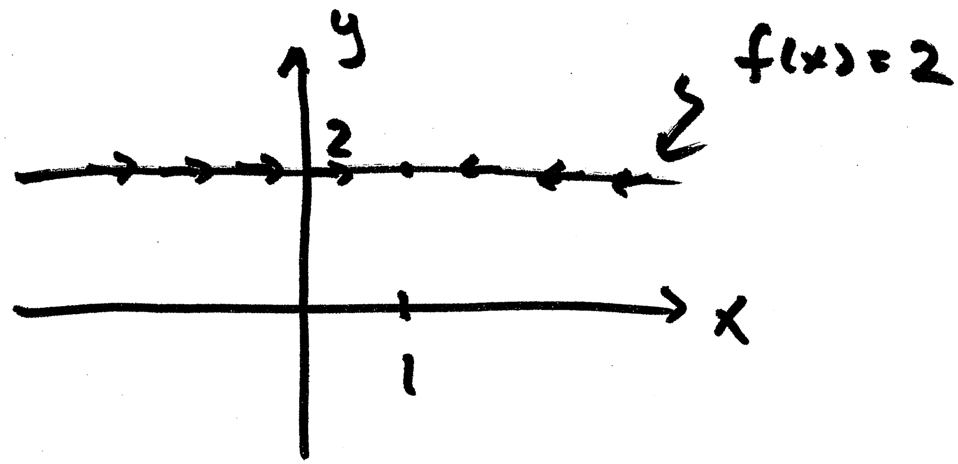
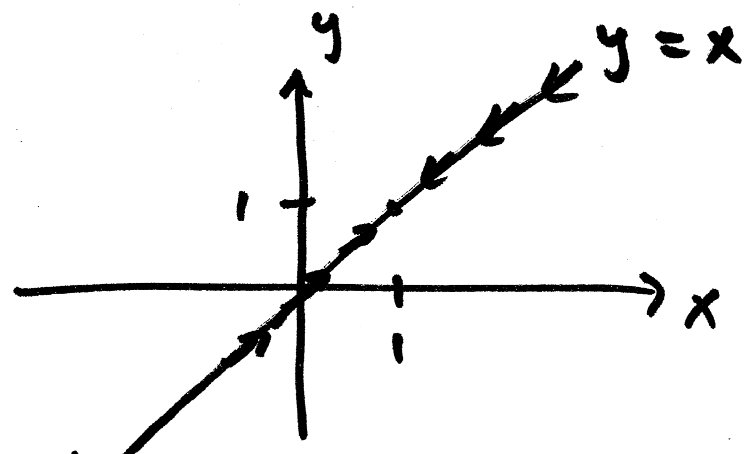
2.3 Calculating limits using limit laws

$$\lim_{x\to 1} 2 = 2$$



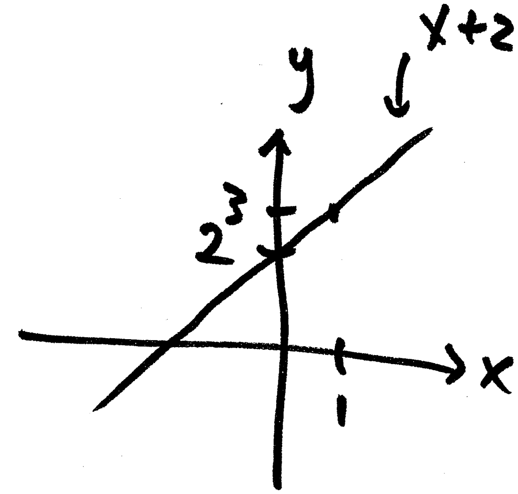
$$\boxed{\lim_{x\to a} c = c}$$

$$\lim_{x\to 1} x = 1$$



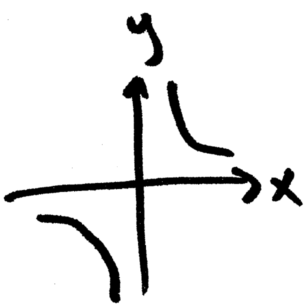
$$\lim_{x\to 1} 2 + \lim_{x\to 1} x = \lim_{x\to 1} (2+x)$$

