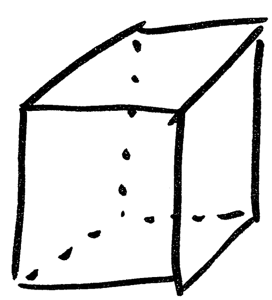
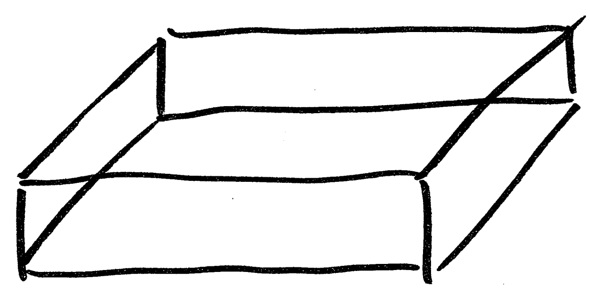
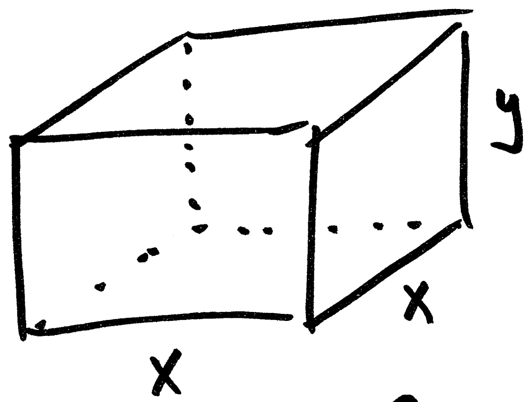
4.7 Optimization Problems (part 2)

Example 1

- A rectangular storage container with a closed top and a square base is to have a volume of 10 m^3. Material for the base costs \$5 per square meter. Material for the lid and the sides costs \$3 per square meter. Find the dimensions of the cheapest possible box.

since base is a square

volume = 10

$$x^2 y = 10$$

quantity to max/min → minimize cost

$$C = \underbrace{(x^2)(5)}_{\text{base}} + \underbrace{(x^2)(3)}_{\text{top}} + 4\underbrace{(x)(y)(3)}_{\text{each side}}$$

4 sides

$$\boxed{C = 8x^2 + 12xy}$$

two variables → choose to eliminate the one appears least often from $x^2y = 10 \rightarrow y = \frac{10}{x^2}$

$$C = 8x^2 + 12x\left(\frac{10}{x^2}\right)$$

$$C = 8x^2 + 120x^{-1}$$

$$C' = 16x - 120x^{-2} = 0$$

$$16x = \frac{120}{x^2}$$

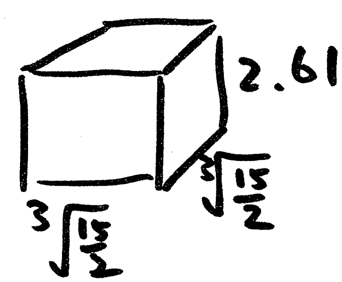
$$x^3 = \frac{120}{16} = \frac{15}{2}$$

$$x = \sqrt[3]{\frac{15}{2}} \approx 1.96$$

$$C'' = 16 + 240x^{-3}$$

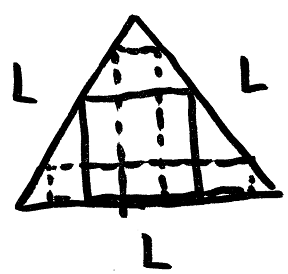
$C''\left(\sqrt[3]{\frac{15}{2}}\right) > 0$ so C is minimized when $x = \sqrt[3]{\frac{15}{2}}$

