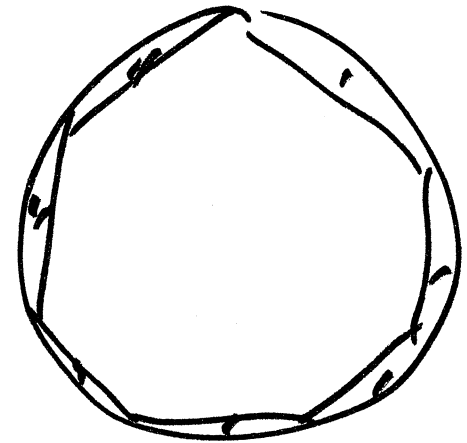
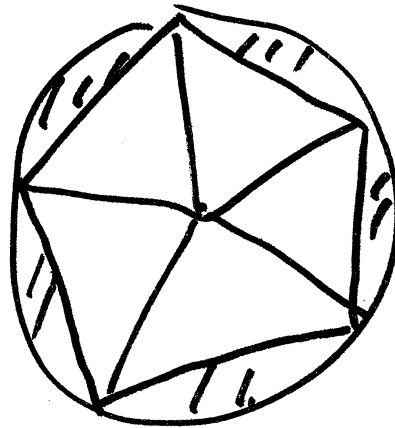
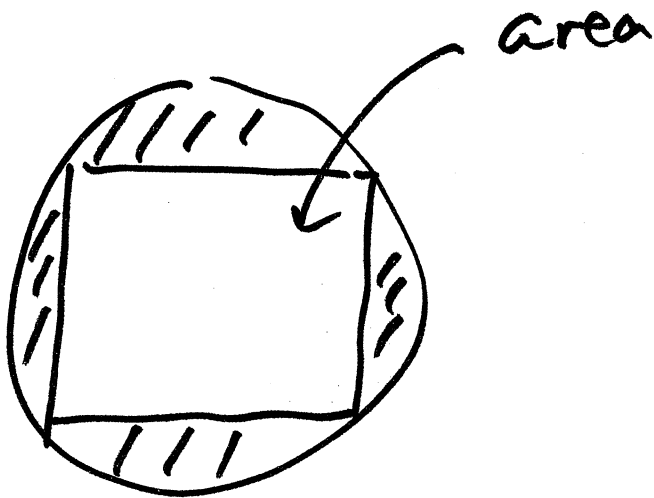


~~$$A = \pi r^2$$~~

How to calculate the
area of circle with
just a ruler and a compass



Limits

Area unaccounted for gets smaller
eventually polygon area \approx area of circle

$$f(x) = \frac{1}{x^2} \quad \text{domain: } (-\infty, 0) \cup (0, \infty)$$

$f(0)$ is undefined

What happens to $f(x)$ as x approaches 0?

	→ approach 0				← approach 0		
x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	100	10,000	1,000,000	X	1,000,000	10,000	100

unbounded
approach ∞

the limit of the function
 $f(x) = \frac{1}{x^2}$ as x approaches 0
is ∞

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$

in general, $\lim_{x \rightarrow a} f(x) = L$

as we make x arbitrarily close to a
(without getting to a), we make
 $f(x)$ arbitrarily close to L .

if $L \neq \pm \infty$

$$f(x) = \frac{1}{x} \quad \lim_{x \rightarrow 0} f(x) = ?$$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
f(x)	-10	-100	-1000	X	1000	100	10



