4.9 Antiderivatives

anti $\rightarrow$ against, opposite, reverse, etc

antiderivative $\rightarrow$ reverse (undo) the process of differentiation

given $f'(x)$, find $f(x)$.

$f(x) = x^2$ $\qquad$ $f'(x) = 2x^1$

from $f'(x) = 2x^1$ how to recover x^2?

$$f(x) = \frac{2}{2} x^{1+1}$$

~~divi~~

add one to existing power, divide by new power

another one: $f'(x) = 3x^2$

$$f(x) = \frac{3x^{2+1}}{2+1} = x^3$$

an arbitrary constant

Power Rule

if $f'(x) = x^n$

then $f(x) = \dfrac{x^{n+1}}{n+1} + C$

$\boxed{n \neq -1}$

Why + C ?

$f(x) = x^2$ $\qquad$ $f'(x) = 2x$

$f(x) = x^2+1$ $\qquad$ $f'(x) = 2x$

$f(x) = x^2+3$ $\qquad$ $f'(x) = 2x$

any constant f(x)
had disappears
after differentiation

no info on
missing constant
→ +C to
account for that.

example

$$f'(x) = x^2 + x + 1 \quad \text{find } f(x)$$

x^0

$$f(x) = \frac{x^{2+1}}{2+1} + \frac{x^{1+1}}{1+1} + x + C$$

$$= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + C$$

check: $$\frac{d}{dx}\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c\right)$$

$$= \frac{1}{3}(3x^2) + \frac{1}{2}(2x) + 1 + 0$$

$$= x^2 + x + 1$$

example

$$f'(x) = 7x^9 - 4x^6 + 12x^3$$

$$f(x) = 7\left(\frac{x^{10}}{10}\right) - 4\left(\frac{x^7}{7}\right) + 12\left(\frac{x^4}{4}\right) + c$$

