4.7 Optimization Problems (part 1)

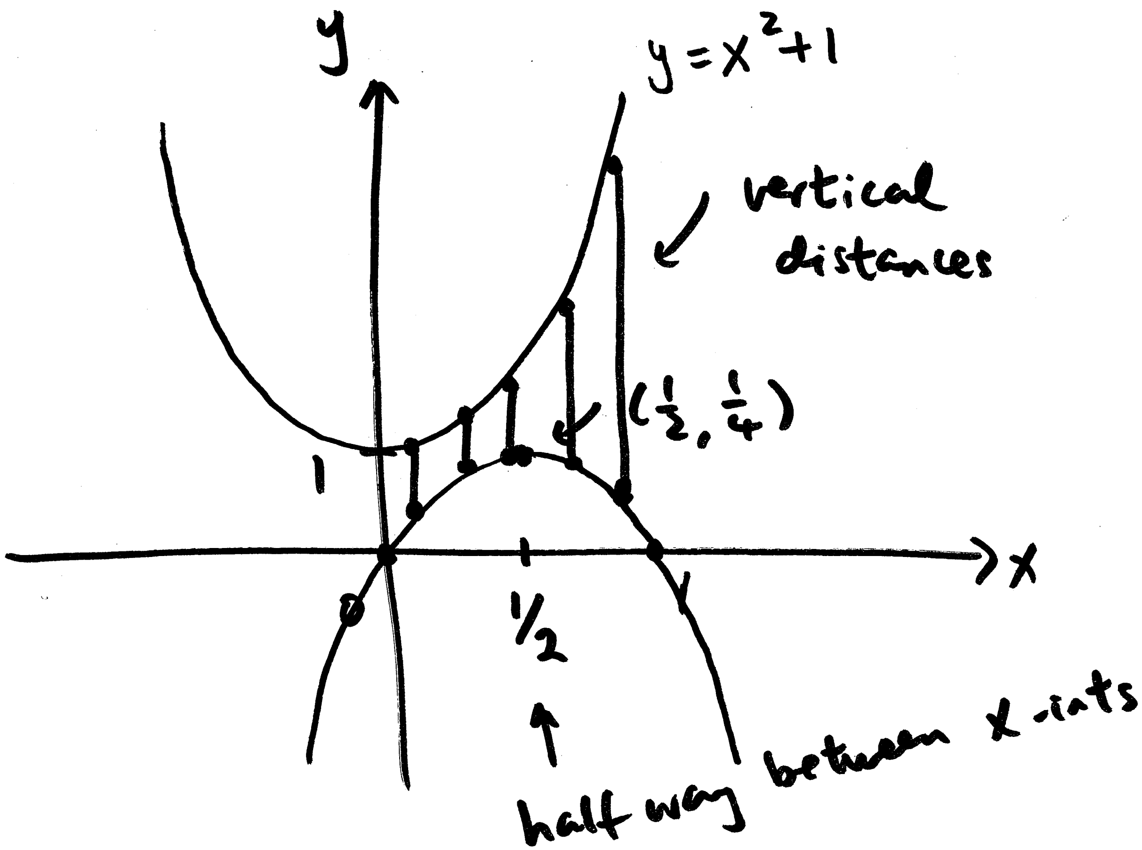
NOT on exam 3

Applying First/Second Deriv. Test to

application problems.

- Example 1. What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?

Whenever possible, sketch.



$y = x - x^2$

find x-ints

$0 = x - x^2$

$= x(1 - x)$

$x = 0, \quad x = 1$

$y(\frac{1}{2}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

express vertical distance as function of x

$$V(x) = \underbrace{(x^2+1)}_{\text{top}} - \underbrace{(x-x^2)}_{\text{bottom}}$$

