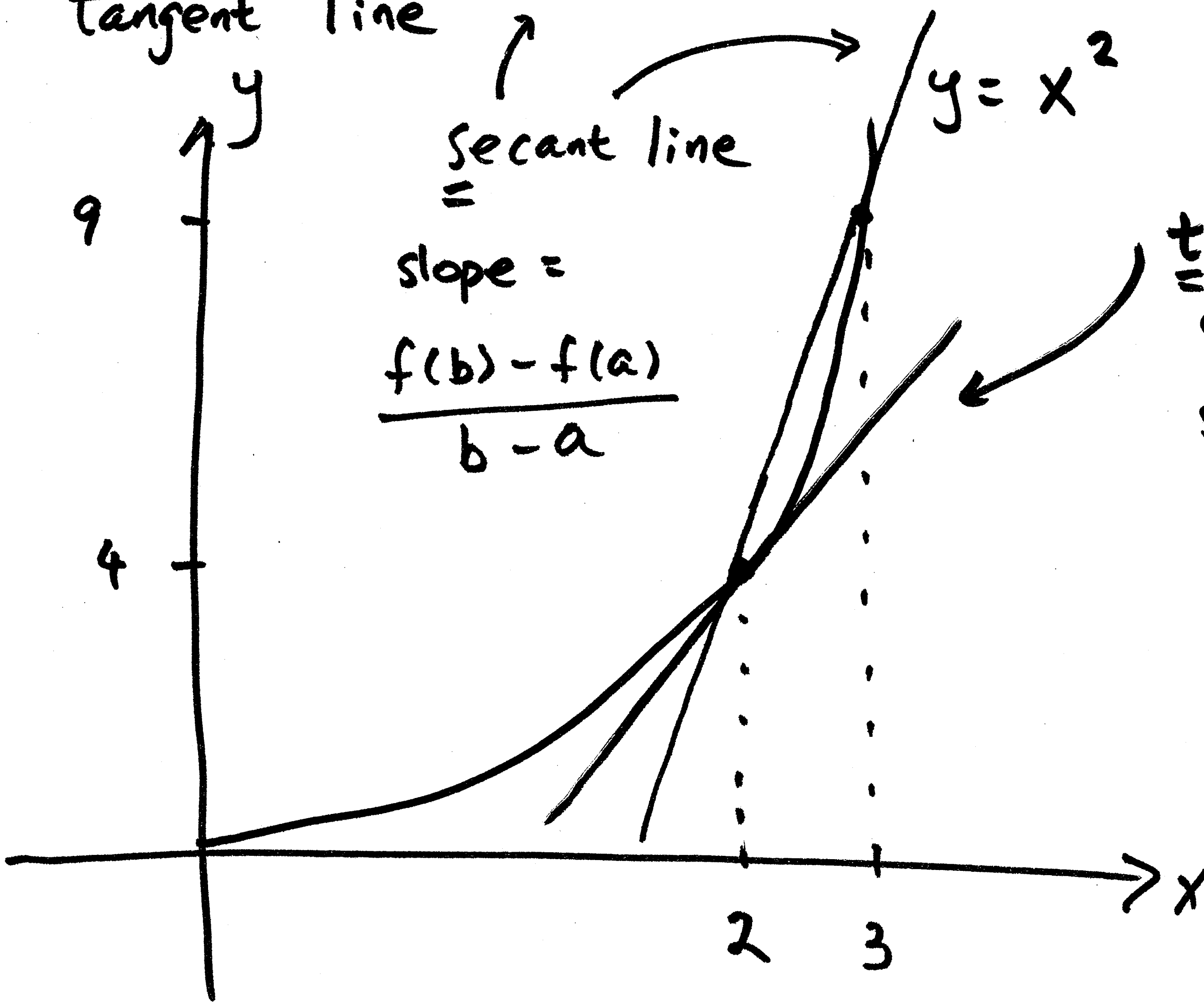
2.1 The Tangent and Velocity Problems

tangent line



secant line "to cut/slice"

