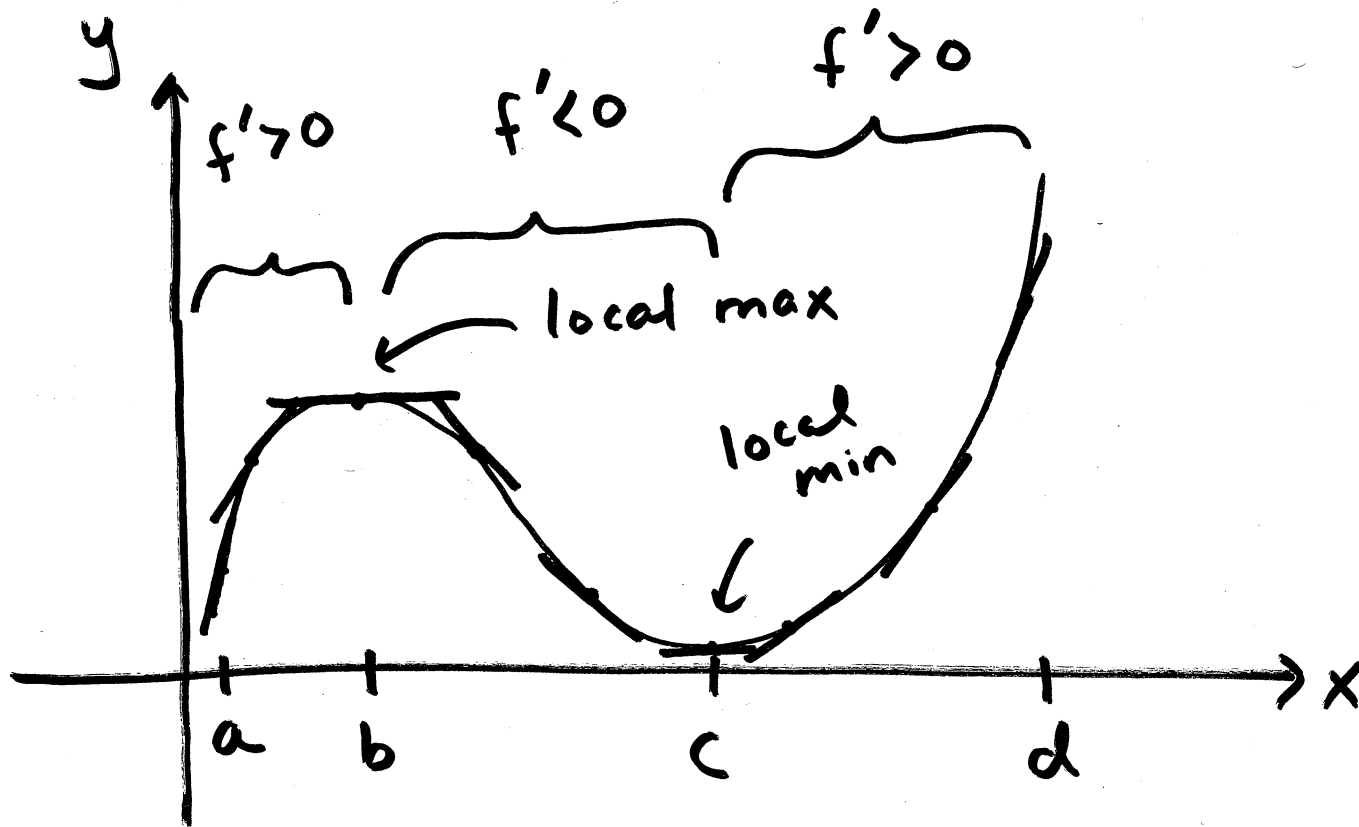


4.3 Derivatives and Shape of Graphs



increasing on intervals $(a, b) \cup (c, d) \rightarrow f' > 0$
decreasing on interval $(b, c) \rightarrow f' < 0$

