longrightarrow \cancel{x = 0}$$

outside $\left[\tfrac{1}{2}, 8\right]$

$$f\left(\tfrac{1}{2}\right) = \frac{1}{2} + \frac{1}{\frac{1}{2}} = \frac{1}{2} + 2 = \frac{5}{2} \qquad f(8) = \frac{65}{8}$$

abs. ~~min~~ max

$$f(1) = 2 \longleftarrow \text{abs. min}$$

example: $f(x) = \ln(x^2+2x+7)$ on $[-2, 2]$

$$f'(x) = \frac{1}{x^2+2x+7} \cdot (2x+2) = \frac{2x+2}{x^2+2x+7}$$

$f'(x) = 0$ $\quad \frac{2x+2}{x^2+2x+7} = 0$

$2x+2 = 0 \cdot x^2+2x+7 = 0$

fraction $= 0 \rightarrow$ numerator $= 0$

$x = -1$

$f'(x)$ DNE $\quad x^2+2x+7 = 0$ $\quad$ (denom $= 0$)

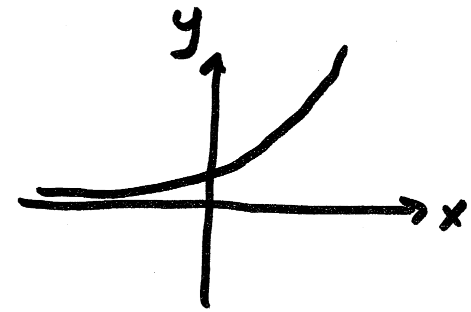
no solutions

$f(-2) = \ln(7)$ $\qquad$ $f(2) = \ln(15)$ abs. max

$f(-1) = \ln(6)$ abs. min (also loc. min) example: $f(x) = x e^{-x^2/8}$ $[-1, 4]$

$$f'(x) = x \cdot e^{-x^2/8} \cdot -\frac{1}{4}x + e^{-x^2/8} \cdot 1$$

$$f'(x) = e^{-x^2/8}\left(-\frac{1}{4}x^2 + 1\right)$$



$f'(x) = 0$ $\quad e^{-x^2/8}\left(-\frac{1}{4}x^2+1\right) = 0$

$e^{\text{anything}} \neq 0$ ever

$$-\frac{1}{4}x^2 + 1 = 0$$

$x^2 - 4 = 0$ $\quad \cancel{x = -2}$ $\quad x = 2$

$f'(x)$ DNE $\longrightarrow$ Never

(exponential defined on $(-\infty, \infty)$
polynomial " " ")

$f(-1) = -e^{-1/8} = -\frac{1}{e^{1/8}}$ $\longrightarrow$ abs. min

$f(2) = 2e^{-1/2} = \frac{2}{e^{1/2}} = 1.21$ $\rightarrow$ abs. max and loc. max

$f(4) = 4e^{-2} = \frac{4}{e^{2}} = 0.54$