approach large
negative number



approach large
positive number

no common goal in sight

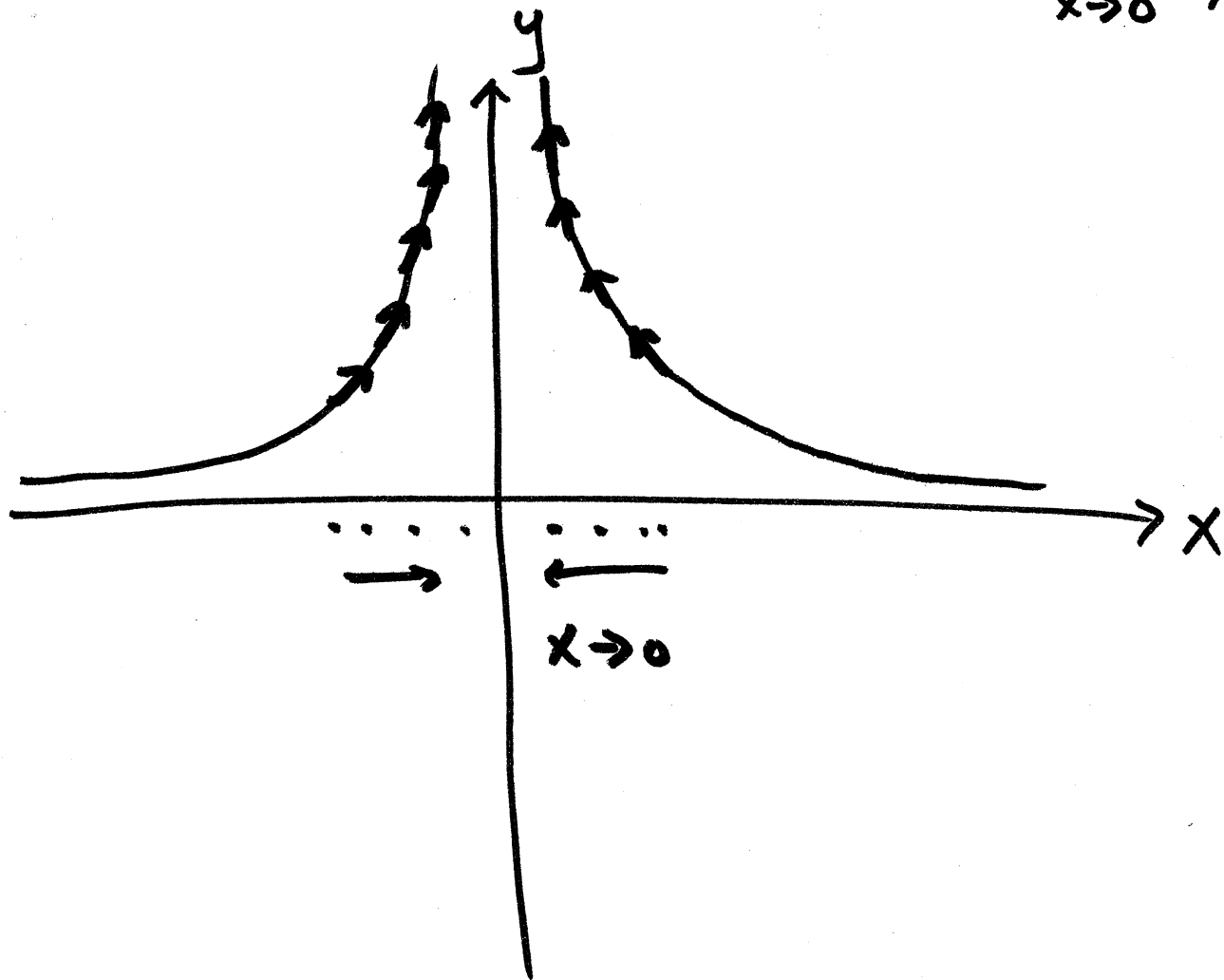
limit does not exist

$$\lim_{x \rightarrow 0} \frac{1}{x} \text{ DNE}$$

