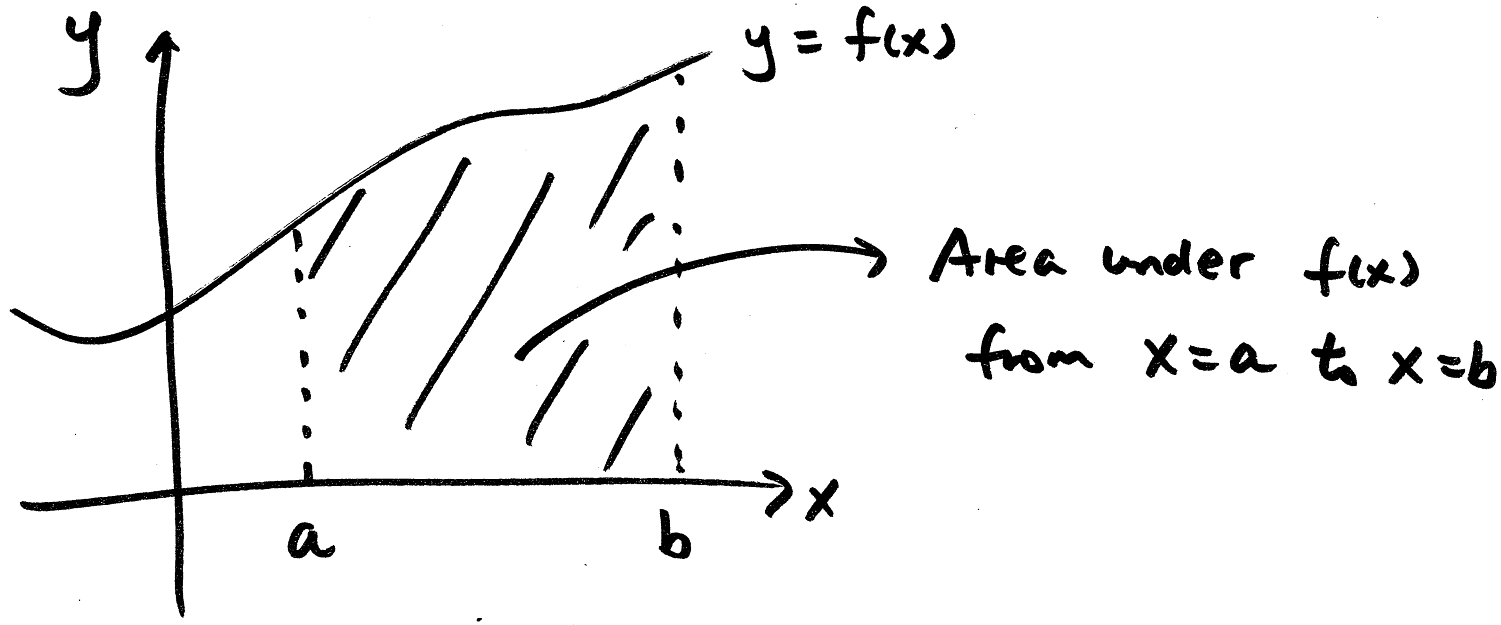
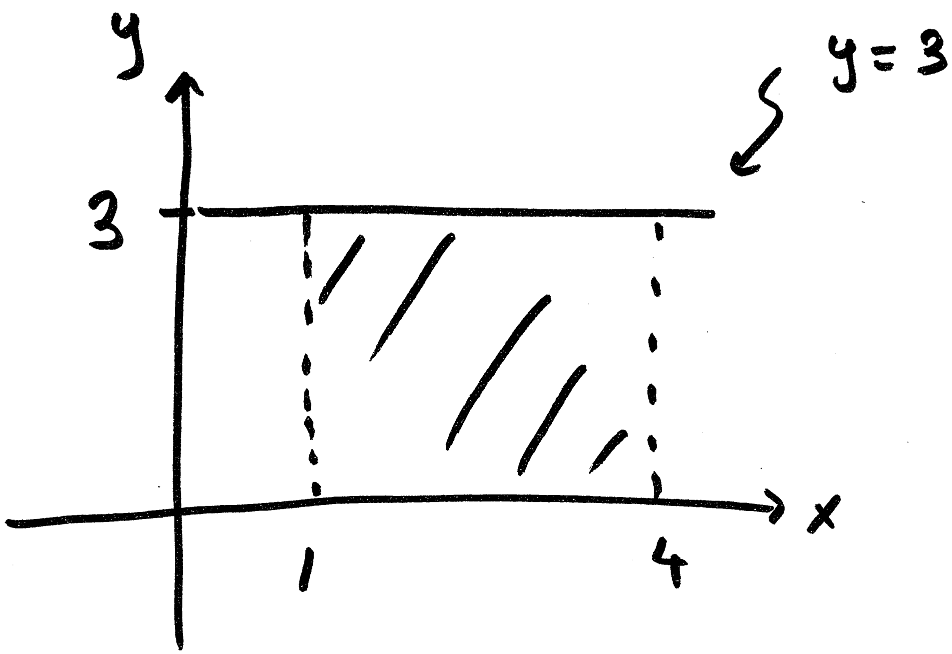
5.1 + 5.2 Areas, Distances, and the Definite Integral