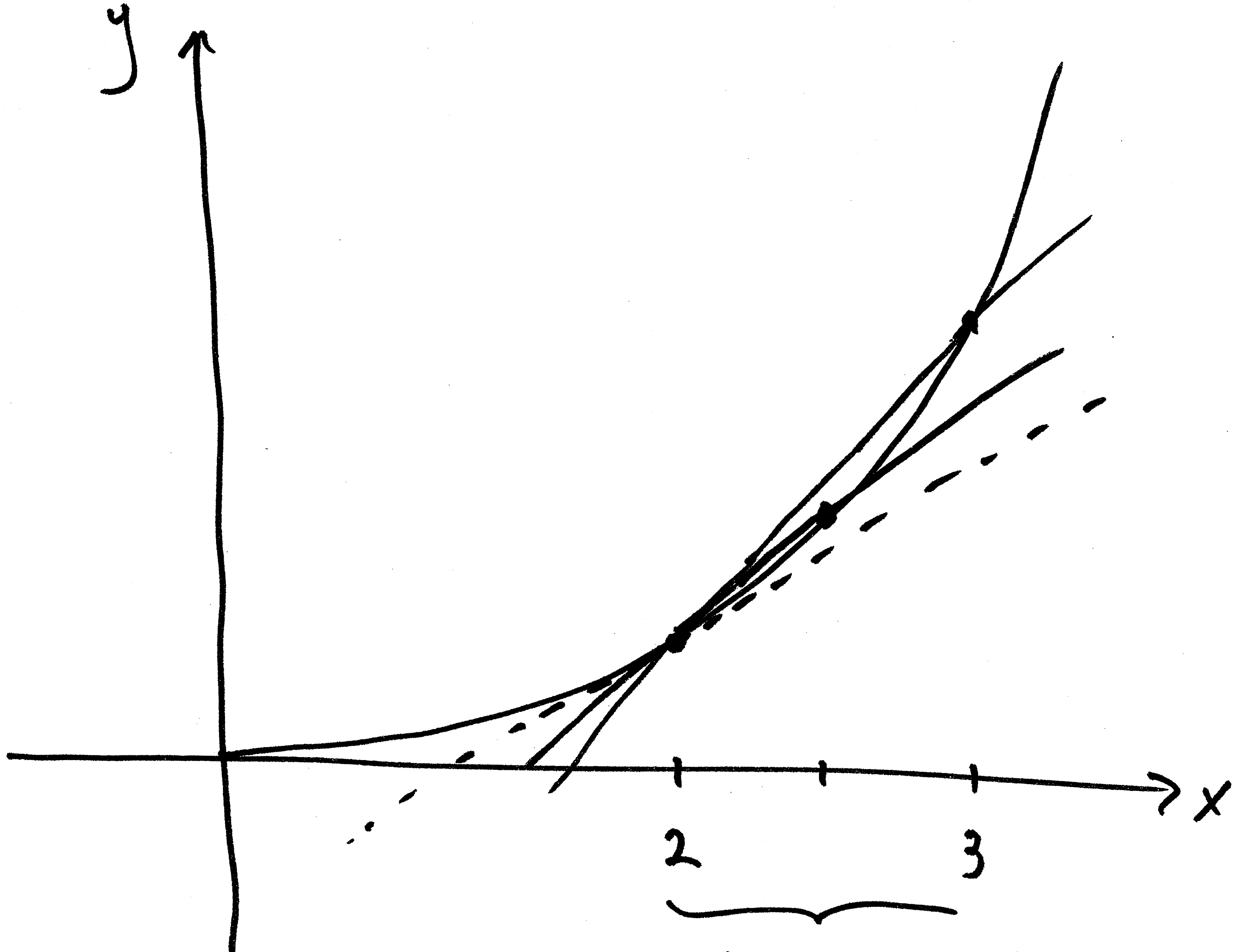
= slope = $\dfrac{f(b)-f(a)}{b-a}$

tangent line "to touch"