$$= x^2+1-x+x^2 = 2x^2-x+1 = V(x)$$

find x to minimize $V(x)$

$$V'(x) = 4x-1 = 0 \qquad x = \frac{1}{4} \quad \text{critical \#}$$

First Deriv. Test

V' $-\ -$ 0 $+\ +$

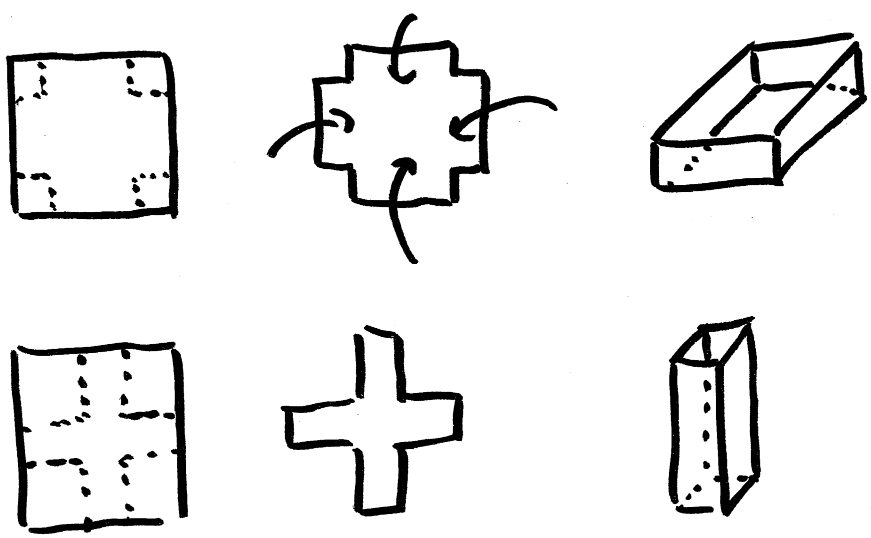
$\searrow$ $\tfrac{1}{4}$ $\nearrow$

minimum V at $x = 1/4$

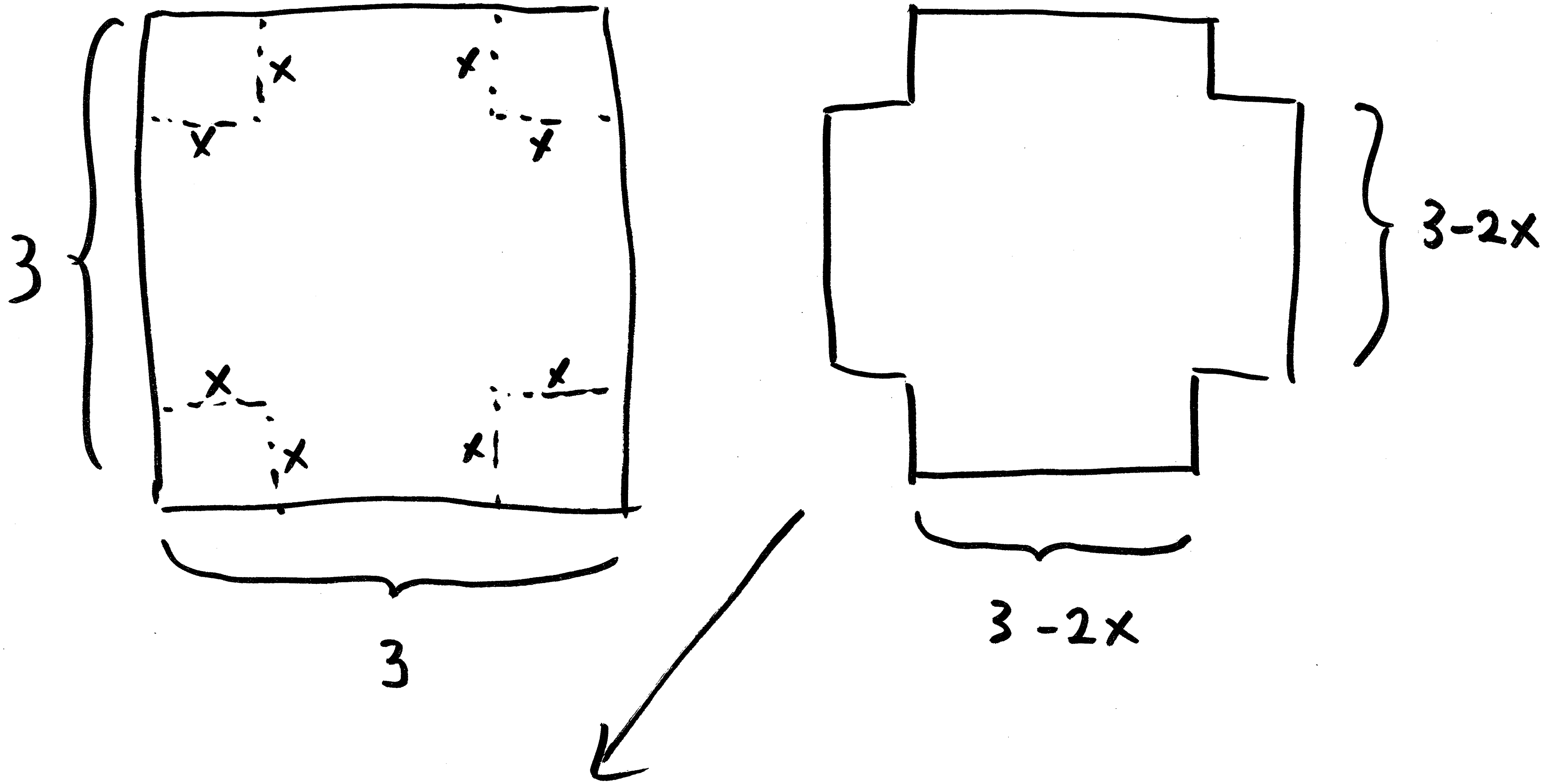
the min V is

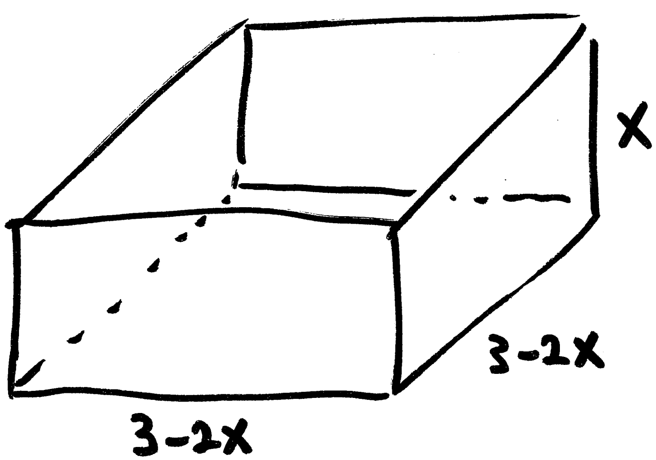
$$V\left(\tfrac{1}{4}\right) = \boxed{\frac{7}{8}}$$

- Example 2. A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.



how to cut to get the largest box by volume?





$$V(x) = x\,(3-2x)^2$$

volume.

$V(x) = x(3-2x)^2$ domain?