NOTE: NOT $\lim_{x \rightarrow 0} \frac{1}{x} = \text{DNE}$

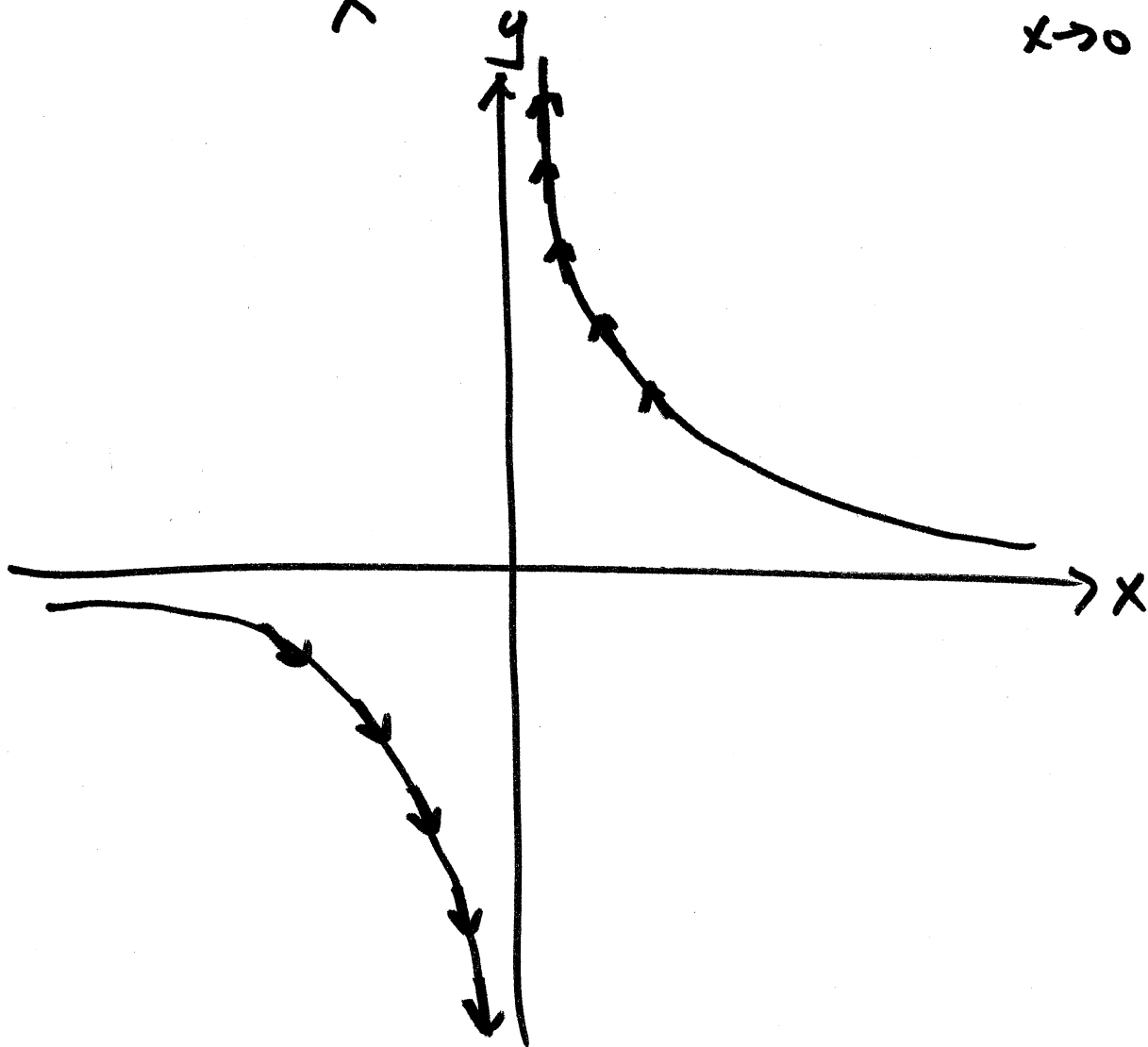
$$f(x) = \frac{1}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$$