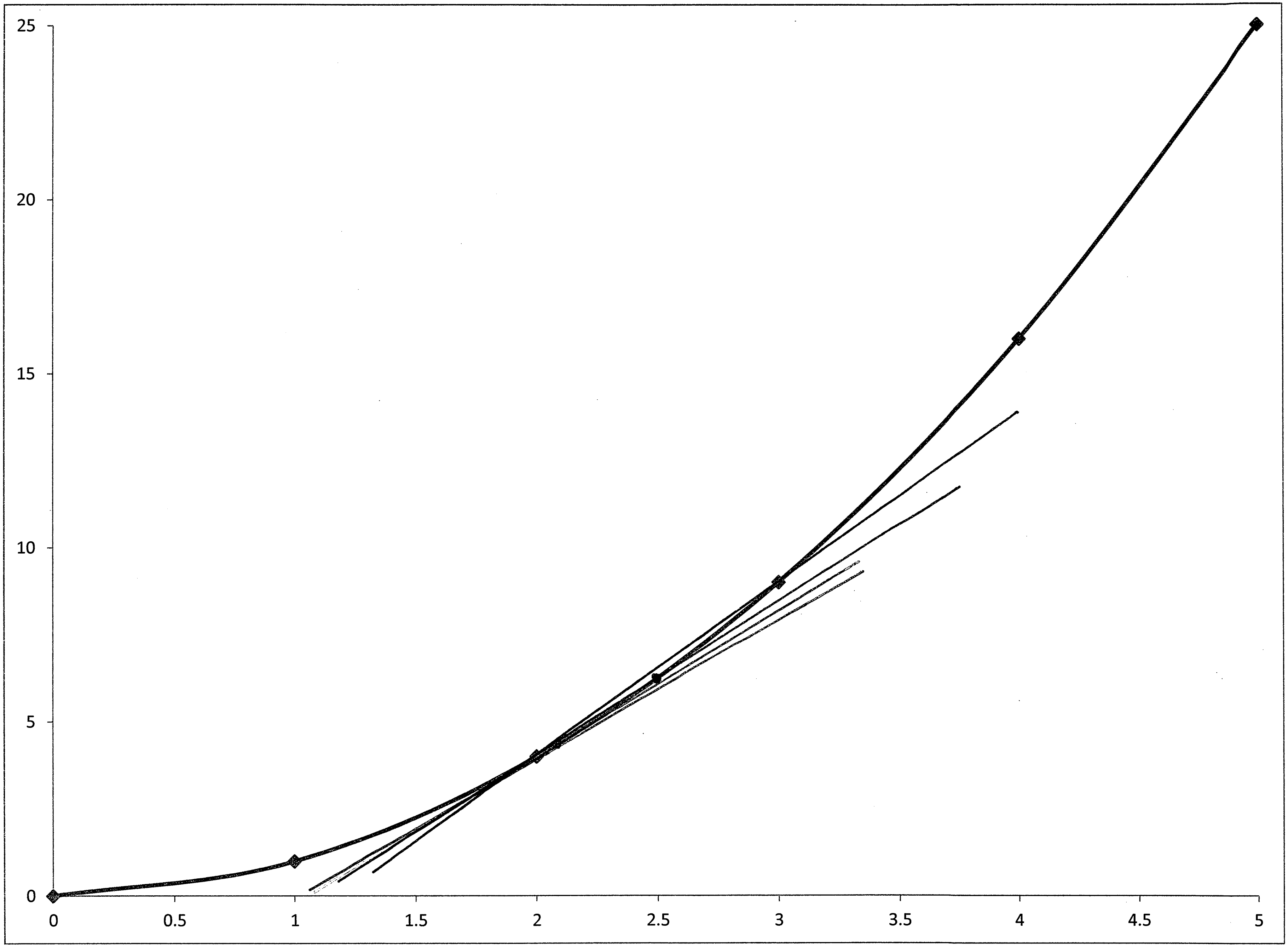
= at $x = 2$

slope = ?

Start moving second point toward (2, 4)



Shrink this distance to make secant more like tangent


25
20
15
10
5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

$y = x^2$ estimate slope of tangent line at $x=2$

$P(2,4)$

start at $Q(3,9)$ $\quad m_{PQ} = \dfrac{9-4}{3-2} = \underline{5}$

$Q(2.5, 6.25)$ $\quad m_{PQ} = \dfrac{6.25-4}{2.5-2} = \underline{\underline{4.5}}$

$Q(2.01, 4.0401)$ $\quad m_{PQ} = \dfrac{4.0401-4}{2.01-2} = \underline{\underline{4.01}}$

$Q(2.001, 4.004001)$ $\quad m_{PQ} = \underline{\underline{4.001}}$

note m_{PQ} changed by very little after a while

tangent line slope at $(2,4)$ is probably 4

A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

P

t (min)	5	10	15	20	25	30
V (gal)	694	444	250	111	28	0

(a) If P is the point (15, 250) on the graph of V, find the slopes of the secant lines PQ when Q is the point on the graph with $t = 5, 10, 20, 25$, and 30.

