

4.5 Curve Sketching

NOT on exam 3

Guidelines

- A. Domain
 - B. Intercepts ← useful
 - C. Symmetry
 - D. Asymptotes ← useful
 - E. Increasing/decreasing intervals
 - F. Local max/min
 - G. Concave up/down, inflection pts
- } useful

example $y = x^3 - 18x^2 + 81x$

A. Domain: $(-\infty, \infty)$

B. Intercepts:

x-int \rightarrow make $y = 0$

x-ints: $x = 0, x = 9$

$$0 = x^3 - 18x^2 + 81x$$

$$= x(x^2 - 18x + 81)$$

$$= x(x - 9)(x - 9)$$

y-int \rightarrow make $x = 0$

y-int: $y = 0$

$$y = 0$$

C. Symmetry \rightarrow typical for even-powered polynomials (x^2, x^4, etc)

don't spend time on this

D. Asymptotes : none

(typically only for rational functions)

E. Inc/dec

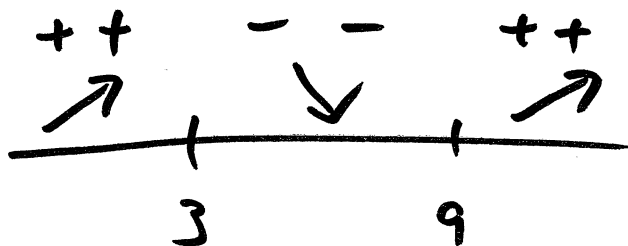
$$y = x^3 - 18x^2 + 81x$$

$$y' = 3x^2 - 36x + 81 = 0$$

$$x^2 - 12x + 27 = 0$$

$$(x - 3)(x - 9) = 0$$

$$\text{CN: } x = 3, x = 9$$



