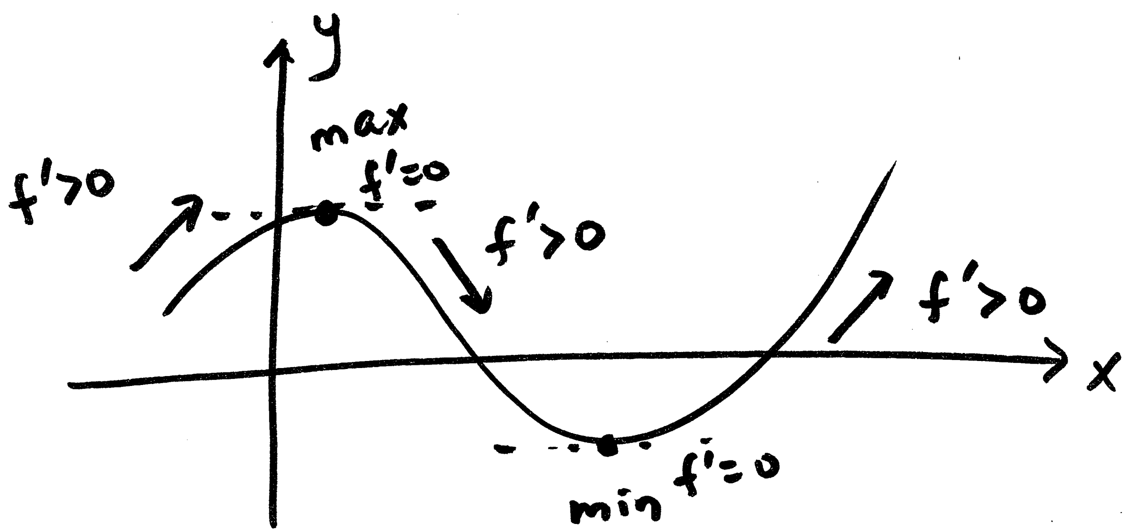
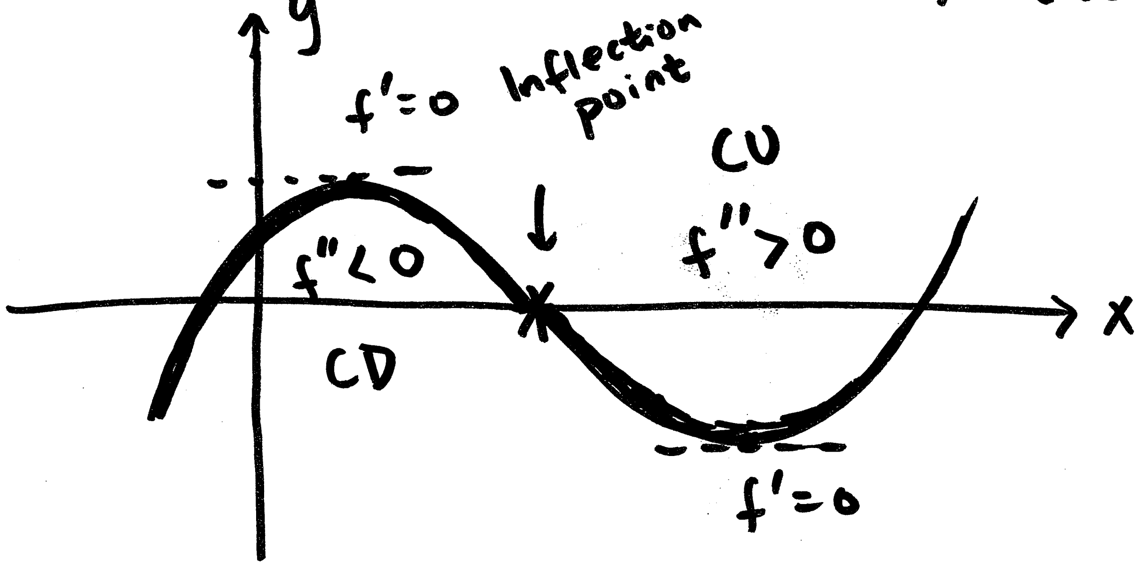
4.3 Derivatives and Shapes (part 2)

