1.5 Inverse Functions and Logarithms

a function is <u>one-to-one</u> if every output value is associated with one and only one input value.

example $f(x) = x^2$ is <u>NOT</u> one-to-one

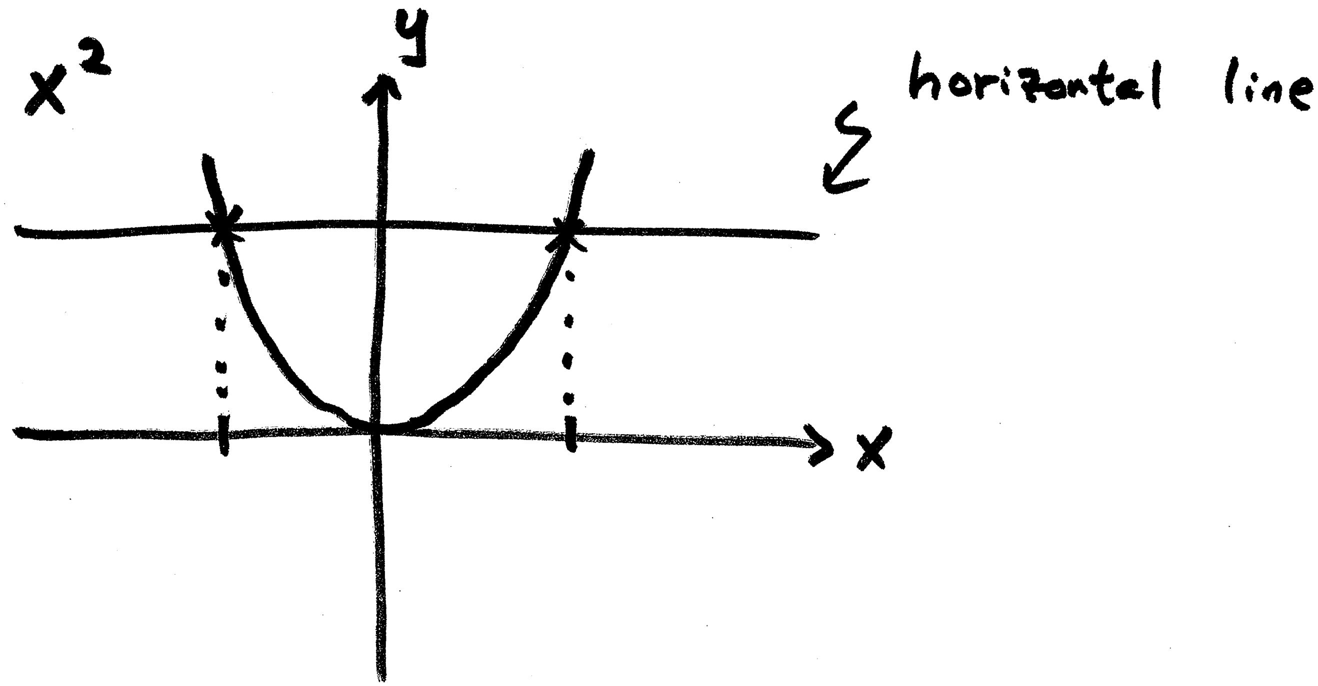
$f(-1) = f(1) = 1$

$f(x) = x$ is one-to-one

because $f(x_1) = f(x_2)$ if $x_1 = x_2$

graph : easy way to tell if not one-to-one

$f(x) = x^2$



if horiz. line crosses more than once

$\rightarrow$ NOT one-to-one

if $f(x)$ is one-to-one, then it has an

<u>inverse</u> $f^{-1}(x)$

$$x \longrightarrow \boxed{f} \longrightarrow f(x) \longrightarrow \boxed{f^{-1}} \longrightarrow x$$

\$1 bill → four 25¢ → \$1 bill

$A = \pi r^2$ area of a circle radius r

$r \geq 0$

$r = \sqrt{\dfrac{A}{\pi}}$ ~~area~~ radius of a circle area A

roles of input and output are <u>reversed</u>

domain of $f(x)$ is range of $f^{-1}(x)$

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<u>example</u> $f(x) = \dfrac{x}{x+1}$ find $f^{-1}(x)$