F. Local max/min

$$\text{max: } (3, 108)$$

$$\text{min: } (9, 0)$$

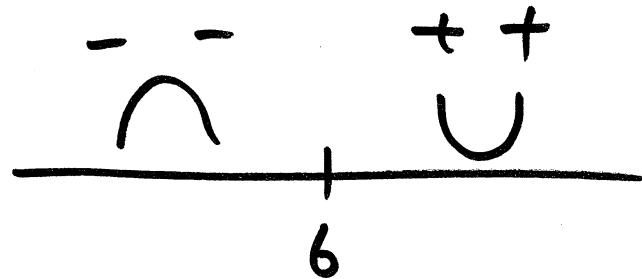
G. CU/CD

$$y = x^3 - 18x^2 + 81x$$

$$y' = 3x^2 - 36x + 81$$

$$y'' = 6x - 36 = 0 \rightarrow x = 6$$

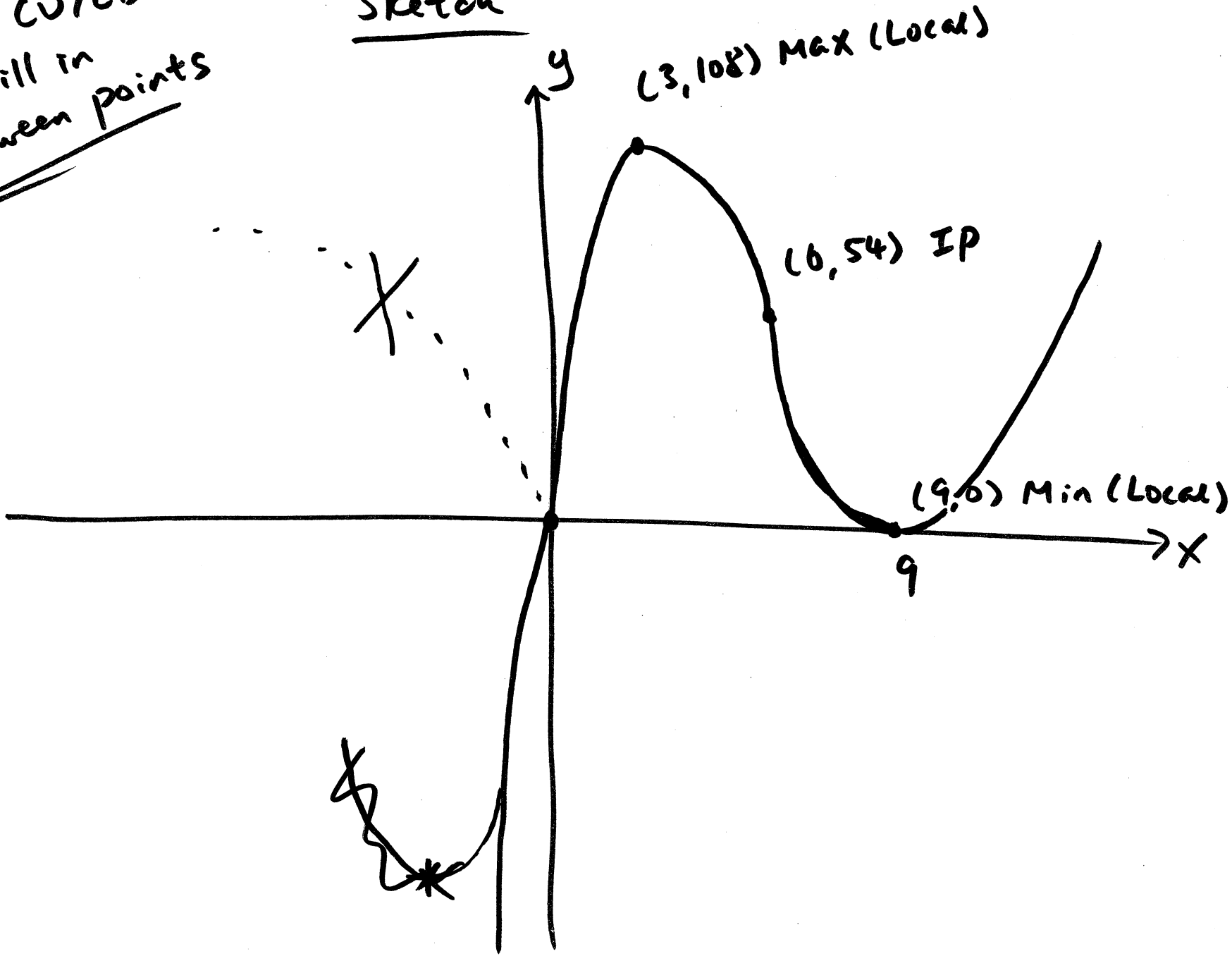
possible
inflection pt



$$IP: (6, 54)$$

use CV/CD
to fill in
between points

Sketch



example

$$y = \frac{x}{x-1}$$

A. Domain: $(-\infty, 1) \cup (1, \infty)$ ←

B. x-int: $0 = \frac{x}{x-1} \rightarrow$ x-int: $x=0$

y-int: $y = \frac{0}{-1} \rightarrow$ y-int: $y=0$

C. Symmetry skip

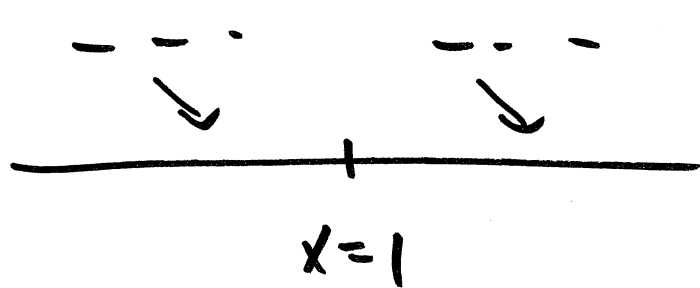
D. Asymptotes: horiz. $\rightarrow \lim_{x \rightarrow \infty} \frac{x}{x-1} \stackrel{L}{=} \lim_{x \rightarrow \infty} \frac{1}{1} = 1$
vert. $\rightarrow x=1$

E. Inc/dec: $y' = \frac{\cancel{1}}{(1-x)^2} = \frac{-1}{(x-1)^2}$

CN: $x=1$ (y' DNE)

↳ local max/min?

NO, not in domain!

