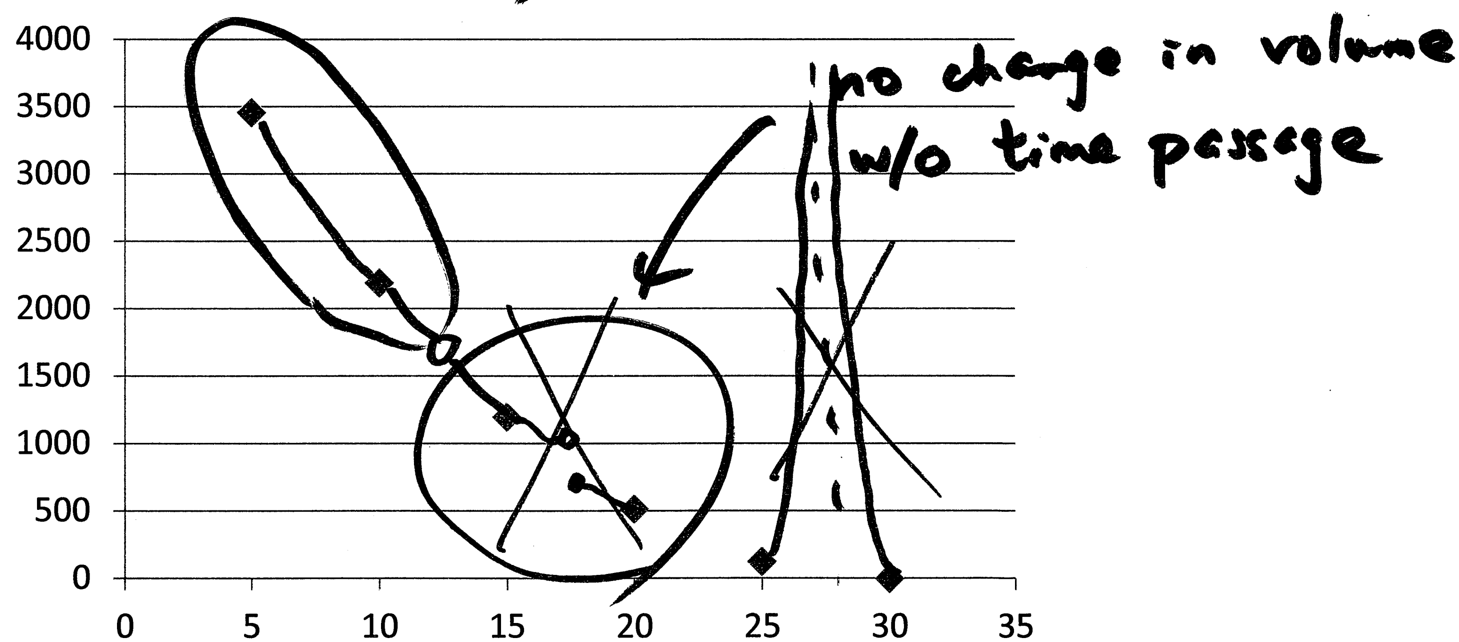
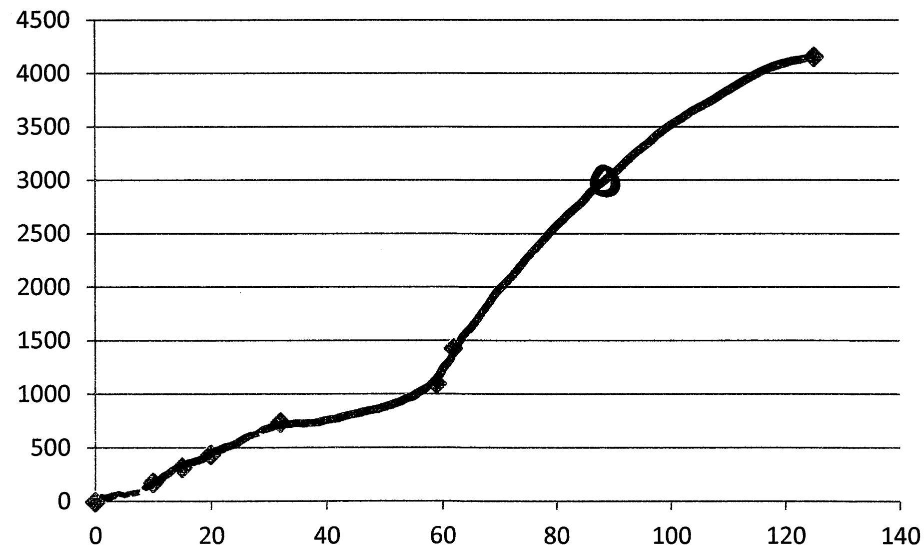
- The volume of water in a tank *t* minutes after it starts draining is