(b) Estimate the slope of the tangent line at P by averaging the slopes of two secant lines.

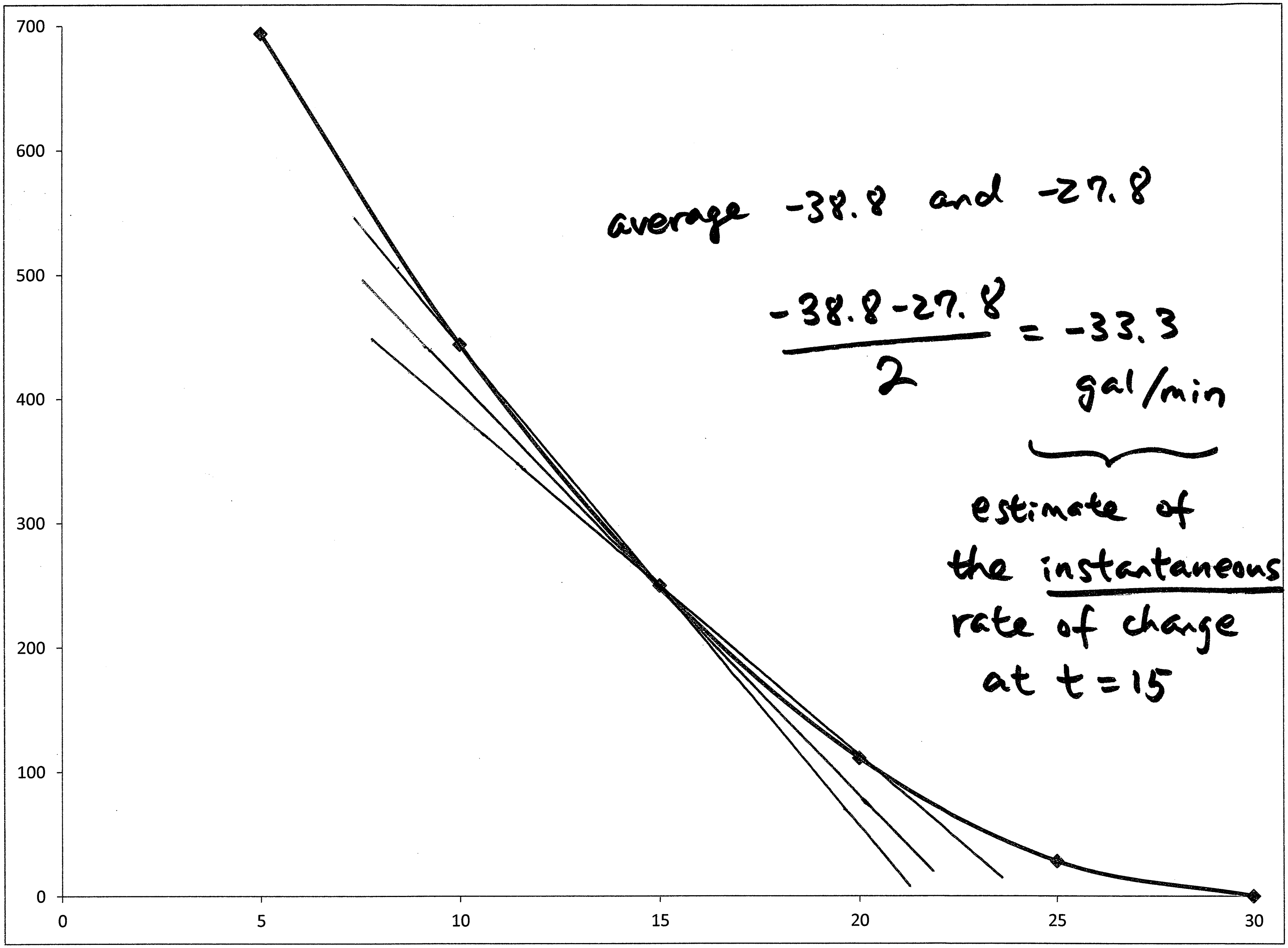
$P(15, 250)$

$Q(5, 694)$ $\quad m_{PQ} = \dfrac{694-250}{5-15} = \underbrace{-44.4 \text{ gallons/min}}$

between $t=5$ and $t=15$
water is leaving the
tank at an <u>average rate</u>
of 44.4 gal/min