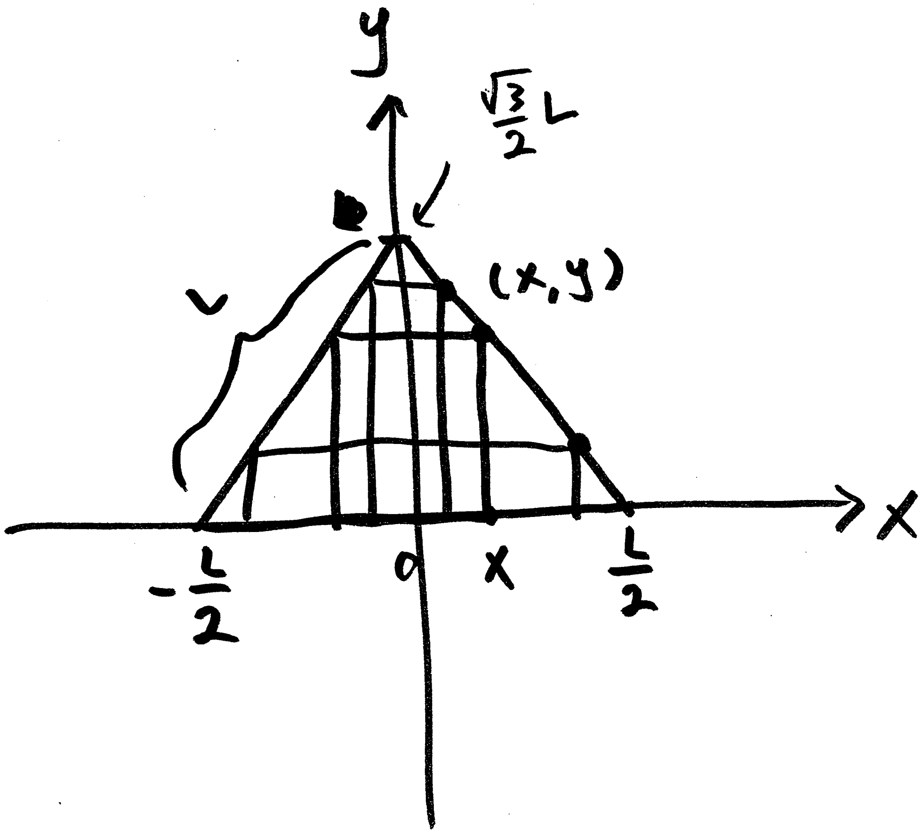
$y \approx 2.61$



Example 2

- Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.





height of rectangle : y

width of rectangle : $2x$

y-intercept of triangle?