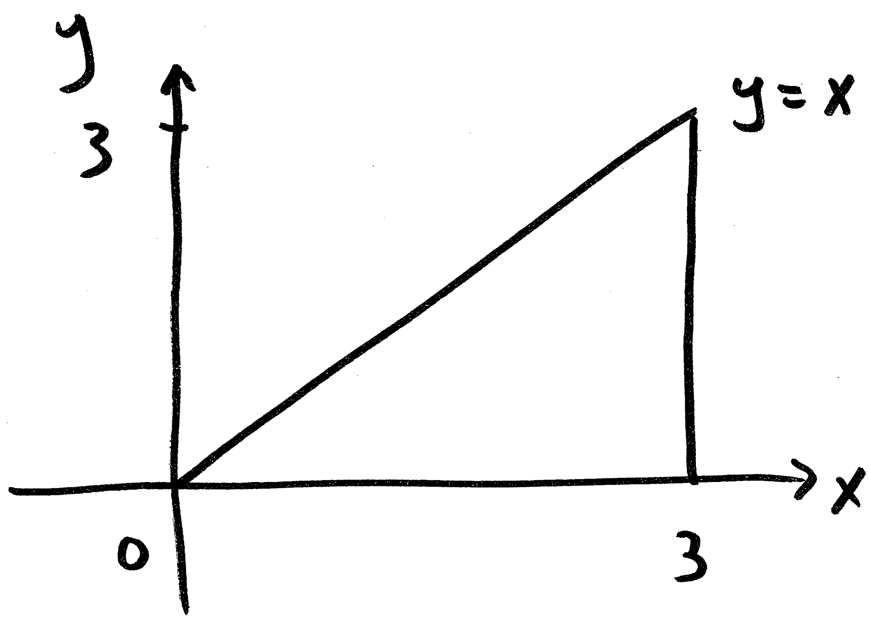
The Area Problem



Some are trivial



$A = 3 \cdot 3 = 9$ (rectangle)