t (min)	5	10	15	20	25	30
V (gal)	3460	2195	1200	515	125	0

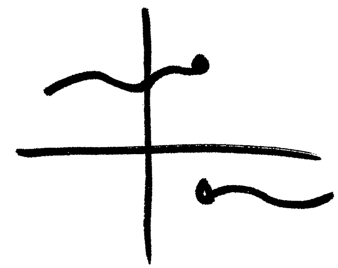


- Velocity history of the space shuttle

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	180
End roll maneuver	15	319
Throttle to 89%	20	442
Throttle to 67%	32	742
Throttle to 104%	59	1100
Maximum dynamic pressure	62	1430
Solid rocket booster separation	125	4151



2.5 Continuity



water tank, space shuttle velocity

→ continuous

no jump, no vertical asymptote, no hole

→ trace the graph w/o having to lift hand

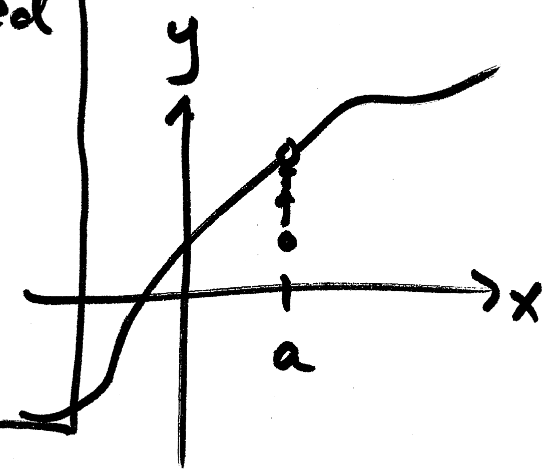
continuous at $x = a$

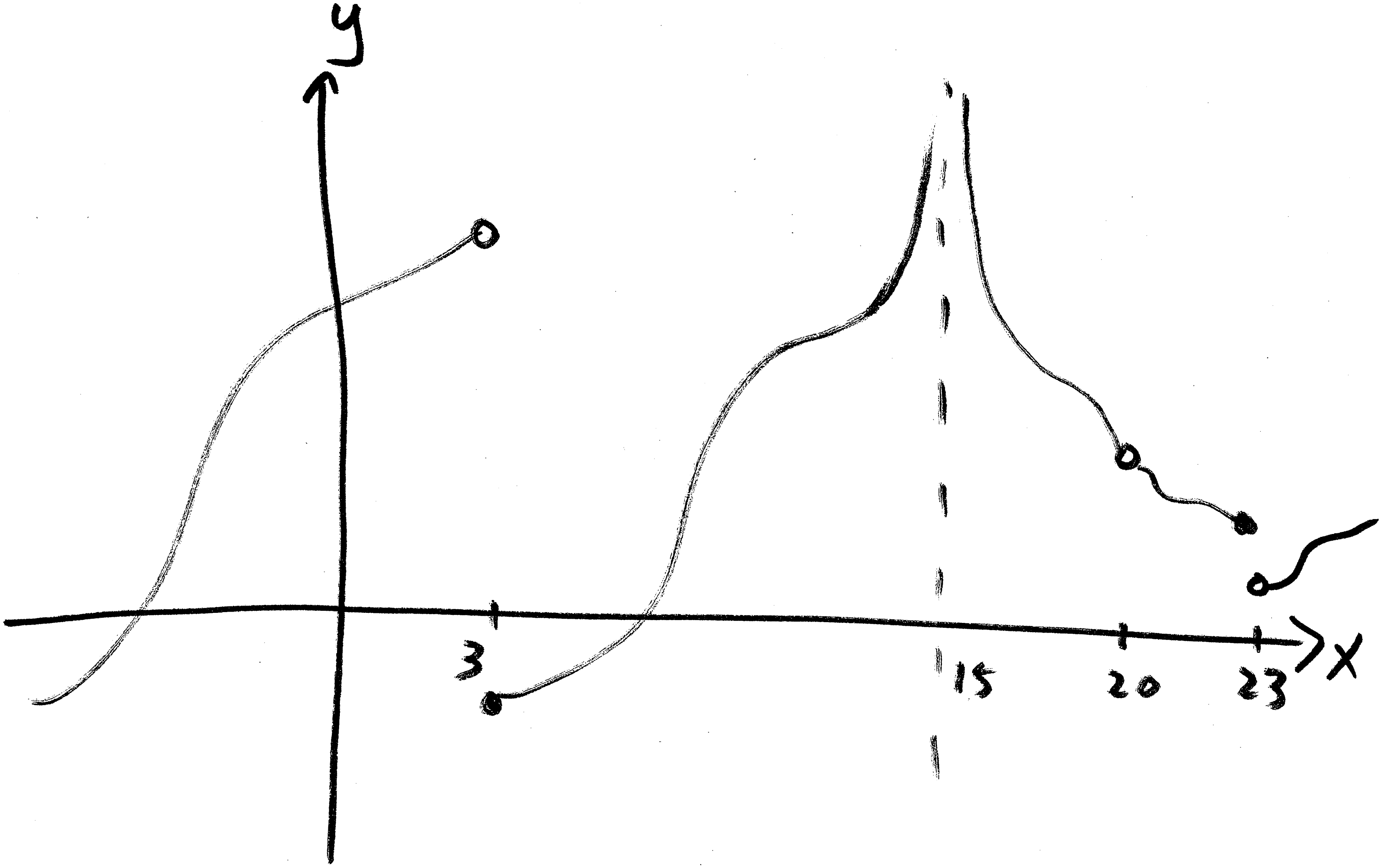
no open circle
no asymptote → $f(a)$ is defined

no jump → $\lim_{x \to a} f(x)$ exists

3 requirement

$$\lim_{x \to a} f(x) = f(a)$$





@ $x=3$ discontinuous but continuous from RIGHT
(can arrive at $x=3$ from right)

@ $x=15$ discontinuous (not continuous from either side)

@ $x=20$ " " " "

an open circle is a "mild" discontinuity that can be "fixed" or removed.