$0 \leq x \leq 3/2$

closed-interval problem

find CN. $x = c$

$V(0)$

$V(c)$

$V(3/2)$

$V(x) = 4x^3 - 12x^2 + 9x$

$V'(x) = 12x^2 - 24x + 9 = 0$

$4x^2 - 8x + 3 = 0$

$(2x-3)(2x-1) = 0$

CN: $x = 3/2$, $x = 1/2$

$V(0) = 0$

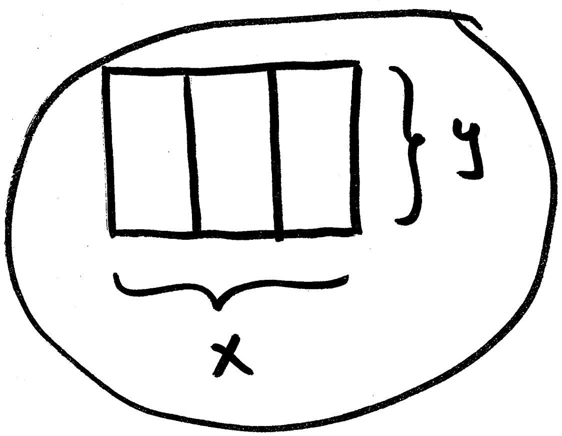
$V(1/2) = 2$

$V(3/2) = 0$

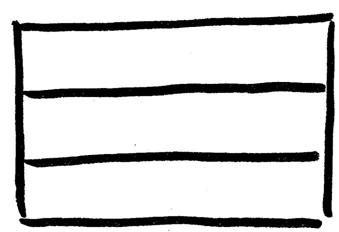
max volume is 2 ft^3

cut away $1/2 \times 1/2$ squares

- Example 3. A farmer with 20 ft of fencing would like to enclose a rectangular region and then divide it into 3 pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the 3 pens?



or



either is fine when setting up.