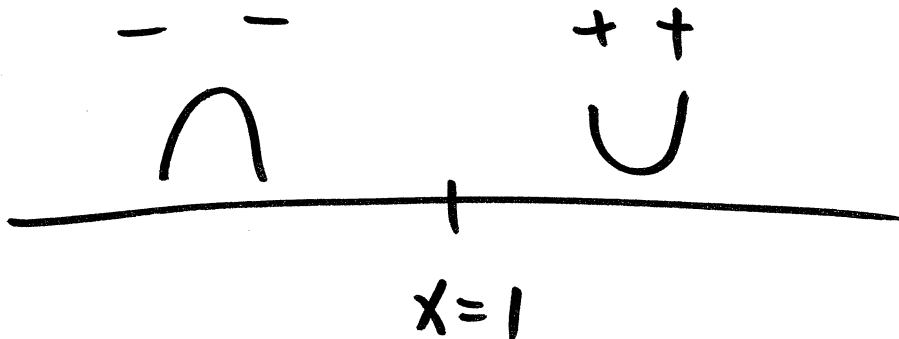


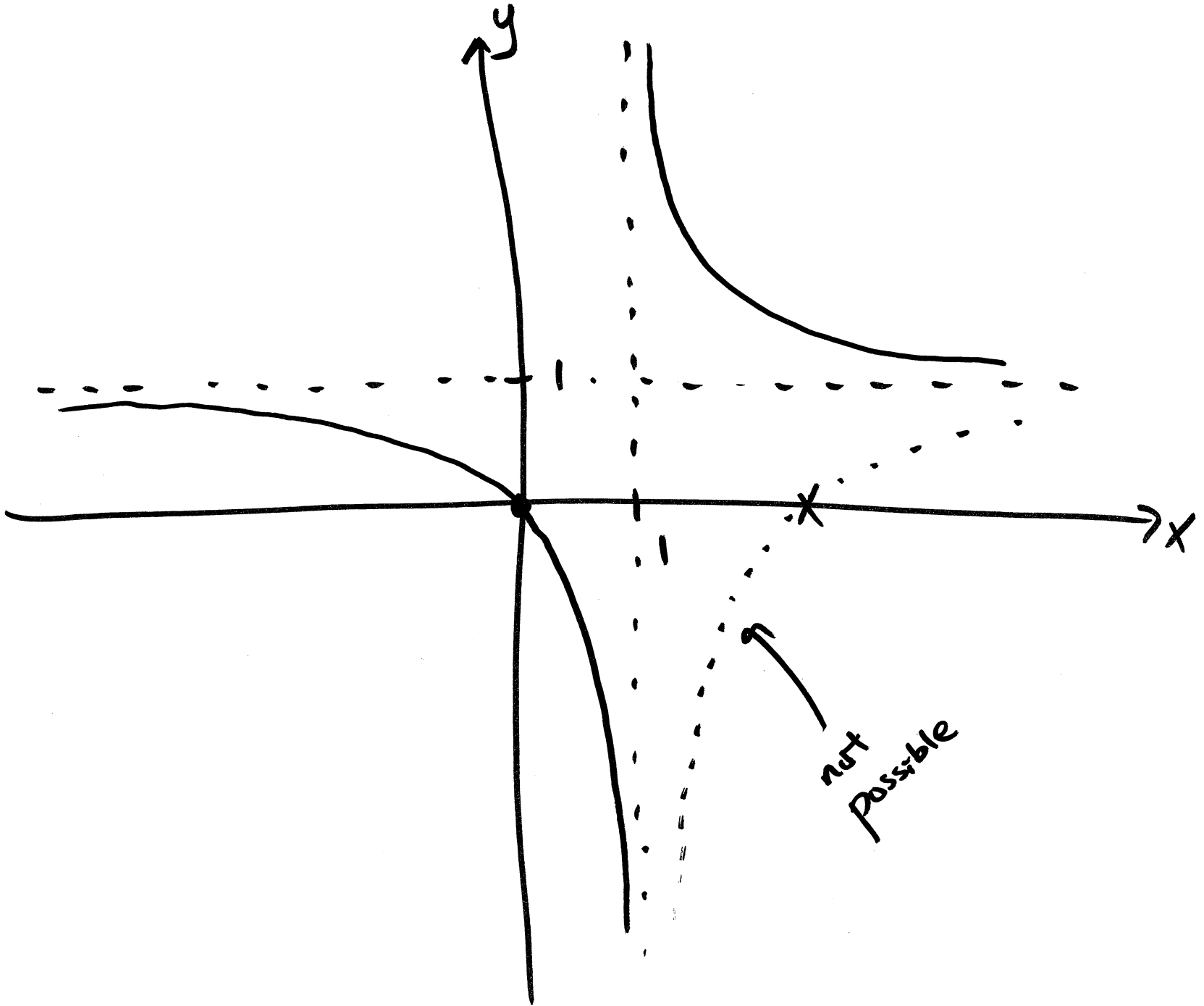
$$y' = \frac{-1}{(x-1)^2} < 0 \text{ for all } x$$