$$2 \quad + \quad 1 \quad = 3$$



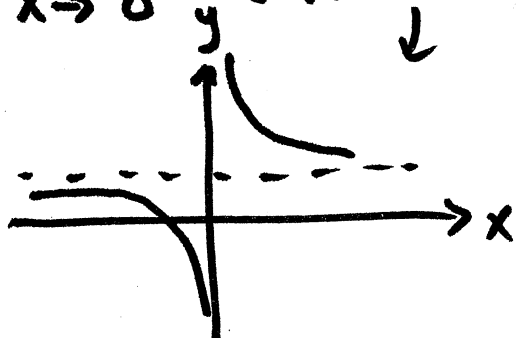
$$\boxed{\lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = \lim_{x\to a}\left[f(x)\pm g(x)\right]}$$

$$\lim_{x\to -3}\left(x^2-2x+5\right) = \lim_{x\to -3} x^2 - \lim_{x\to -3} 2x + \lim_{x\to -3} 5$$



$$= 9-(-6)+5 = 20$$

$$\lim_{x\to 0}\left(\frac{1}{x}+1\right) = \underbrace{\lim_{x\to 0}\left(\frac{1}{x}\right)}_{\text{DNE}} + \underbrace{\lim_{x\to 0} 1}_{1} \quad \text{DNE}$$



$$\lim_{x\to a} x^n = \left[\lim_{x\to a} x\right]^n \qquad \lim_{x\to -3} x^2 = \left[\lim_{x\to -3} x\right]^2$$

$$\cancel{\lim_{x\to a} f(x)}.$$

direct replacement of $x = -2$

example: $$\lim_{x\to -2} \frac{x^4 - 1}{2x^2 - 3x + 6} = \frac{(-2)^4 - 1}{2(-2)^2 - 3(-2) + 6} = \frac{3}{4}$$

$$= \frac{\lim_{x\to -2} x^4 - \lim_{x\to -2} 1}{\lim_{x\to -2} 2x^2 - \lim_{x\to -2} 3x + \lim_{x\to -2} 6} = \frac{16 - 1}{8 + 6 + 6}$$

$$= \frac{15}{20} = \boxed{\frac{3}{4}}$$

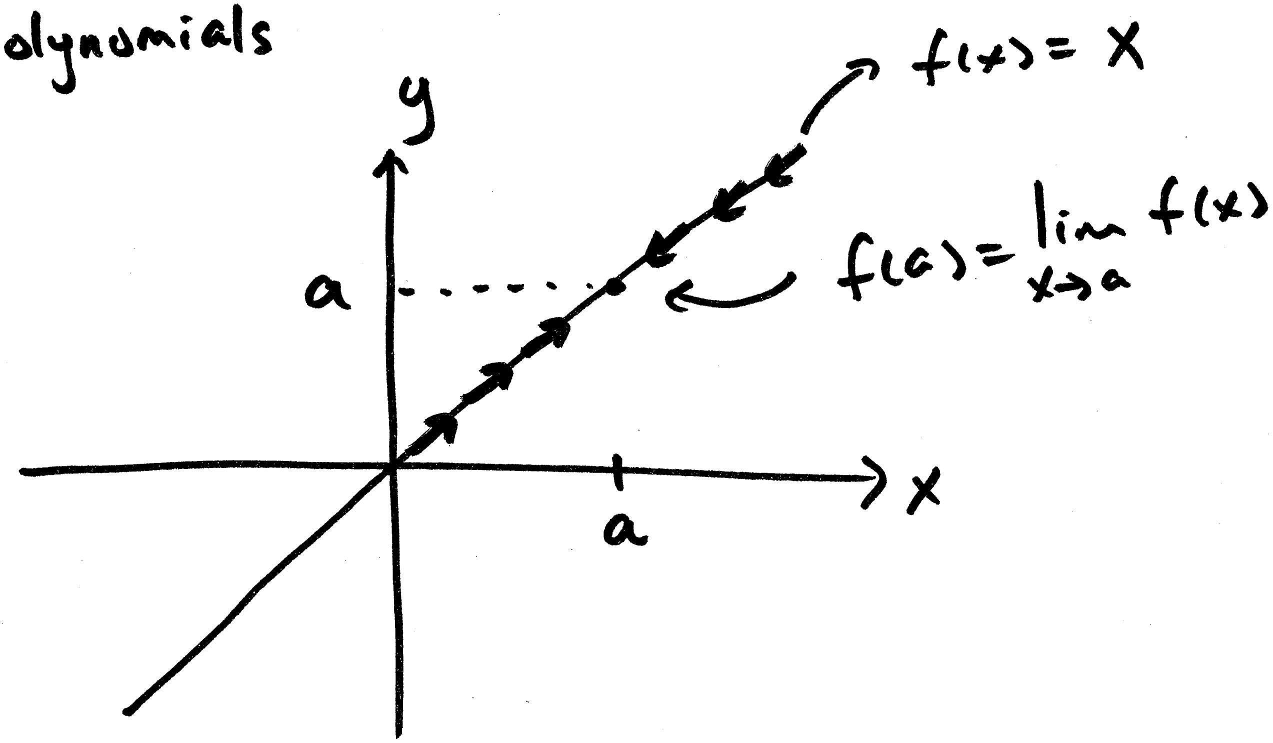
if $X = a$ is in the domain of a polynomial