$$= \frac{7}{10}x^{10} - \frac{4}{7}x^7 + 3x^4 + c$$

example

$$f'(x) = x \cdot (6 - x)^2$$ exponentiation

multiplication

for now, we can only handle x^n

→ turn into x^n

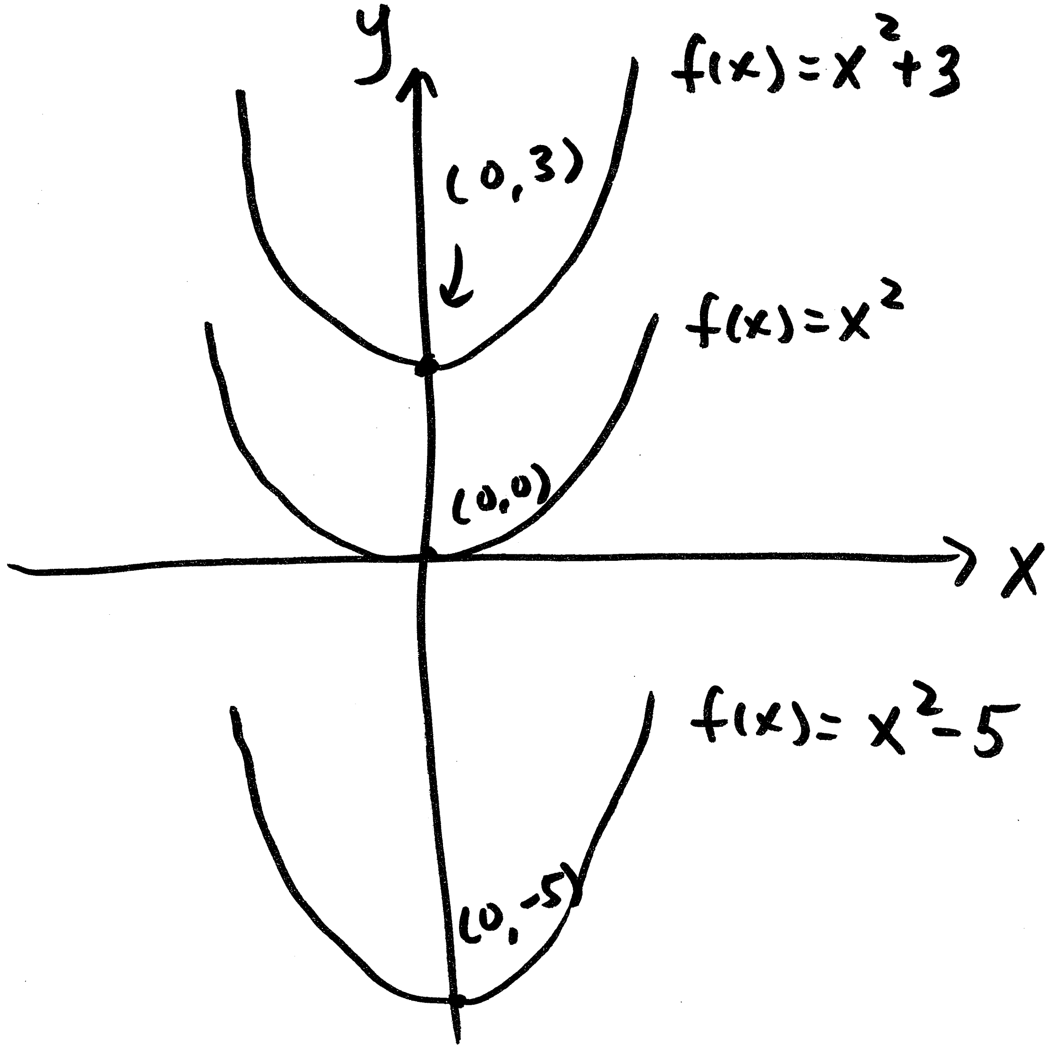
$$f'(x) = x \cdot (36 - 12x + x^2)$$

$$f'(x) = 36x - 12x^2 + x^3$$

now apply power rule

$$\boxed{f(x) = 18x^2 - 4x^3 + \frac{1}{4}x^4 + C}$$

to find the value of C, we need one point ~~that~~ on the graph of f(x)



all three have

$f'(x) = 2X$

$f(x) = x^2 + C$

if we know

$f(0) = 3$

any point on f(x)

$3 = (0)^2 + C$

$C = 3$

so $f(x) = x^2 + 3$

<u>example</u>

$$f'(x) = \frac{6+x+x^2}{\sqrt{x}} \qquad f(0)=1$$

find $f(x)$

$$f'(x) = \frac{6+x+x^2}{x^{1/2}} = \frac{6}{x^{1/2}} + \frac{x^1}{x^{1/2}} + \frac{x^2}{x^{1/2}}$$

$$= 6x^{-1/2} + x^{1/2} + x^{3/2}$$

$$f(x) = 6\left(\frac{x^{1/2}}{1/2}\right) + \frac{x^{3/2}}{3/2} + \frac{x^{5/2}}{5/2} + C$$

$$f(x) = 12x^{1/2} + \frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2} + C$$

$$1 = 0 + 0 + 0 + C$$

so $\boxed{f(x) = 12x^{1/2} + \frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2} + 1}$

what if $n = -1$

this means $f'(x) = \frac{1}{x}$

$f(x) = \ln x$ could give us $f'(x) = \frac{1}{x}$

$$\boxed{\text{if } f'(x) = \frac{1}{x}, \text{ then } f(x) = \ln|x| + C}$$

example If $f''(x) = \frac{1}{x^2}$, find $f(x)$

$$f''(x) = x^{-2}$$

$$f'(x) = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$$

$$\boxed{f(x) = -\ln|x| + Cx + D}$$

2nd constant since we undid the deriv. 2nd time

two unknown constants

→ need Two points that $f(x)$ passes through

→ or one point on $f'(x)$ AND one point on $f(x)$

example

$$f''(x) = \cos x + \sinh x + e^x$$

find $f(x)$

$$f'(x) = \sin x + \cosh x + e^x + C$$

$$\boxed{f(x) = -\cos x + \sinh x + e^x + Cx + D}$$