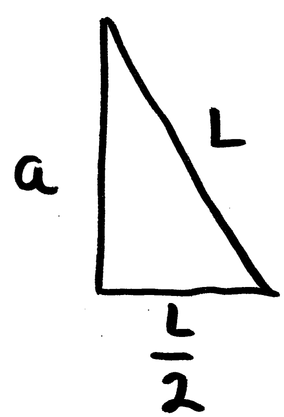
rectangle area

$$A = 2xy$$

two ways to relate x, y

→ Similiar triangles

→ equation of line that contains the upper right corner of rectangle

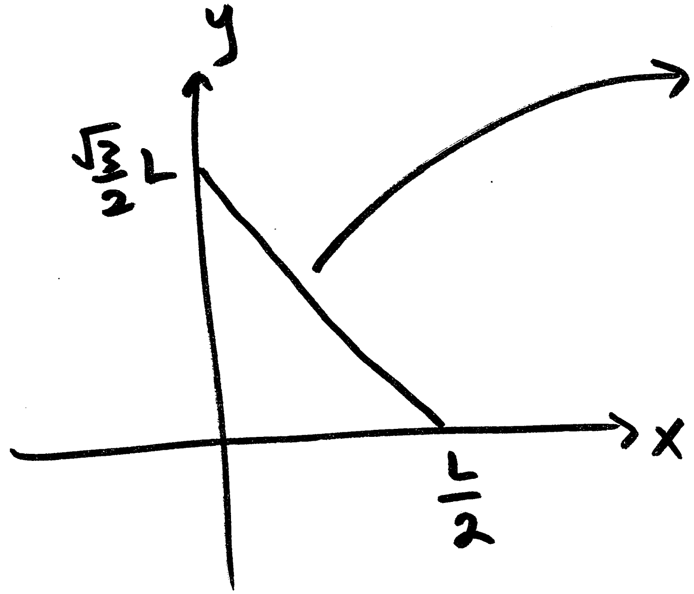


$$a^2 + \left(\frac{L}{2}\right)^2 = L^2$$

$$a^2 + \frac{1}{4}L^2 = L^2$$

$$a^2 = \frac{3}{4}L^2$$

$$a = \frac{\sqrt{3}}{2}L$$



y-int: $\frac{\sqrt{3}}{2}L$

$$m = \frac{-\frac{\sqrt{3}}{2}L}{\frac{1}{2}L}$$

$$m = -\sqrt{3}$$

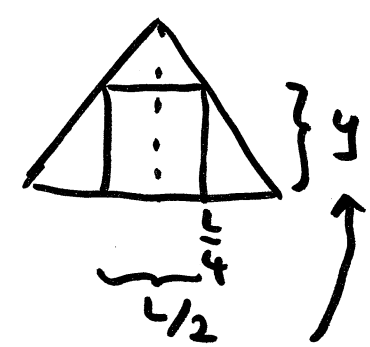
equation: $y = -\sqrt{3}\,x + \frac{\sqrt{3}}{2}L$

$$A = 2xy = 2x\left(-\sqrt{3}\,x + \frac{\sqrt{3}}{2}L\right) = \cancel{-2\sqrt{3}\,x^2 + \frac{\sqrt{3}}{2}L\,x}$$