The First Derivative Test

local max/min at critical numbers $f'=0$ or DNE

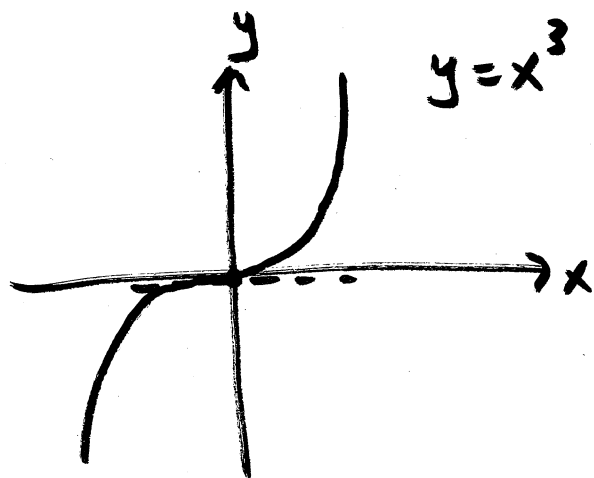
If $f' < 0$ BEFORE $x=c$ $f' > 0$ AFTER

→ local min at $x=c$

If $f' > 0$ BEFORE $x=c$ $f' < 0$ AFTER

→ local max at $x=c$

If f' does not change sign → neither max nor min



example $f(x) = x^4 - 8x^2 + 2$

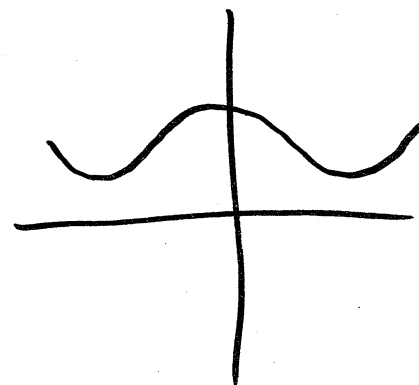
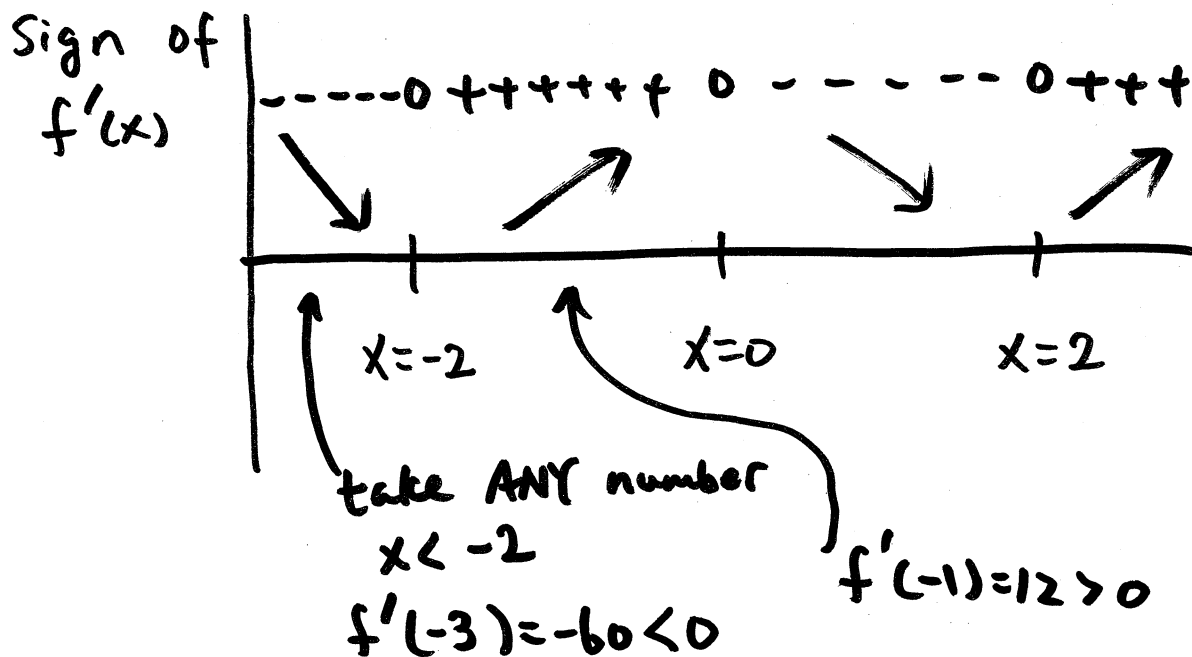
Find: intervals of inc/dec
local max/min