<u>example</u>

$$f(x) = \frac{x^2-4}{x-2}$$

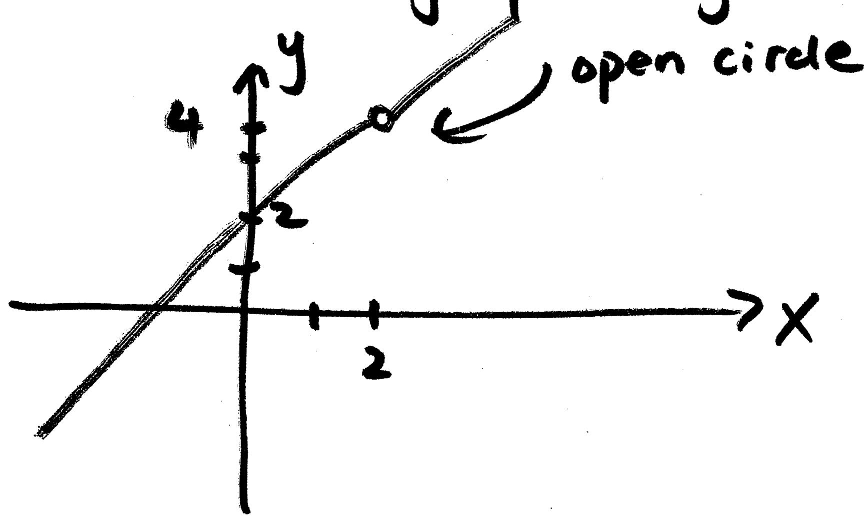
$\underbrace{\text{domain: } x \neq 2}$

so $f(2)$ not defined
not continuous there

$$= \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} \quad (x \neq 2)$$

$$= x+2 \quad (x \neq 2)$$

they have same graph anywhere $x \neq 2$



remove discontinuity by defining $\underbrace{f(2)=4}$

the missing dot is $(2,4)$

so $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$

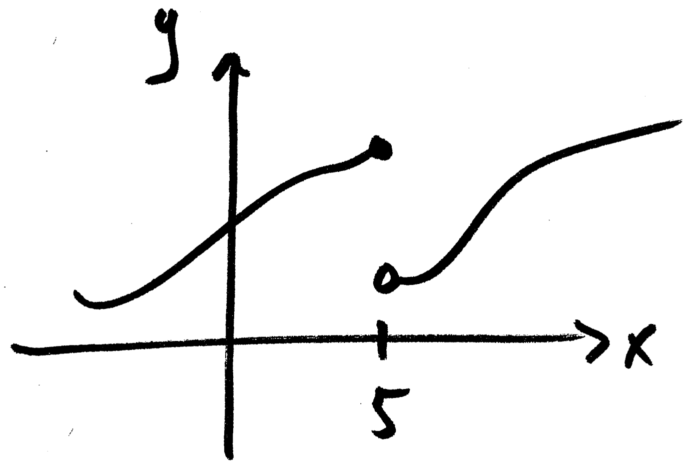
not same function that we started with

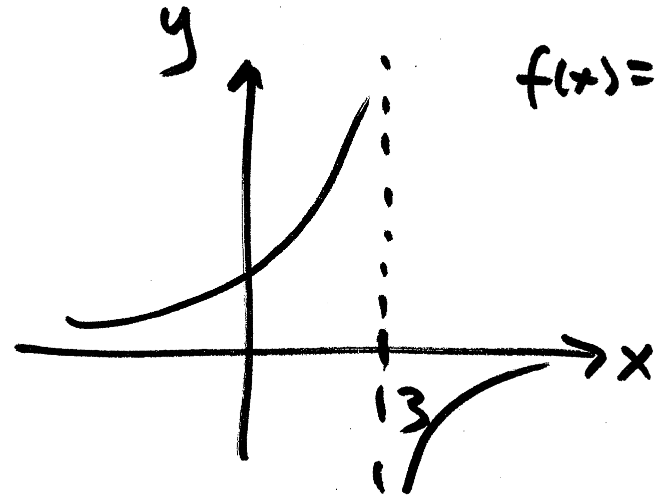
→ modify to remove discontinuity

equal to $f(x) = x+2$

open circle is a <u>removable discontinuity</u>

can't fix jump discontinuities or asymptotes





$f(x) = \frac{1}{3-x}$

many functions are <u>continuous on their domains</u>

polynomial

rational

root

trig $\longrightarrow$ $\tan x$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

inverse trig

exponential

logarithmic

<u>example</u> Is this function continuous?

$$f(x) = \begin{cases} x+2 & \text{if } x<0 \\ e^x & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x>1 \end{cases}$$

individual pieces are continuous

any discontinuity must be at where they branch

<u>@ $x=0$</u>

$f(0) = e^0 = 1$

$\lim_{x\to 0} f(x)$ exists? NO

NOT continuous b/c $\lim_{x\to 0} f(x)$ DNE

$$\left.\begin{aligned} \lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-}(x+2) = 2 \\ \lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+} e^x = 1 \end{aligned}\right\} \text{not equal}$$

@ $x = 1$

NOT continuous

$f(1) = e$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} e^x = e$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \cancel{2}(2-x) = 1$$

$$\lim_{x \to 1} f(x) \quad \text{DNE}$$

continuous from LEFT