First Derivative Test → use f' (slope) for max/min



Second Derivative Test → use f'' (concavity) for max/min

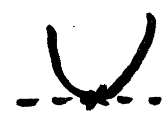


Second Deriv. Test

1. Find critical numbers ($f' = 0$ or f' DNE)

2. Check concavity near each critical #
   by evaluating f''

   a. If $f'' < 0$ $\longrightarrow$ local max 

   b. If $f'' > 0$ $\longrightarrow$ local min 

   c. If $f'' = 0$ $\longrightarrow$ test fails

   Still could be max/min
   but this test can't tell.

<u>example</u> Use the 2DT to find local max/min of $f(x) = 2 + 3x^2 - 2x^3$

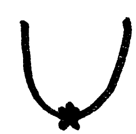
Find CN : $f' = 0$

$$f'(x) = 6x - 6x^2 = 0$$

$$6x(1 - x) = 0$$

$$x = 0, \quad x = 1$$

Check concavity (f'') $\qquad f''(x) = 6 - 12x$

$f''(0) = 6 > 0 \rightarrow$ CU  local min

$f''(1) = -6 < 0 \rightarrow$ CD  local max

local min : $f(0) = 2$

local max : $f(1) = 3$

<u>example</u>

$f(x) = x^5 \cdot \ln x$ Use $\underline{\underline{\text{2DT}}}$

Domain: $(0, \infty)$

$$f'(x) = x^5 \cdot \frac{1}{x} + (\ln x) \cdot (5x^4)$$

$$= x^4 + 5x^4 \ln x = 0$$

$$x^4 (1 + 5 \ln x) = 0$$

$$x^4 = 0 \quad \text{or} \quad 1 + 5\ln x = 0$$

$\cancel{x = 0}$ $\ln x = -\frac{1}{5}$

not in domain of $f(x)$

$$x = e^{-1/5} > 0$$

in domain, keep

$$f'(x) = x^4 + 5x^4 \ln x$$

$$f''(x) = 4x^3 + 5x^4 \cdot \frac{1}{x} + 20x^3 \ln x$$

$$= 4x^3 + 5x^3 + 20x^3 \ln x$$

$$= 9x^3 + 20x^3 \ln x$$

$$= x^3 (9 + 20 \ln x)$$

CN: $x = e^{-1/5}$

$$-\frac{1}{5}$$

$$f''(e^{-1/5}) = (e^{-1/5})^3 (9 + 20 \ln e^{-1/5})$$

$$= e^{-3/5} (9 - 4) = 5e^{-3/5} > 0$$

CU

local min at $x = e^{-1/5}$

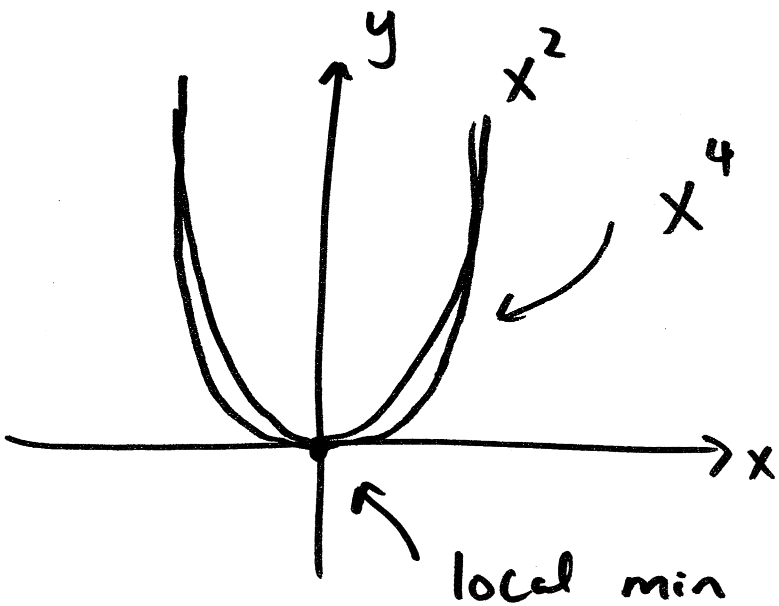
local min value is $f(e^{-1/5}) = -\frac{1}{5e}$