$Q(10, 444)$ $\quad m_{PQ} = -38.8$

$Q(20, 111)$ $\quad m_{PQ} = -27.8$



average -38.8 and -27.8

$$\frac{-38.8-27.8}{2} = \underbrace{-33.3 \text{ gal/min}}_{\text{estimate of the \underline{instantaneous} rate of change at } t=15}$$

If a rock is thrown upward on Mars with a velocity of 10 m/s, its height in meters t seconds later is $y = 10t - 1.86t^2$

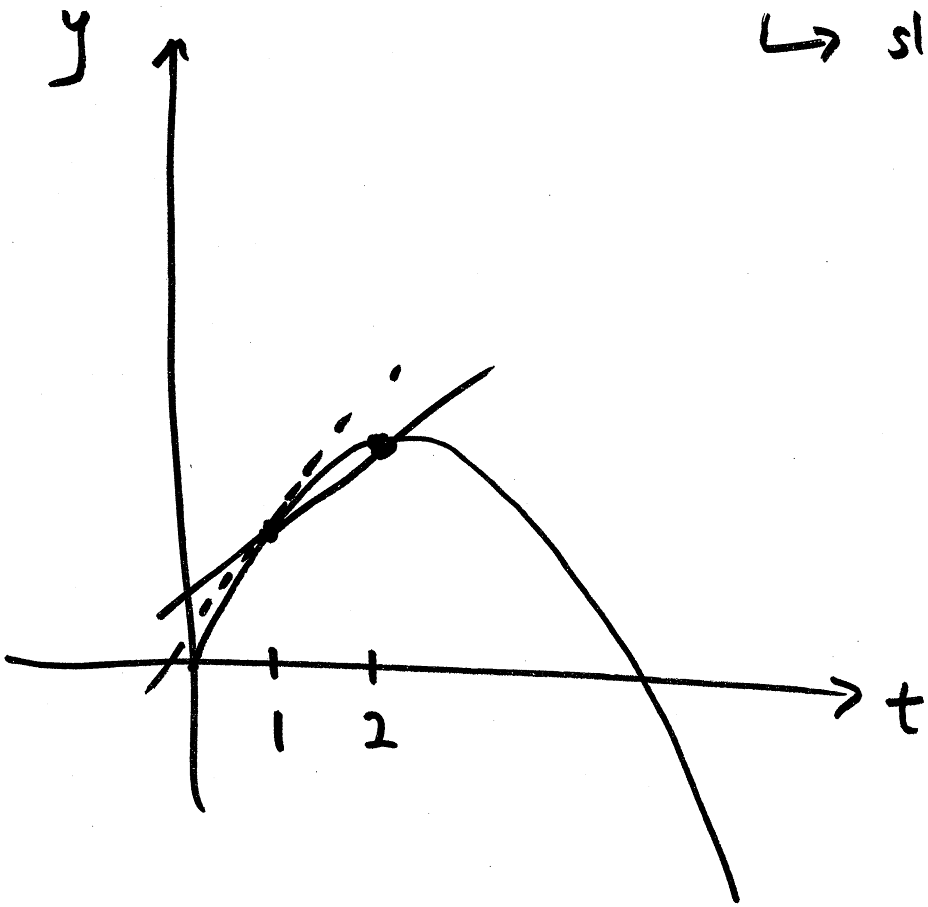
moves vertex

parabola

negative: opens down

Find: the velocity at the instant when $t = 1$

↳ slope of tangent at $t = 1$



~~ai~~ $P(1, 8.14)$ $\quad Q(t, y)$

Q at $t=2$ $\quad \dfrac{y(2)-y(1)}{2-1} = 4.42$ m/s (avg velocity between $t=1$ and $t=2$)

Q at $t=1.5$ $\quad \dfrac{y(1.5)-y(1)}{1.5-1} = 5.35$ m/s

Q at $t=1.1$ $\quad m_{PQ} = 6.09$

Q at $t=1.001$ $\quad m_{PQ} = 6.26$

so tangent line slope at $t=1$ is around 6.26

secant line : between 2 points

average rate of change

tangent line : at one point only

instantaneous rate of change

speedometer

secant line $\approx$ tangent line if interval is small