$$= -2\sqrt{3}\,x^2 + (\sqrt{3}L)x$$

$$A' = -4\sqrt{3}\,x + \sqrt{3}L = 0$$

$$x = \frac{\sqrt{3}L}{4\sqrt{3}} = \frac{1}{4}L$$

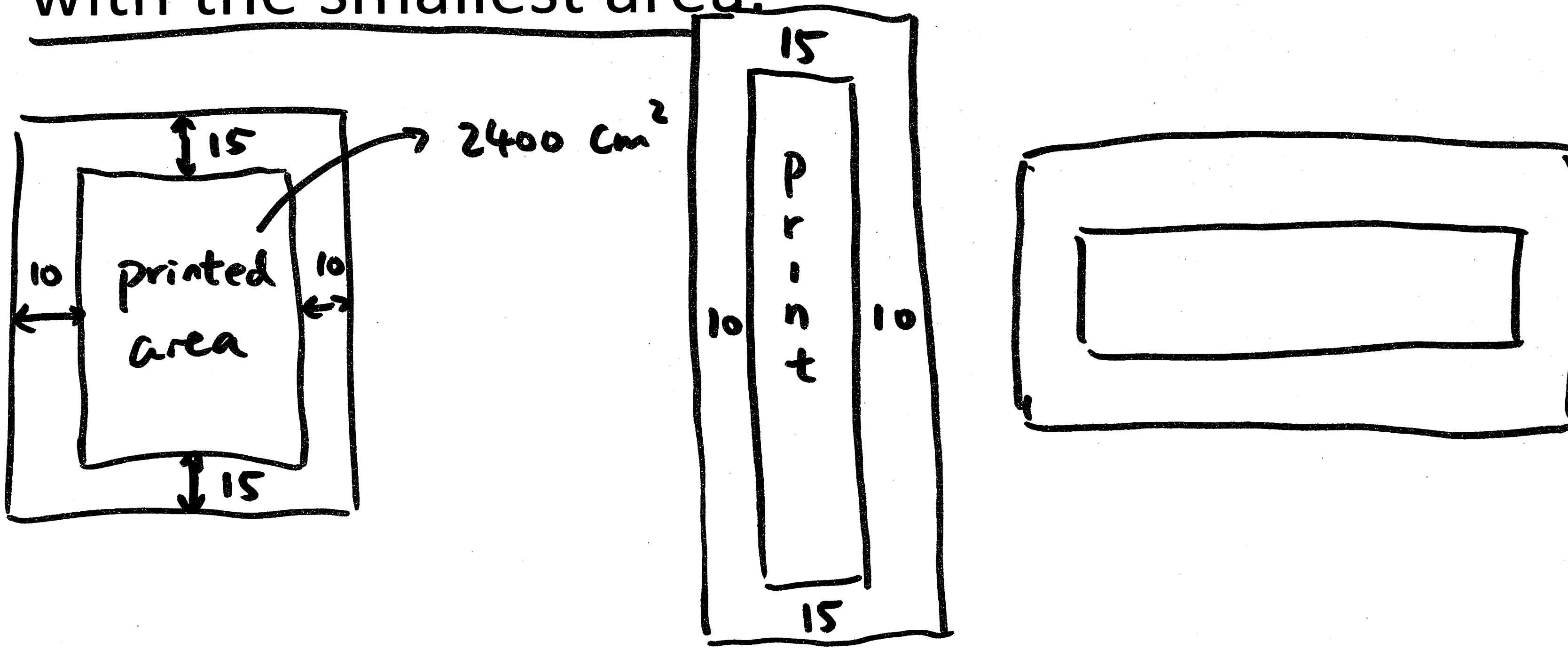


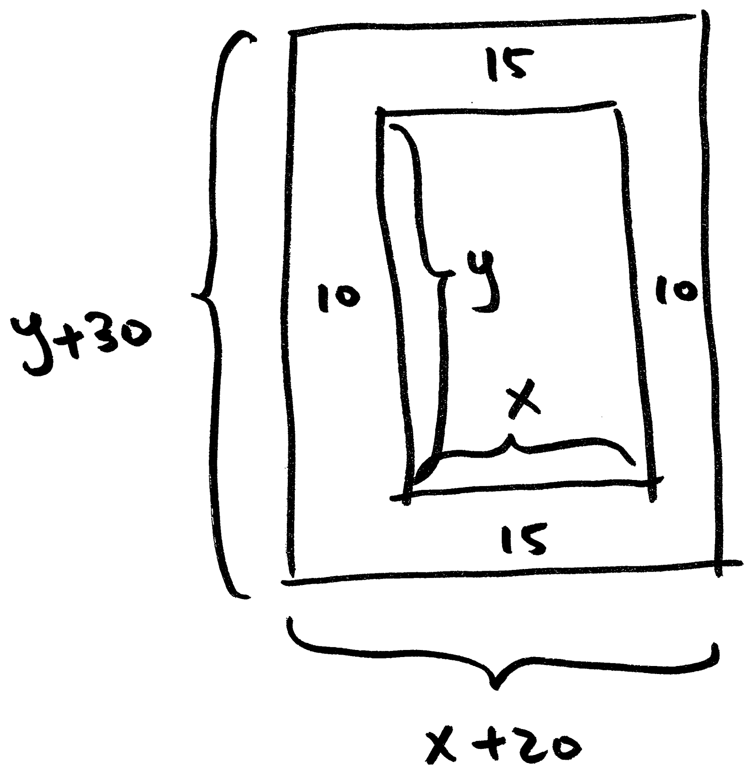
$$A'' = -4\sqrt{3} < 0 \rightarrow \text{max } A \text{ at } x = \boxed{\frac{1}{4}L}$$

$$y = -\frac{\sqrt{3}}{4}L + \frac{\sqrt{3}}{2}L = \boxed{\frac{\sqrt{3}}{4}L}$$

Example 3

- The top and bottom margins of a poster are each 15 cm and the side margins are each 10 cm. If the area of printed material is fixed at 2400 cm^2, find the dimensions of the poster with the smallest area.





$A_{print} = xy = 2400$

$A_{poster} = (x+20)(y+30)$

$y = \frac{2400}{x}$

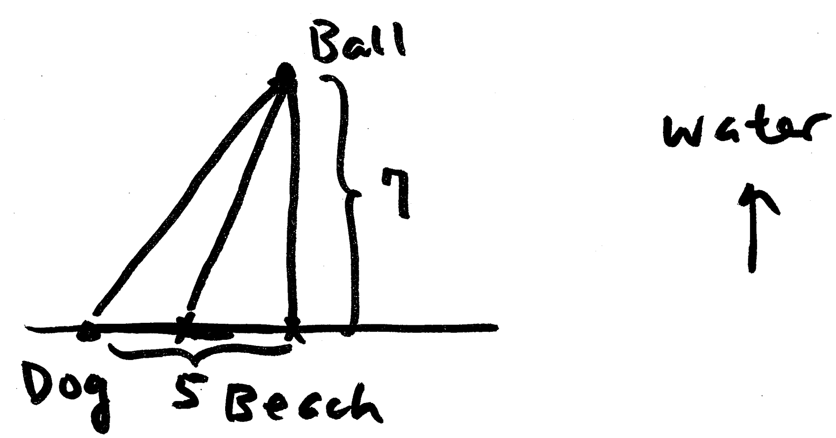
$$\boxed{A_{poster} = (x+20)\left(\frac{2400}{x} + 30\right)}$$