Find critical numbers \rightarrow not inc/dec

$$f'(x) = 4x^3 - 16x = 0$$

$$4x(x^2 - 4) = 0$$

$$x = -2, 0, 2$$



inc: $(-2, 0) \cup (2, \infty)$

dec: $(-\infty, -2) \cup (0, 2)$

↙ max value

local max: at $x=0$ $f(0) = 2$

↑
location

local min: at $x = -2$ $f(-2) = -14$

at $x = 2$ $f(2) = -14$

example

$$f(x) = 4x^3 + 3x^2 - 6x + 9$$

find: inc/dec intervals

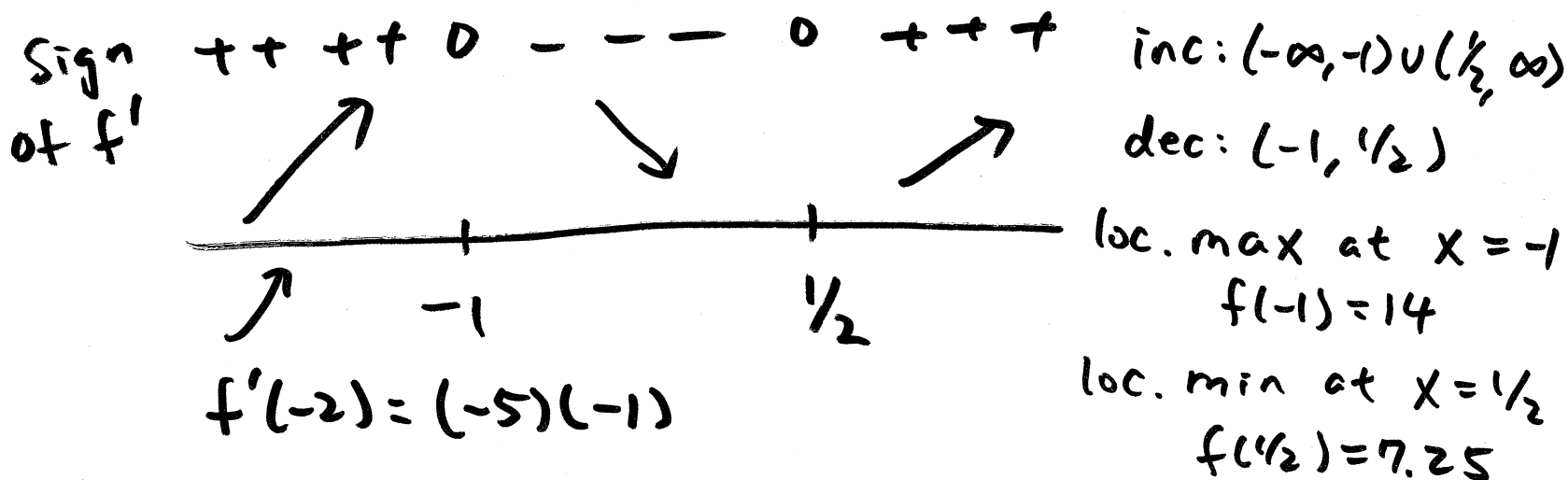
local max/min

$$f'(x) = 12x^2 + 6x - 6 = 0$$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