$A = \frac{1}{2}(3)(3) = \frac{9}{2}$

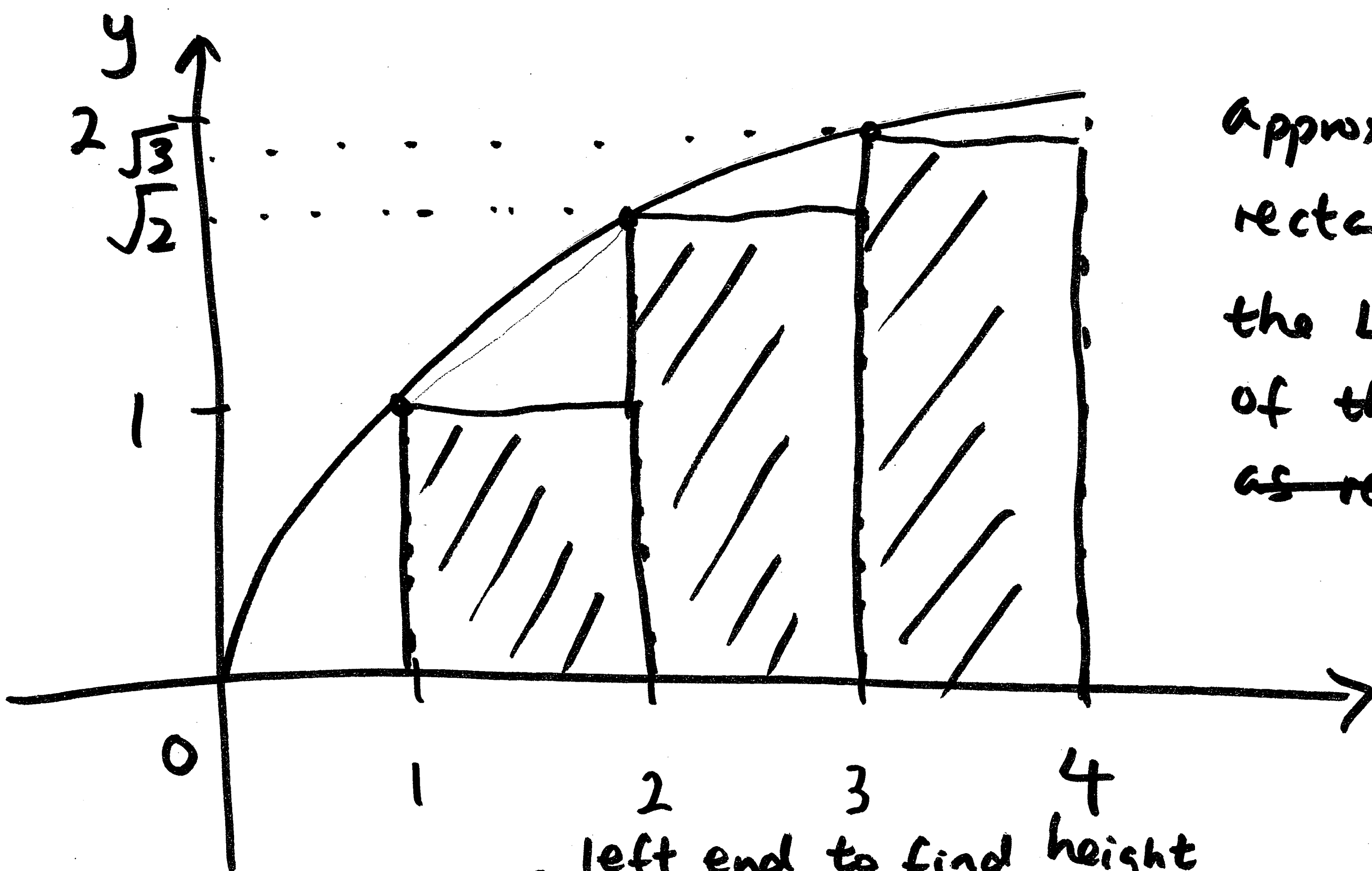
(triangle)

most are not doable by geometric means

approximate by a bunch rectangles

$y = \sqrt{x}$

(Riemann Sum)



approximated by rectangles using the LEFT end of the subintervals ~~as rect~~ to find rectangle height