G. CU/CD $y'' = \frac{2}{(x-1)^3}$

$$y'' = 0 \rightarrow \text{never}$$

$$y'' \text{ DNE} \rightarrow x=1 \quad \text{IP? No!}$$





example

$$y = x + \cos x$$

Domain: $(-\infty, \infty)$

X-int: $0 = x + \cos x$ not solvable by hand

$$x = -\cos x$$

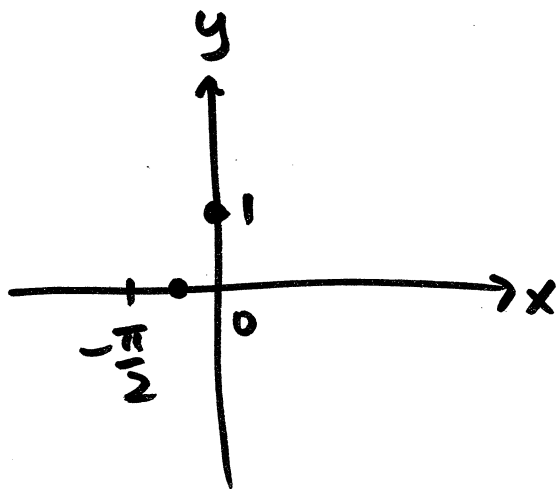
narrow down location

$$y = x + \cos x \quad \text{at } x = -\frac{\pi}{2}, y = -\frac{\pi}{2}$$

$$\text{at } x = 0, y = 1$$

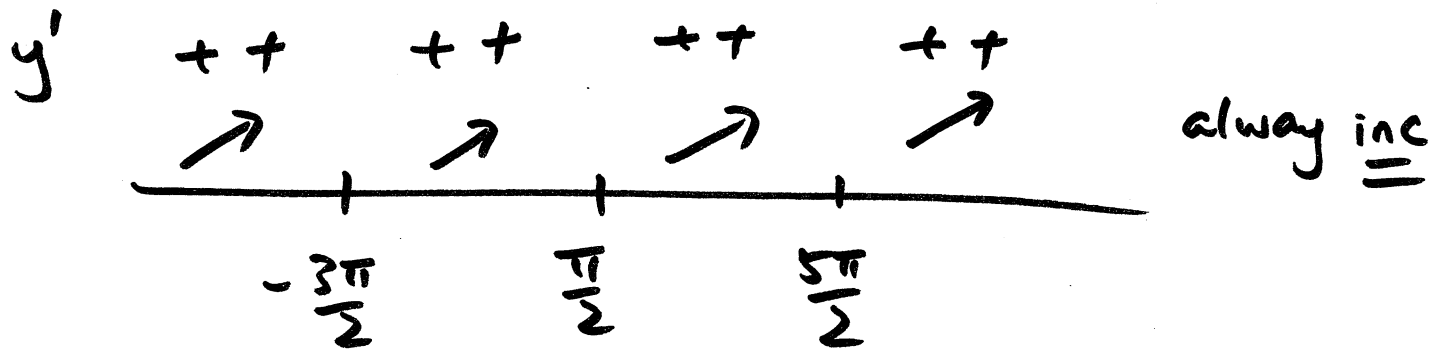
X-int is between $-\frac{\pi}{2}$ and 0

$$y\text{-int: } y = 1$$



inc/dec: $y' = 1 - \sin x = 0$ $\sin x = 1$

CN: $x = \frac{\pi}{2} + 2n\pi$
 $= -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \text{ etc}$



local max/min? NONE

cu/cd

$y'' = -\cos x = 0$ $x = \frac{\pi}{2} + n\pi$

