5.3 + 5.4 Indefinite Integral

Fundamental Theorem of Calculus (FTC)

FTC 1:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

example

$$g(x) = \int_2^x e^{5t^2-2t}\,dt$$

$$g'(x) = ?$$

$$\boxed{g'(x) = e^{5x^2-2x}}$$

example

$$y = \int_{2-3x}^{5} \frac{t^3}{1+t^3}\,dt$$

$y' = ?$

$$y = -\int_{5}^{2-3x} \frac{t^3}{1+t^3}\,dt$$

chain rule since upper limit is not just x

let $u = 2-3x$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{d}{du}\left(-\int_5^u \frac{t^3}{1+t^3}\,dt\right)\frac{d}{dx}(2-3x)$$

$$\frac{dy}{dx} = -\frac{u^3}{1+u^3} \cdot -3$$

sub u out

$$\boxed{\frac{dy}{dx} = \frac{3(2-3x)^3}{1+(2-3x)^3}}$$

<u>FTC 2</u>

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F'(x) = f(x)$

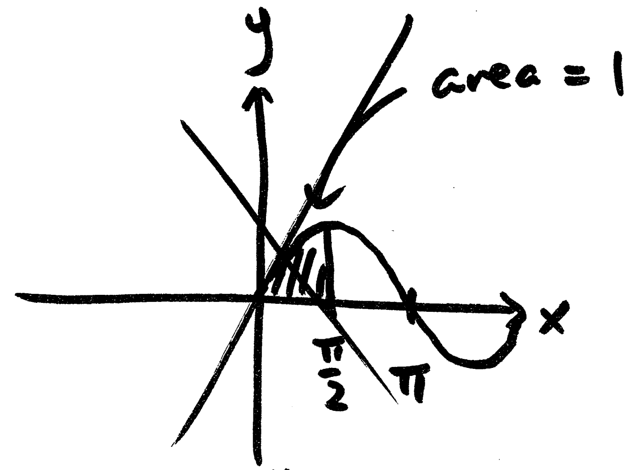
__example__

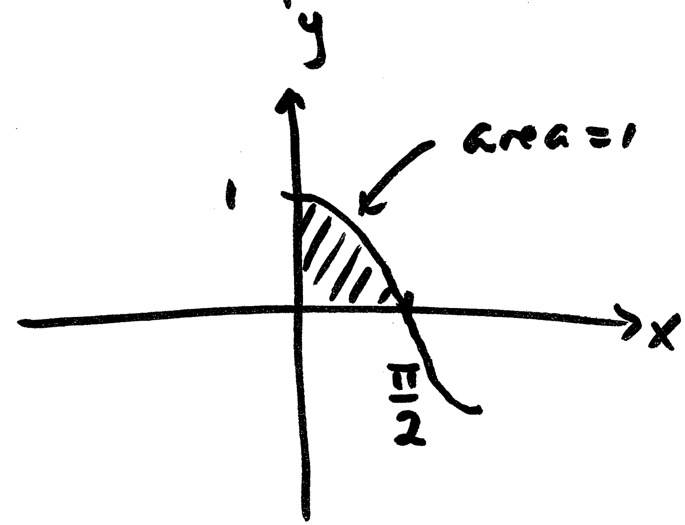
$$\int_0^{\pi/2} \cos x \, dx$$

$$= \sin x \Big|_0^{\pi/2}$$

$$= \sin(\pi/2) - \sin(0)$$

$$= \boxed{1}$$





Indefinite Integral

$\int f(x)\,dx$ integration limits are missing

$= F(x) + C$ the general antideriv. of $f(x)$

(has $+C$)

def. integration gives a number

indef. integration gives a function

example

$$\int \left(3\sqrt{x^3} + 4\sqrt[3]{x^2}\right) dx$$

$$= \int \left(3x^{3/2} + 4x^{2/3}\right) dx$$

capital C in webassign

$$= 3 \cdot \frac{x^{3/2+1}}{\frac{3}{2}+1} + 4 \cdot \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C$$

check by differentiating

$$= 3 \cdot \frac{x^{5/2}}{5/2} + 4 \cdot \frac{x^{5/3}}{5/3} + C$$

$$= 3 \cdot \frac{2}{5} x^{5/2} + 4 \cdot \frac{3}{5} x^{5/3} + C = \boxed{\frac{6}{5} x^{5/2} + \frac{12}{5} x^{5/3} + C}$$

example

$$\int \left(5x^2 + 4 + \frac{4}{x^2+1}\right) dx \longrightarrow 4 \cdot \frac{1}{x^2+1}$$

$$= 5\frac{x^3}{3} + 4x + 4 \cdot \tan^{-1} x + C$$

$$= \boxed{\frac{5}{3}x^3 + 4x + 4\tan^{-1} x + C}$$

example

$$\int \frac{2 + 5\cos^2 x}{\cos^2 x}\, dx$$

recall $\dfrac{1}{\cos x} = \sec x$

$$= \int \left(\frac{2}{\cos^2 x} + \frac{5\cos^2 x}{\cos^2 x} \right) dx$$

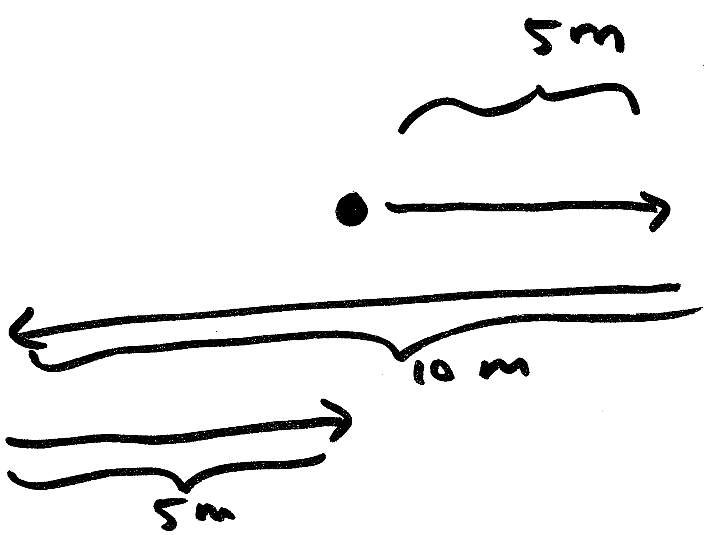
$$= \int \left(2\sec^2 x + 5 \right) dx$$

$$= \boxed{2\tan x + 5x + C}$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

FTC 2 aka the <u>Net Change Theorem</u>

Some applications: displacement (not necessarily the same as distance)



move right
" left
" right

displacement = 0 (back to original position)
distance = 20 m

if $v(t)$ is velocity function

position function

then $$\int_a^b v(t)\,dt = \underbrace{S(b) - S(a)}_{\text{net change of position}}$$

(no info on ~~during~~ between begin/end)

displacement

(distance only if $v(t)$ never changes sign)

Work (physics)

$$W = \int_a^b \overbrace{f(x)}^{\text{force}}\,dx$$ = work done in moving an object from $x=a$ to $x=b$ in the presence of a force $f(x)$

$$\underbrace{W = F(b) - F(a)}$$

many forces disregards the middle part (conservative force)

e.g. gravity