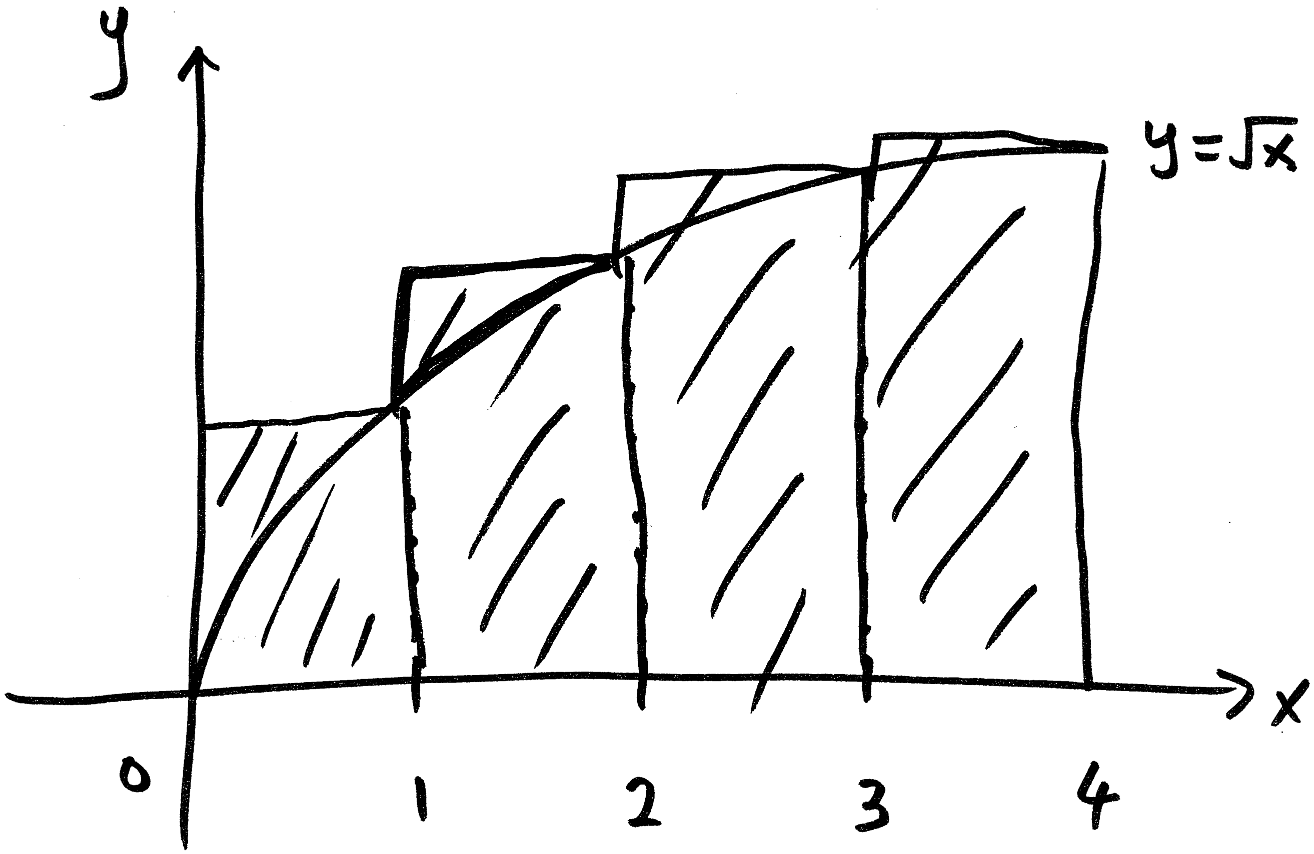
left end to find height

under-estimate

$$A \approx L_4 = (1)(0) + (1)(1) + (1)(\sqrt{2}) + (1)(\sqrt{3}) \approx \boxed{4.1463}$$