$$f(x) = \frac{1}{x}$$

$$\lim_{x \rightarrow 0} \frac{1}{x} \text{ DNE}$$



$$f(x) = \frac{1}{x}$$

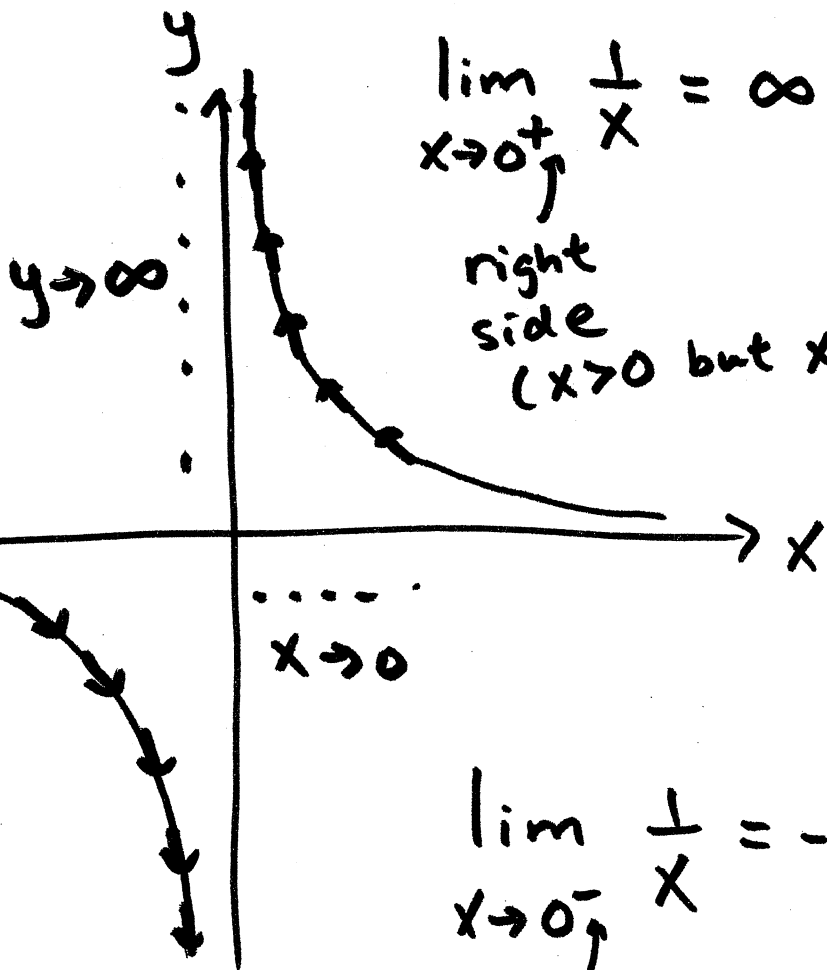
but only pay attention to one side
of $x=0$ one at a time

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$$

one-sided limit

right
side

($x > 0$ but $x \rightarrow 0$)