example $f(x) = x^4$

$f'(x) = 4x^3 = 0$ CN: $x = 0$

$f''(x) = 12x^2$

$f''(0) = 0$ 2DT fails



Fall back to 1DT

sign of f' $- - - -$ 0 $+ + + +$

$x=0$

1DT says:

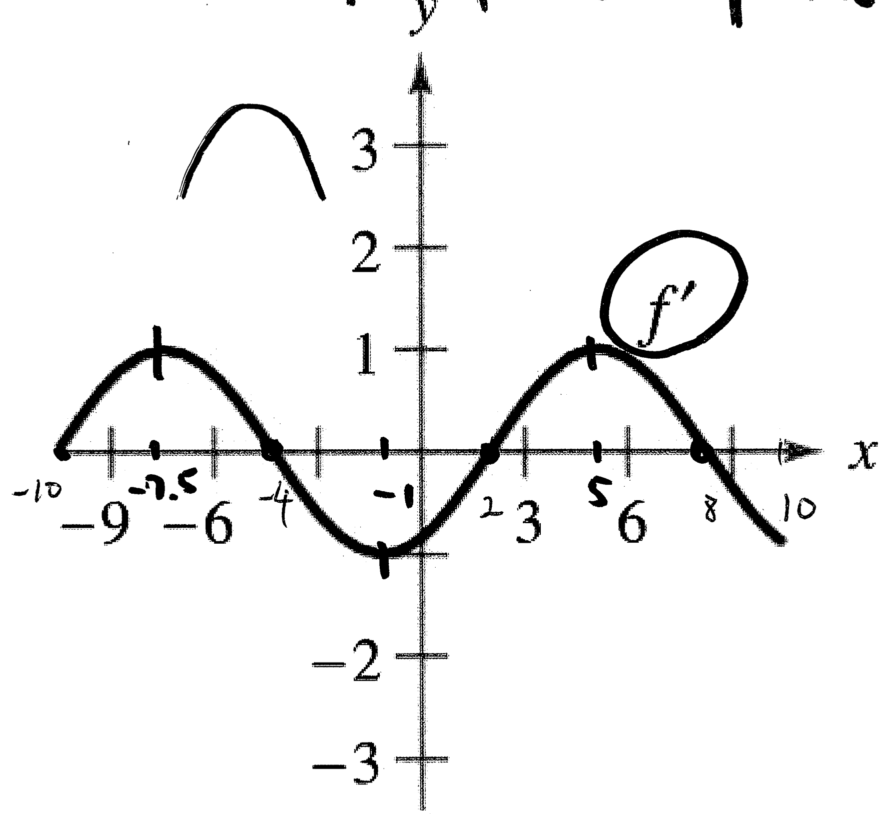
local min at $x=0$

Find: intervals of inc / dec (for f)

x values of local max / min

intervals of CU / CD (for f)

x value(s) of inflection point(s)



Inc: $f' > 0 \rightarrow$ above x-axis

$(-10, -4) \cup (2, 8)$

dec: $(-4, 2) \cup (8, 10)$

local max @ -4, 8

local min @ 2

$f' < 0$ before
$f' > 0$ after

CU: $f'' > 0 \rightarrow (f')' > 0 \rightarrow f'$ increasing

$(-10, -7.5) \cup (-1, 5)$

CD: $f'' < 0 \rightarrow (f')' < 0 \rightarrow f'$ decreasing

$(-7.5, -1) \cup (5, 10)$

IP: f'' changes sign $\rightarrow f'$ from inc to dec or vice versa @ $-7.5, -1, 5$