4 subintervals

right end point

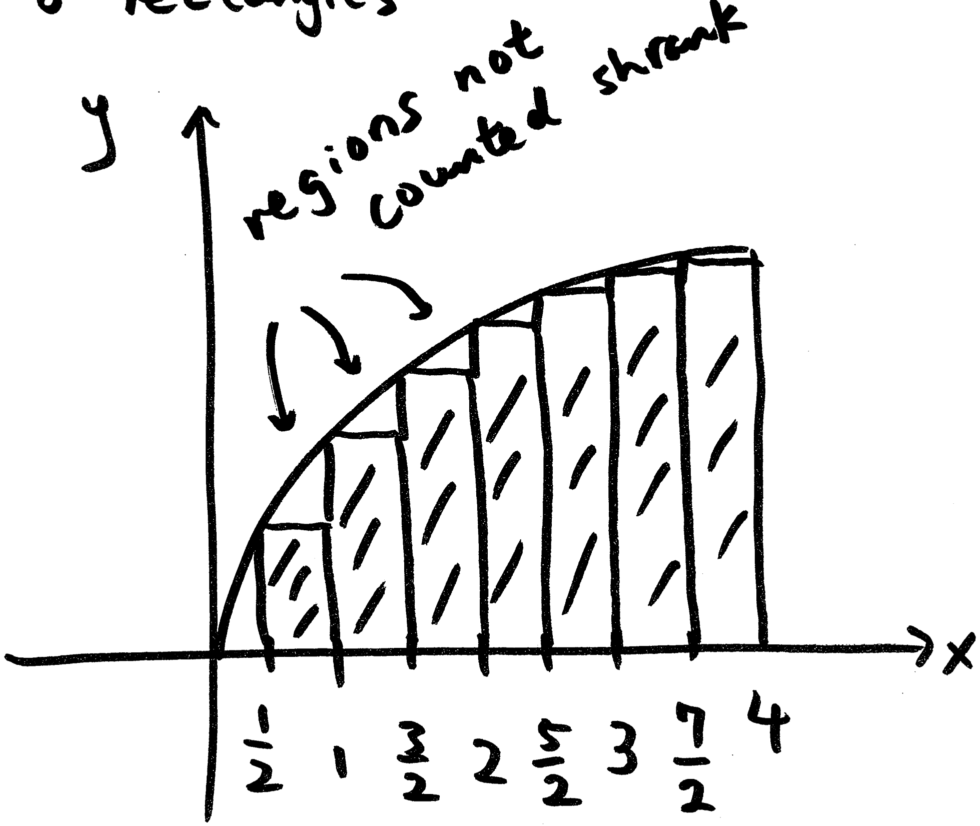


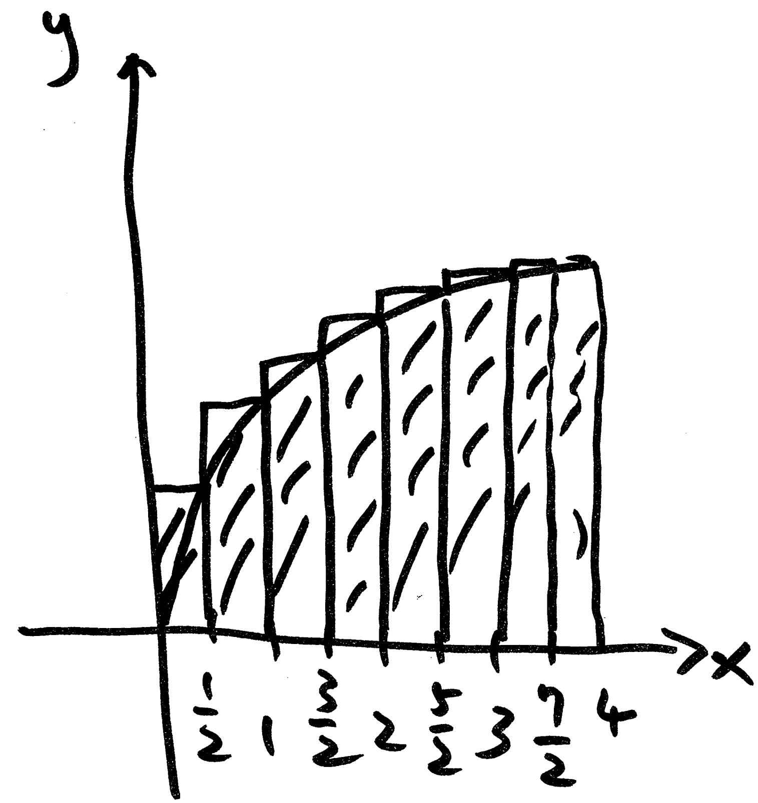
over-estimate

$$A \approx R_4 \approx (1)(1) + (1)(\sqrt{2}) + (1)(\sqrt{3}) + (1)(2) \approx \boxed{6.1423}$$

$$\boxed{\text{true area: } \quad 4.1463 < A < 6.1423}$$

8 rectangles





$L_8 = \left(\frac{1}{2}\right)\left(\sqrt{0} + \sqrt{\frac{1}{2}} + \sqrt{1} + \sqrt{\frac{3}{2}} + \sqrt{2} + \sqrt{\frac{5}{2}} + \sqrt{3} + \sqrt{\frac{7}{2}}\right)$

$= 4.7650$

$R_8 = 5.7650$

true area

$4.7650 < A < 5.7650$

interval decreases with more rectangles

we trap the area by using more rectangles

area is <u>exact</u> if we use infinitely-many rectangles

$$A = \lim_{n\to\infty} \left[f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x \right]$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

as n becomes large ($n \to \infty$), the choice of left/right endpoint becomes irrelevant

