# 5.5 The Substitution Rule (part 1)

$y = \frac{1}{3}(10+x^4)^3$ want $y'$

$y' = \frac{1}{3} \cdot 3(10+x^4)^2 \cdot 4x^3 = 4x^3(10+x^4)^2$

<u>chain rule</u>!

How to find antiderivative (undo chain rule)?

$$\int 4x^3(10+x^4)^2\,dx$$

make a substitution $u = 10 + x^4$

$$\frac{du}{dx} = 4x^3$$

$$du = 4x^3\,dx$$

$$\int 4x^3 (10 + x^4)^2 \, dx$$

$u = 10 + x^4$

$du = 4x^3 dx$

Sub these into the integral

$$= \int \underbrace{(10 + x^4)^2}_{u^2} \cdot \underbrace{(4x^3 dx)}_{du}$$

$$= \int u^2 \, du$$

same as $\int x^2 dx$

$$= \frac{u^3}{3} + C = \boxed{\frac{1}{3}(10 + x^4)^3 + C}$$

sub u out

example

$$\int x^2 \sqrt{x^3+4}\, dx = \int \boxed{x^2} \cdot (\boxed{x^3+4})^{1/2}\, dx$$

note the two main parts of the integrand

$$x^2 \quad \text{vs} \quad x^3+4$$

often $u$ is the more complicated part

choose one to be $u$

→ the one that has a derivative that is related to the other

$$\frac{d}{dx}(x^3+4) = 3\boxed{x^2}$$

↳ the other part

let $u = x^3+4$

$$\frac{du}{dx} = 3x^2 \qquad du = 3x^2 dx$$

$$\int x^2 \cdot (x^3+4)^{1/2}\,dx$$

$$u = x^3+4$$

$$du = 3x^2\,dx$$

$$\frac{du}{3} = x^2\,dx$$

$$= \int \underbrace{(x^3+4)^{1/2}}_{u^{1/2}} \underbrace{(x^2\,dx)}_{\frac{1}{3}du}$$

$$= \int \frac{1}{3} u^{1/2}\,du = \frac{1}{3} \cdot \frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \boxed{\frac{2}{9}(x^3+4)^{3/2} + C}$$

example

$$\int \frac{dx}{1-x} = \int \frac{1}{1-x}\,dx$$

compare $1$ and $1-x$

choose $u = 1-x$

$$\frac{du}{dx} = -1$$

$$du = -dx \longrightarrow -du = dx$$

$$\int \frac{1}{u} \cdot -du = \int -1 \cdot \underbrace{\frac{1}{u}}_{\to\, u^{-1}}\, du$$

$$= -\ln|u| + c = \boxed{-\ln|1-x| + c}$$

example

$$\int \cos x \cdot \sin x \, dx$$

compare $\cos x$ and $\sin x$

deriv. of $u$ = ~~somet~~ some constant multiple of the other

if $u = \sin x$ $\quad du = \underbrace{\cos x}_{\text{the other (ok)}} \, dx$

if $u = \cos x$ $\quad du = \underbrace{-\sin x}_{-1 \cdot \text{the other (ok)}} \, dx$

BOTH are acceptable choice of $u$

$$\int \cos x \sin x \, dx$$

$$u = \sin x \qquad du = \cos x \, dx$$

$$= \int u \cdot du = \frac{u^2}{2} + C$$

$$= \boxed{\frac{\sin^2 x}{2} + C}$$

<u>example</u>

$$\int \frac{(\ln x)^2}{x}\,dx$$

$$= \int (\boxed{\ln x})^2 \cdot \boxed{\frac{1}{x}}\,dx$$

$\ln x$ vs. $\frac{1}{x}$

$u = \ln x$ since $\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

$$= \int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{(\ln x)^3}{3} + C}$$

example

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^2}\,dx$$

if ~~co~~ we compare $\cos\left(\frac{1}{x}\right)$ and $x^2$

neither is appropriate choice of $u$

since neither has deriv. that looks like the other

rearrange integral

$$\int \cos\left(\boxed{\frac{1}{x}}\right)\cdot \boxed{\frac{1}{x^2}}\,dx$$

$\frac{1}{x}$ vs $\frac{1}{x^2}$

deriv. is $-\frac{1}{x^2}$

so $u = \frac{1}{x}$

$$\int \cos\left(\frac{1}{x}\right) \cdot \frac{1}{x^2}\, dx$$

$$u = \frac{1}{x}$$

$$\frac{du}{dx} = -\frac{1}{x^2} \qquad du = -\frac{1}{x^2}\, dx$$

$$= \int -\cos u \cdot du = -\sin u + C$$

$$= -\sin\left(\frac{1}{x}\right) + C$$

example

$$\int \sec^2 x \tan^7 x \, dx$$

$\sec^2 x$ vs $\tan^7 x$ not useful

compare : $\sec^2 x$ vs $\tan x$

$u = \tan x$ since

$du = \sec^2 x \, dx$

$$= \int (\tan x)^7 \cdot (\sec^2 x \, dx)$$

$$= \int u^7 \cdot du = \frac{u^8}{8} + C = \boxed{\frac{\tan^8 x}{8} + C}$$