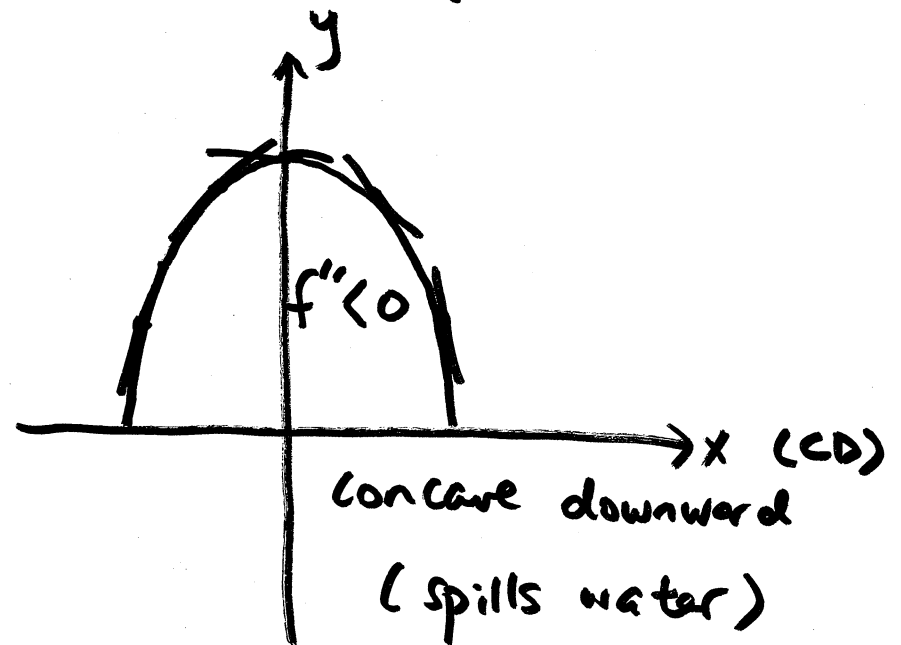
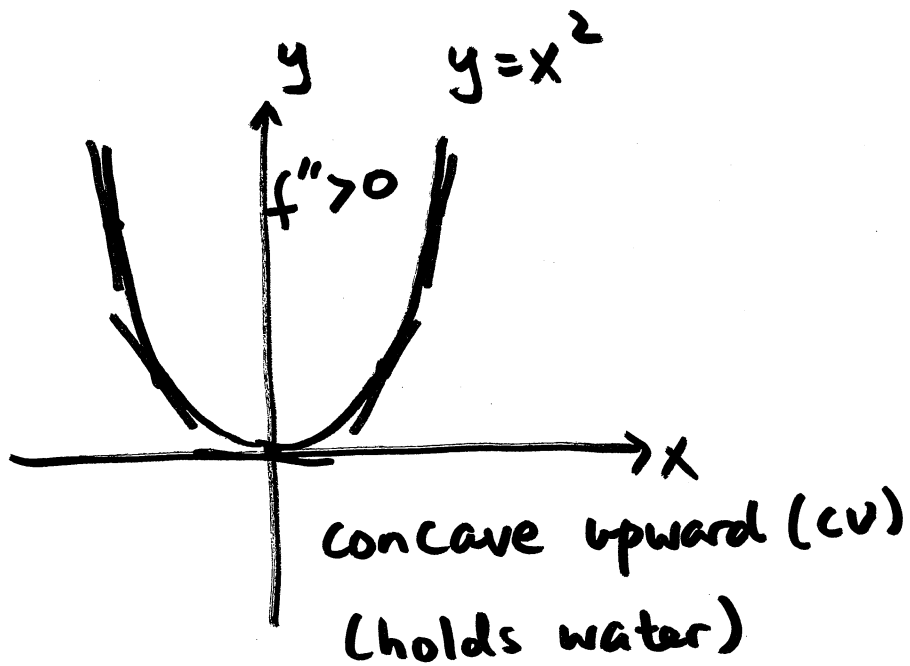
$$x = 1/2, \quad x = -1$$