Distance

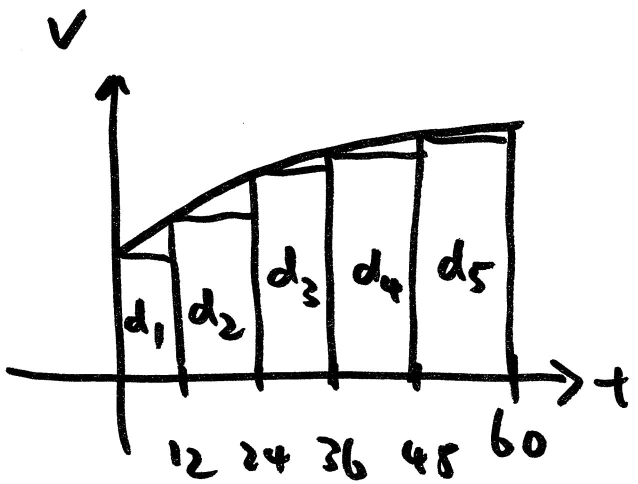
<u>examples</u> Speedometer readings in 12-second intervals

t (s)	0	12	24	36	48	60
v (ft/s)	22	24	25	27	28	30

estimate distance traveled.

distance = velocity · time

use begining velocity for each interval



$d_1 \approx (22)(12) = 264$

$d_2 \approx (24)(12) = 288$

$d_3 \approx 300$

$d_4 \approx 324$

$d_5 \approx 336$

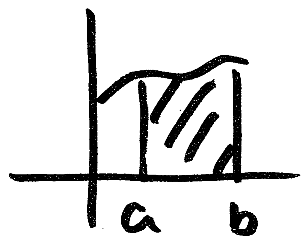
$\boxed{\text{total} \approx 1512}$

underestimate

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x = \boxed{\int_a^b f(x)\,dx}$$

Definite integral

area under $f(x)$ on $a \le x \le b$



$\Delta x \to dx$ as $n \to \infty$

a : lower integration limit

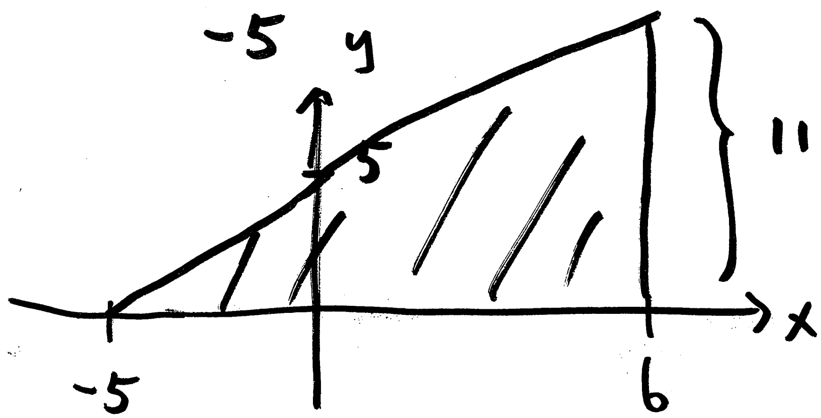
b : upper integration limit

Method of evaluation → next time

for now, think AREAS

<u>example</u>

$$\int_{-5}^{6} (x+5)\,dx = \frac{1}{2}(11)(11) = \boxed{\frac{121}{2}}$$

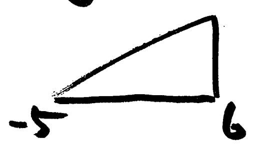


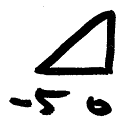
example

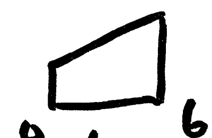
$$\int_{-5}^{0} (x+5)\,dx = \frac{1}{2}(5)(5) = \frac{25}{2}$$

same region,
start at -5
end at 0

$$\int_{0}^{6} (x+5)\,dx = ?$$







$$\int_{-5}^{6} \cancel{f(x)}\,(x+5)\,dx = \int_{-5}^{0} (x+5)\,dx + \int_{0}^{6} (x+5)\,dx$$

$$\frac{121}{2} = \frac{25}{2} + \int_{0}^{6} (x+5)\,dx = \frac{96}{2}$$