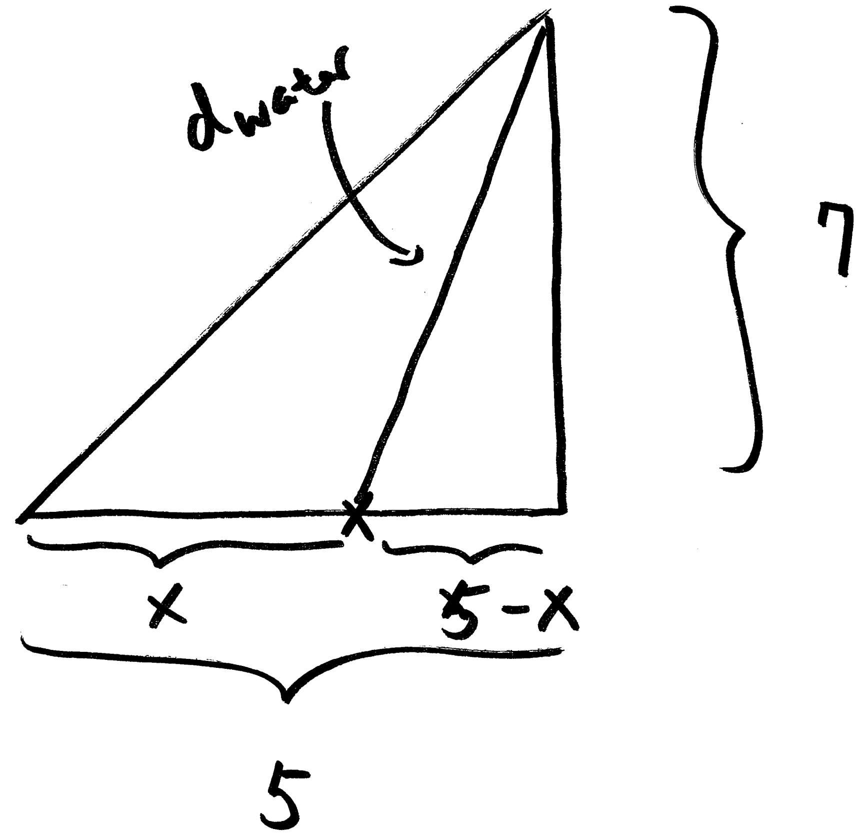
then find CN, check max/min.

Ans: $x = 40$, $y = 60$

Example 4

- A dog is at a beach with its owner. The owner throws a tennis ball into the water and the ball rests at a spot 7 meters from a point on the beach that is 5 meters from the dog. The dog can run with a speed of 28 ~~ft~~ m/s and can swim with a speed of 4 ~~ft~~ m/s. How should the dog approach the ball in the shortest amount of time possible?





assume: dog runs x meters along beach, then swim the rest of way

$0 \le x \le 5$

$$\text{time} = \frac{\text{dist}}{\text{rate}} \qquad T_{land} = \frac{x}{28}$$

$$(d_{water})^2 = (5-x)^2 + 49$$

$$d_{water} = \sqrt{(5-x)^2+49}$$

$$T_{water} = \frac{\sqrt{(5-x)^2+49}}{4}$$

minimize

$$T = \frac{x}{28} + \frac{\sqrt{(5-x)^2+49}}{4} \qquad 0 \le x \le 5$$

find $T' = 0 \rightarrow x = c$

compare: $T(0)$

$T(c)$

$T(5)$

best way : $x \approx 3.99$ m

swim ≈ 6.9 m