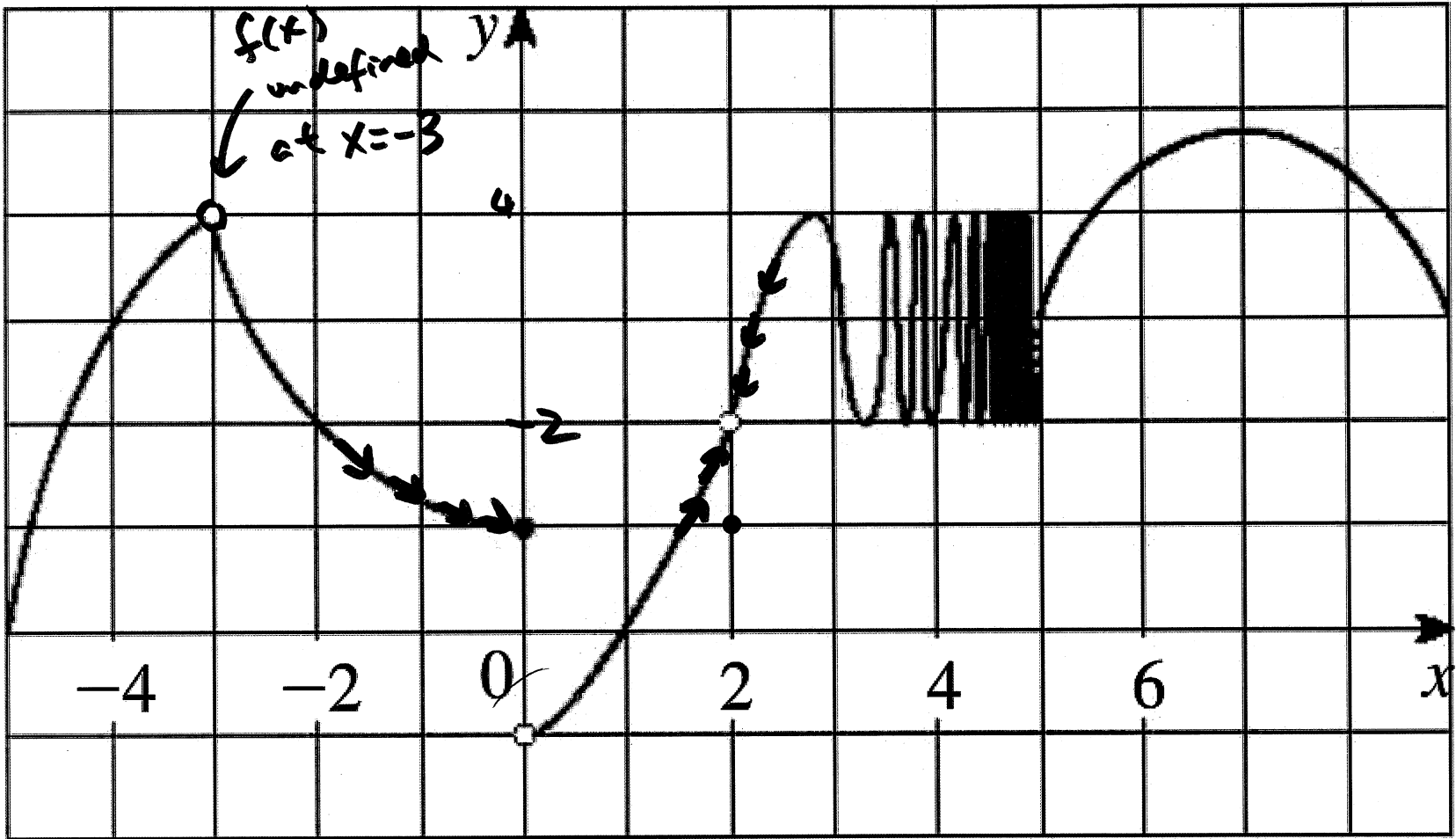


BUT $\lim_{x \rightarrow 0} \frac{1}{x}$ DNE

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$$

left side

($x < 0$ but $x \rightarrow 0$)



Find: $\lim_{x \rightarrow -3} f(x)$
 $= 4$

$\lim_{x \rightarrow 0} f(x)$ DNE
 $\lim_{x \rightarrow 0^+} f(x) = -1$

$\lim_{x \rightarrow 0^-} f(x) = 1$

$\lim_{x \rightarrow 2} f(x) = 2$, $f(2) = 1$

↙
 x close to 2, BUT NOT equal to 2

$$\lim_{x \rightarrow -9^-} \frac{x+10}{x+9} \quad 0 \quad \infty \quad \text{DNE} \quad \boxed{-\infty}$$

as $x \rightarrow -9^-$ ($x < -9$ & but close to -9)

$$\text{so } x+10 > 0$$

so $x+9 < 0$ but very small #

$$\frac{13}{0.000001}$$

$$\frac{\text{pos. \#}}{\text{very small neg. \#}} = -\infty$$

x	-9.1	-9.01	\dots
func			