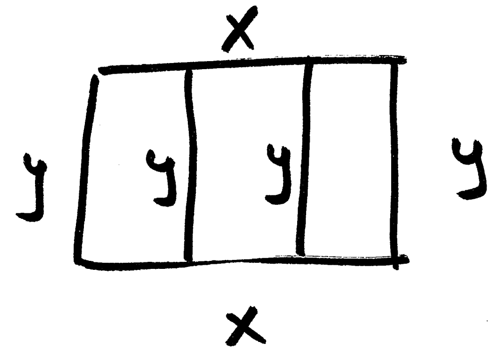
$$A = 3\cdot(y)\left(\frac{x}{3}\right) = xy$$

assume fence occupies no area

two variables
one has to be eliminated

relationship between x and y ⇒ equation

total fencing : 20 ft

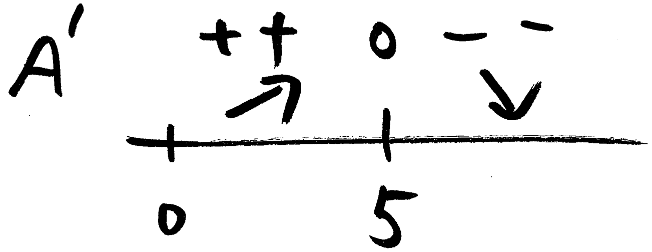


$$20 = 2x + 4y$$

$$10 = x + 2y \longrightarrow 2y = 10 - x \rightarrow y = 5 - \frac{x}{2}$$

$$A = xy = x\left(5 - \frac{x}{2}\right) = 5x - \frac{1}{2}x^2$$

$$A' = 5 - x = 0 \qquad x = 5$$

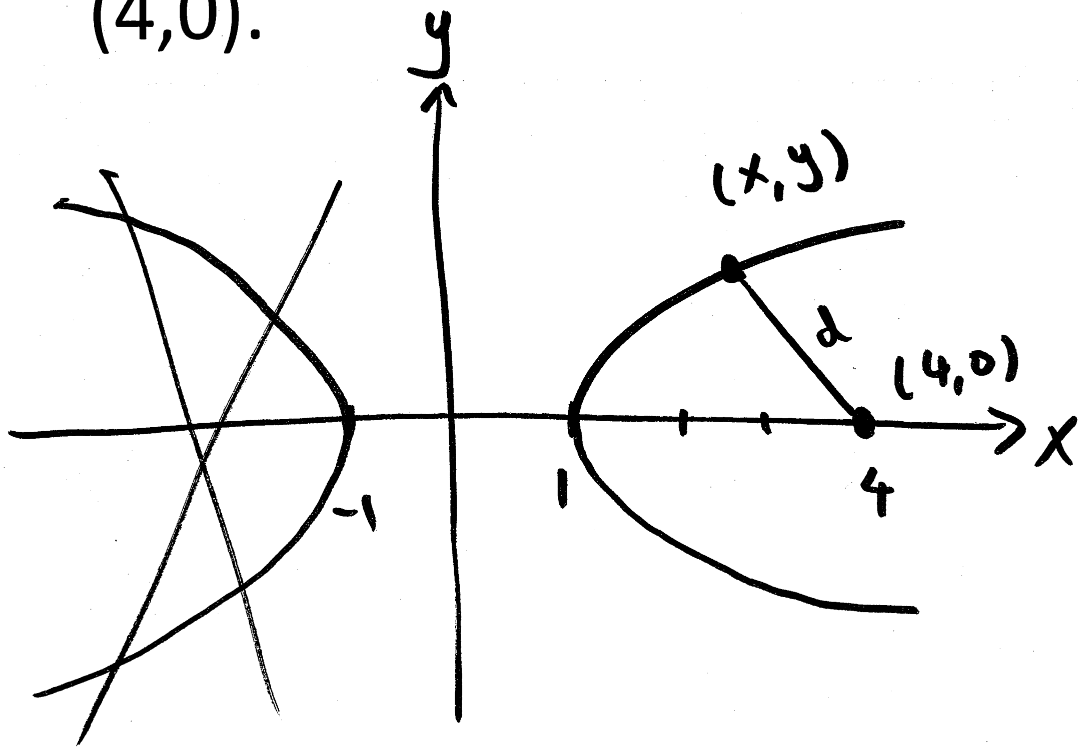


max A is when $x = 5$

$y = \frac{5}{2}$

max $A = \frac{25}{2}$ ft^2

- Example 4. Find the least distance between the hyperbola $\boxed{x^2 - y^2 = 1}$ and the point (4,0).



$$d = \sqrt{(x-4)^2 + (y-0)^2}$$

$$d = \sqrt{(x-4)^2 + y^2}$$

two variables again

from $x^2 - y^2 = 1$

$$y^2 = x^2 - 1$$

$d = \sqrt{(x-4)^2 + (x^2-1)}$ root $\rightarrow$ deriv. is messy

Since $d \geq 0$ and $d^2 \geq 0$

can minimize d^2 instead of d

$S = d^2 = (x-4)^2 + x^2 - 1$

$S' = \cdots = 0$