$$y = \frac{x}{x+1}$$ solve for x

$$(x+1)y = x$$

$$xy + y = x$$

$$xy - x = -y$$

$$x(y-1) = -y$$

$$x = \frac{-y}{y-1}$$

$$\boxed{f^{-1}(x) = \frac{-x}{x-1}}$$

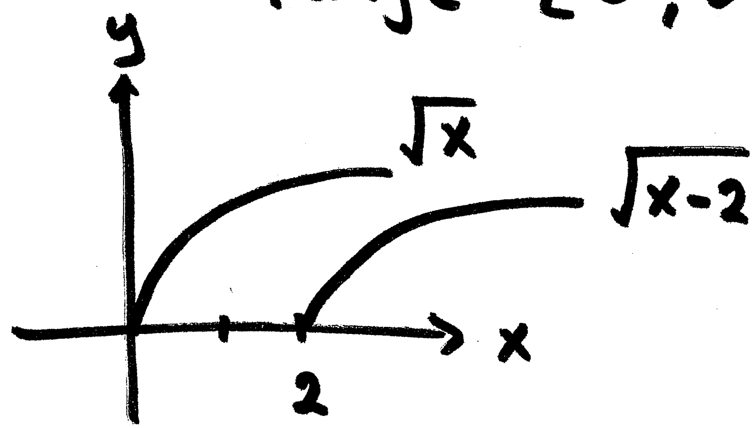
example

$f(x) = \sqrt{x-2}$

find $f^{-1}(x)$ and its domain

$f(x)$: domain $[2, \infty)$

range $[0, \infty)$



so $f^{-1}(x)$ has domain $[0, \infty)$

$y = \sqrt{x-2}$

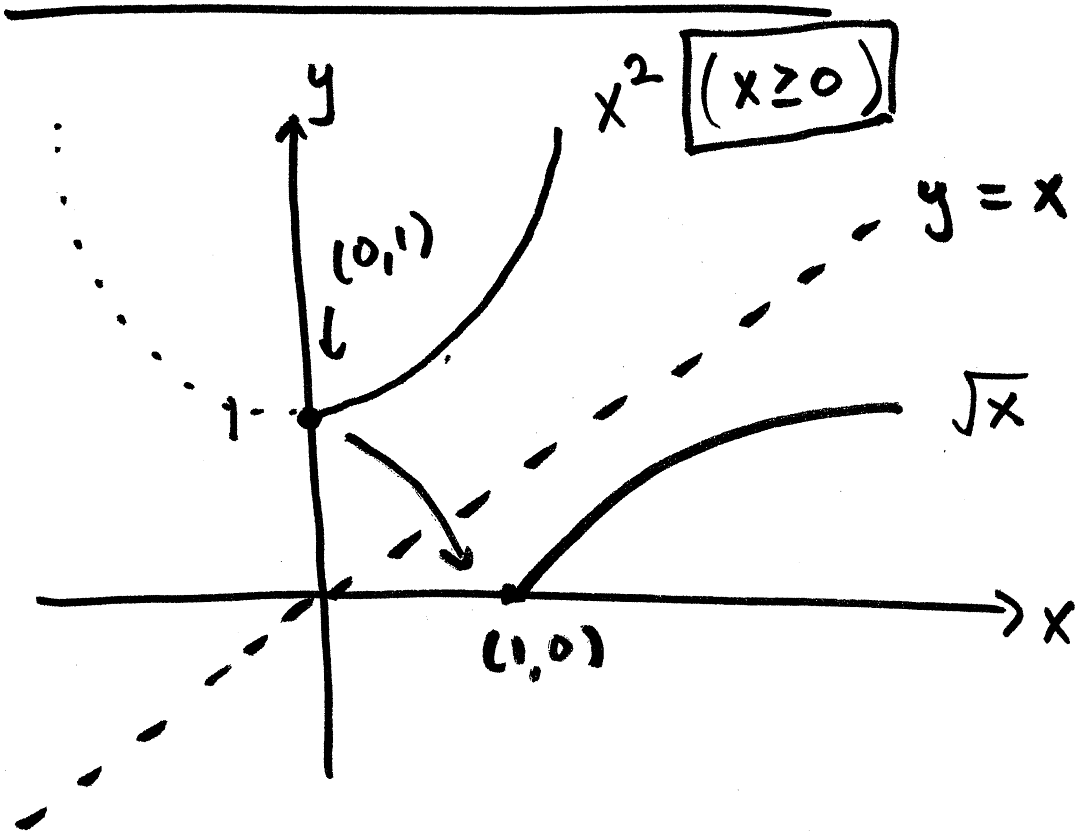
$y^2 = x-2$

$x = y^2+2$

$$\boxed{f^{-1}(x) = x^2+2}$$

note: x^2+2 has domain $(-\infty, \infty)$ if it is not an inverse of f

Graphs of $f(x)$ and $f^{-1}(x)$



$f(x)$ and $f^{-1}(x)$ are reflections of each other across the line $y = x$

Exponentials and Logarithms

$$y = a^x \longleftrightarrow x = \log_a y$$

$$\boxed{\begin{array}{c} \text{domain of } \log_a x \\ \hline (0, \infty) \end{array}}$$