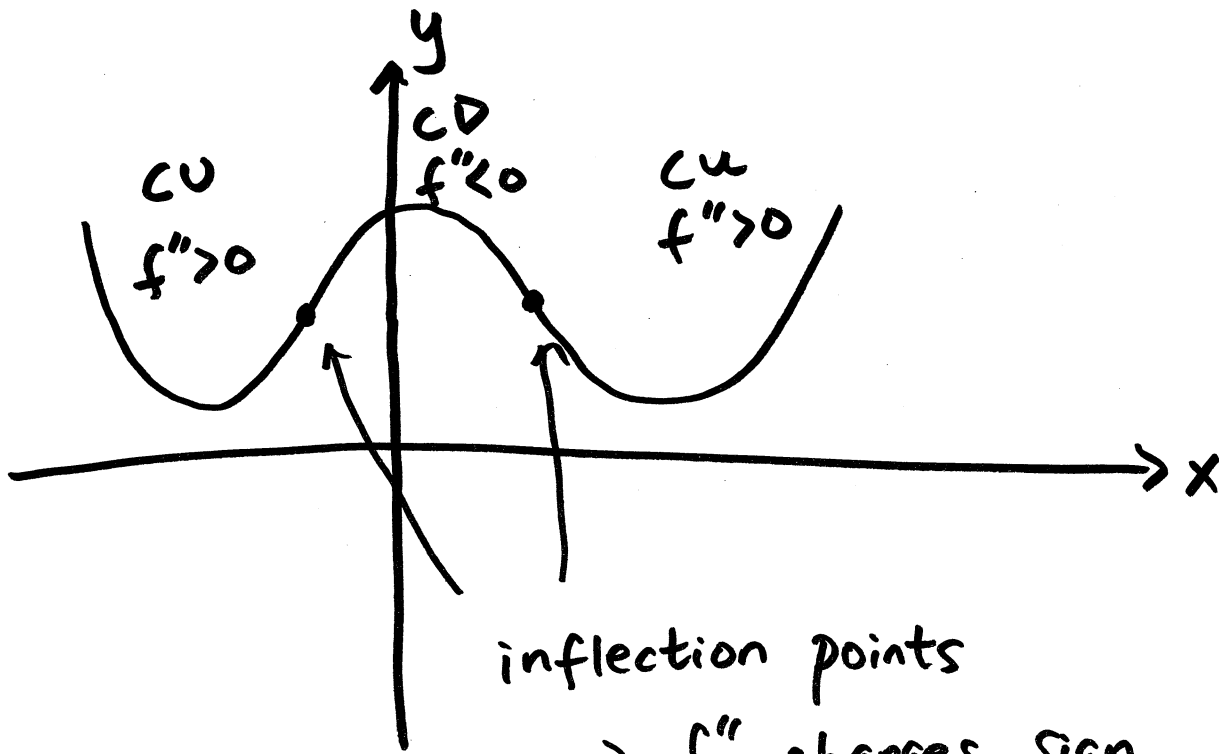


How does f'' affect shape?

$f'' = (f')'$ → rate of change of slope of tangent line

$f'' > 0$ $(f')' > 0$ → tangent line slope is increasing





inflection points

→ f'' changes sign

($f'' = 0$ or DNE)

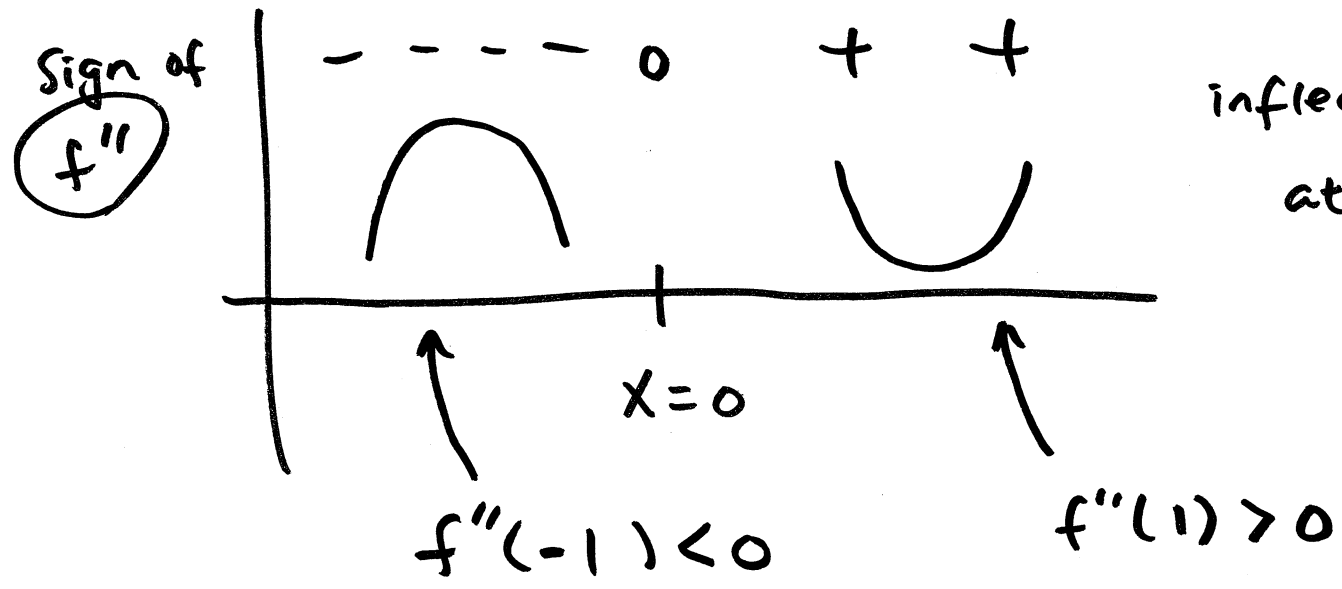
Very similar
to critical &
numbers

Example $f(x) = x^3 - 12x + 2$

find: intervals of CU / CD
inflection point(s)

$$f'(x) = 3x^2 + 12$$

$$f''(x) = 6x = 0 \rightarrow x = 0 \text{ where } f \text{ is neither CU nor CD}$$



inflection point
at $x=0, y=f(0)=2$