or $\boxed{\text{ratio}}$nal function, then the limit

$\lim_{x \to a} f(x)$ is $f(a)$

ratio of polynomials

why?



example

$$\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4} = \frac{16-16}{16-12-4} = \underbrace{\frac{0}{0}}_{\text{NOT limit of this function}}$$

note $x=4$ is not in domain (denominator $= 0$)

indeterminant form

$$\lim_{x\to 4} \frac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+1)} = \lim_{x\to 4} \frac{x}{x+1} = \frac{4}{5}$$

$$\boxed{\lim_{x \to 4} \frac{x^2-4x}{x^2-3x-4} = \lim_{x \to 4} \frac{x}{x+1}}$$

but is $\dfrac{x^2-4x}{x^2-3x-4} = \dfrac{x}{x+1}$ true? <u>NO</u>

because function on the ~~fo~~ left has domain $x \neq 4$ whereas function on the right <u>IS</u> defined at $x=4$

$$\frac{x\cancel{(x-4)}}{\cancel{(x-4)}(x+1)}$$

the cancellation required

$x \neq 4$

limit is ok because $x \to 4$ does NOT mean $x=4$

example : $\lim_{h \to 0} \dfrac{\sqrt{4+h} - 2}{h}$ direct subs : $\dfrac{\sqrt{4}-2}{0} = \dfrac{0}{0}$

$$= \lim_{h \to 0} \left(\frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2} \right)$$

$$= \lim_{h \to 0} \frac{(\sqrt{4+h})^2 - 2^2}{h(\sqrt{4+h} + 2)}$$

$$= \lim_{h \to 0} \frac{4+h-4}{h(\sqrt{4+h}+2)} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{4+h}+2} = \boxed{\frac{1}{4}}$$

<u>example</u> $\lim_{x\to 0}\left(\frac{1}{x\sqrt{1+x}}-\frac{1}{x}\right)$

$$= \lim_{x\to 0}\left(\frac{1}{x\sqrt{1+x}}-\frac{\sqrt{1+x}}{x\sqrt{1+x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{1-\sqrt{1+x}}{x\sqrt{1+x}}\right) \quad \frac{0}{0}$$

$$= \lim_{x\to 0}\left(\frac{1-\sqrt{1+x}}{x\sqrt{1+x}}\cdot\frac{1+\sqrt{1+x}}{1+\sqrt{1+x}}\right)$$

$$= \lim_{x\to 0}\frac{1-(1+x)}{x\sqrt{1+x}\,(1+\sqrt{1+x})} = \lim_{x\to 0}\frac{-\cancel{x}}{\cancel{x}\sqrt{1+x}\,(1+\sqrt{1+x})}$$

$$= \boxed{\frac{-1}{2}}$$

Squeeze (or Sandwich) Theorem

$f(x) \le g(x) \le h(x)$ when x is near a

(except possibly at a) and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$$

then $\lim_{x\to a} g(x) = L$