$$y = e^x \longleftrightarrow x = \ln y$$

Properties of Logarithms

$$\log_a a^x = x$$

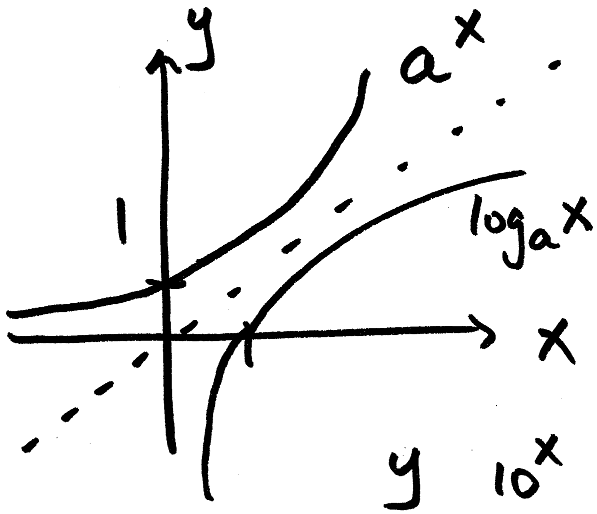
$$a^{\log_a x} = x$$

$$\log_a (xy) = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

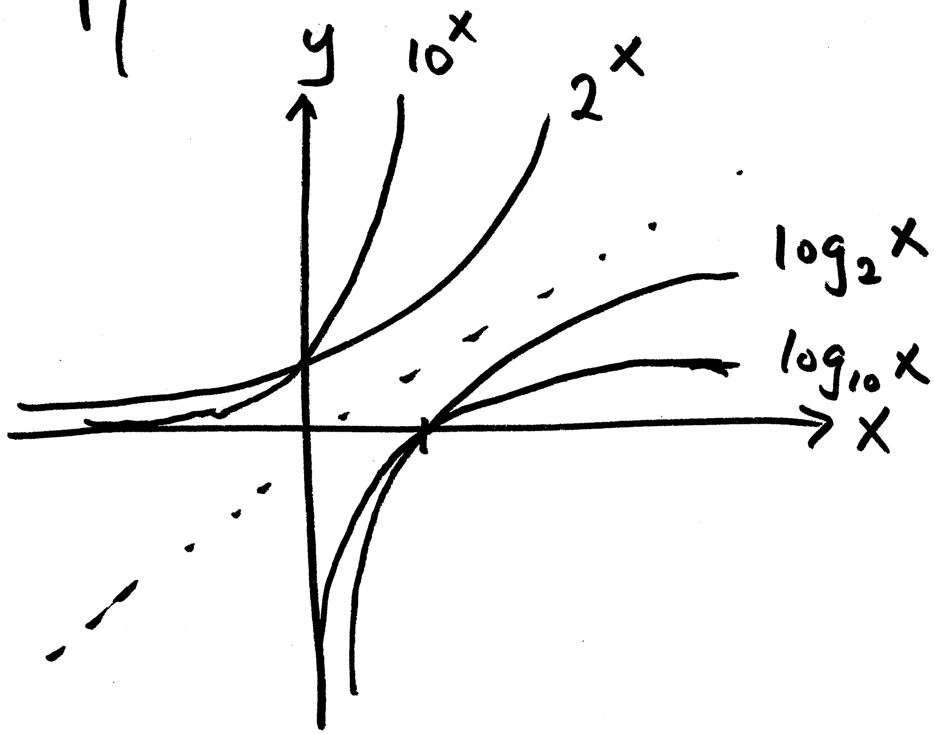
$$\log_a x^r = r \log_a x$$

$$\underbrace{y = a^x}_{} \longleftrightarrow x = \log_a y$$



D: $(-\infty, \infty)$

R: $(0, \infty)$ → domain of $\log_a x$



<u>example</u> $9^{x-4} = 8^{-9x}$ find x

$\ln 9^{x-4} = \ln 8^{-9x}$ log of <u>any</u> base is ok

$(x-4)\ln 9 = -9x \ln 8$

$(\ln 9)x - 4\ln 9 = (-9\ln 8)x$

$(\ln 9)x + (9\ln 8)x = 4\ln 9$

$x(\ln 9 + 9\ln 8) = 4\ln 9$

$$\boxed{x = \frac{4\ln 9}{\ln 9 + 9\ln 8}}$$

$$\ln\left(\frac{2}{e}\right) = \ln 2 - \ln e = \ln 2 - 1$$

$$= \ln(2e^{-1})$$

$$= \ln 2 + \ln e^{-1} = \ln 2 - \ln e$$

$$= \ln 2 - 1$$

$$\log_a a^x = x$$

$$e^{\ln x} = x$$

$$e^{-2\ln 4} = \cancel{e}\ e^{\ln(4^{-2})} = 4^{-2} = \frac{1}{16}$$

Rocket Equation

initial mass

$$\Delta V = V_e \ln \frac{m_0}{m_1}$$

final mass

change in velocity

exhaust velocity

Earth surface to low-Earth orbit

$$\Delta V = 9700 \text{ m/s}$$

$$m_1 = 11.6\,\% \text{ of } m_0$$

single-stage