MA 161 EXAM 1

Tuesday 9/23 6:30 pm (1-hr)

Location: CL 50 224

(the BIG lecture hall)

2.6 Limits at Infinity

$$f(x) = \frac{1}{x}$$

$$\underbrace{\lim_{x \to \infty}}_{\text{very large number}} f(x) \quad \text{or} \quad \underbrace{\lim_{x \to -\infty}}_{\text{very large negative number}} f(x)$$

x	100	1000	100,000	1,000,000,000
$\frac{1}{x}$	0.01	0.001	10^{-5}	10^{-9}

$$\lim_{x \to \infty} \frac{1}{x} = 0 \qquad \lim_{x \to -\infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} \frac{2}{x} = 2\left(\lim_{x \to \infty} \frac{1}{x}\right) = 0$$

$$\lim_{x \to \infty} \frac{1}{x^2} = \left(\lim_{x \to \infty} \frac{1}{x}\right)^2 = 0$$

denominator gets increasingly large while numerator does not

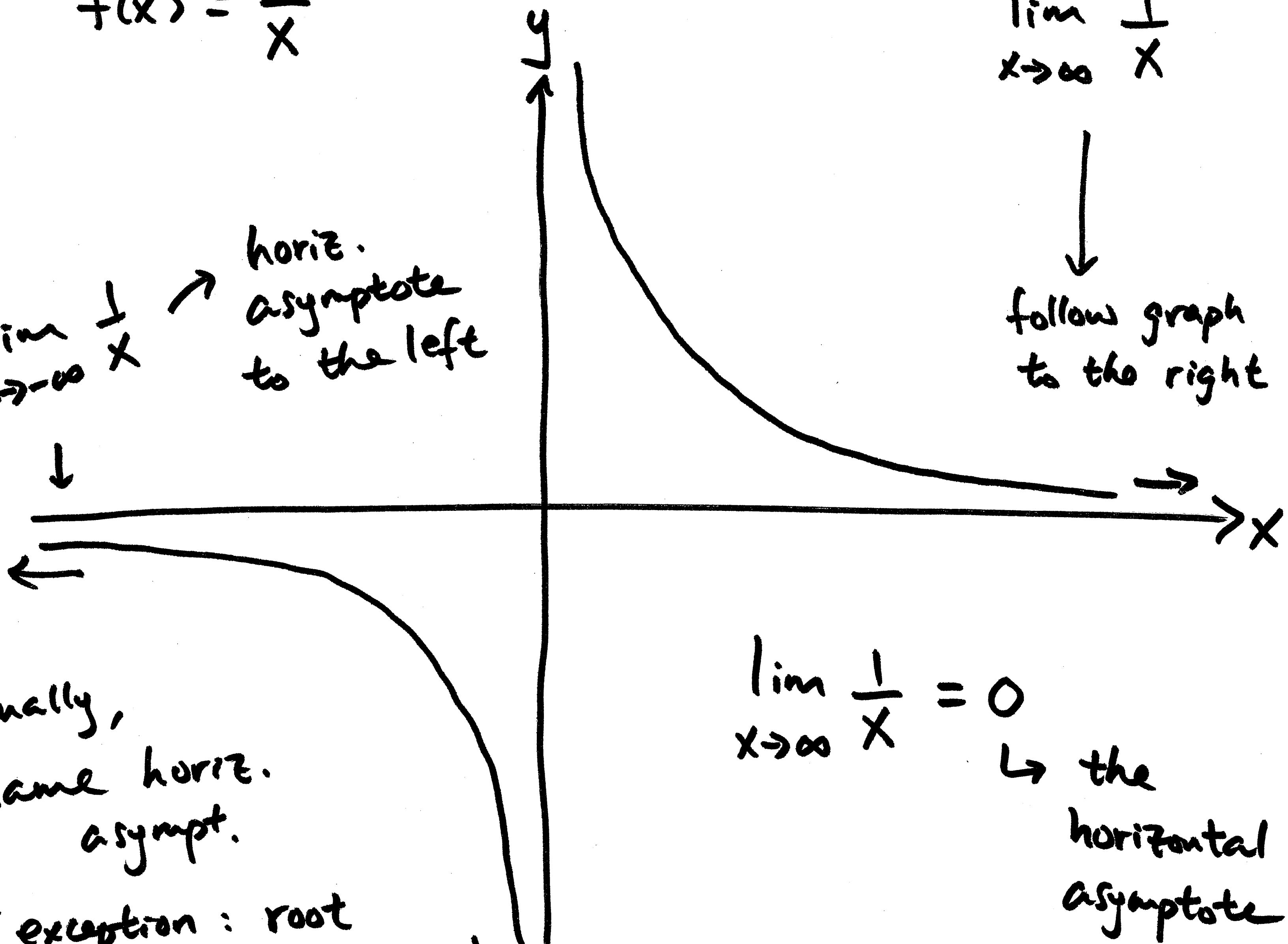
$$f(x) = \frac{1}{x}$$

$$\lim_{x \to \infty} \frac{1}{x}$$

follow graph to the right

$$\lim_{x \to -\infty} \frac{1}{x}$$

horiz. asymptote to the left



usually, same horiz. asympt.

(exception: root functions)

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

↳ the horizontal asymptote (to the right) example :

$$\lim_{x\to\infty} \frac{\frac{1}{x}+\frac{2}{x^3}}{x+5} \qquad \frac{\text{very small \#}}{\text{very large \#}}$$

$$= \frac{\lim_{x\to\infty}\left(\frac{1}{x}+\frac{2}{x^3}\right)}{\lim_{x\to\infty}(x+5)} = \frac{\lim_{x\to\infty}\frac{1}{x}+\lim_{x\to\infty}\frac{2}{x^3}}{\lim_{x\to\infty}x+\lim_{x\to\infty}5}$$

0 means "very small number"

$$= \frac{0+0}{\infty+5 \sim 5} = 0$$

very large number

example

$$\lim_{x\to\infty} \frac{7x^2 - x + 6}{4x^2 + 5x - 9}$$

numerator becomes lage

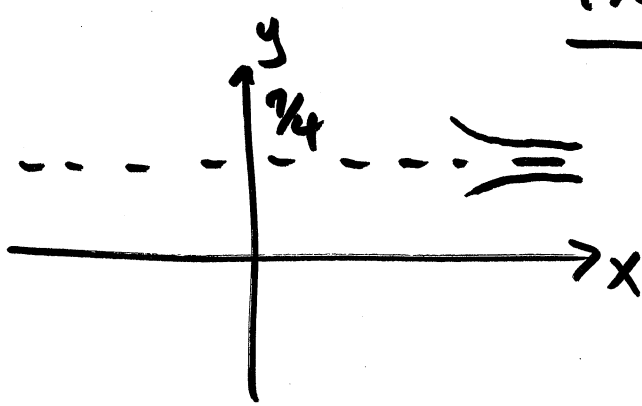
denominator " "

first, want $\dfrac{c}{x^n}$ in rational function

divide top and bottom by x^n
where n is the highest power
of x in denominator

$$= \lim_{x\to\infty} \frac{\dfrac{7x^2 - x + 6}{x^2}}{\dfrac{4x^2+5x-9}{x^2}} = \lim_{x\to\infty} \frac{7 - \frac{1}{x} + \frac{6}{x^2}}{4 + \frac{5}{x} - \frac{9}{x^2}}$$

$$= \boxed{\frac{7}{4}}$$



example:

$$\lim_{x\to-\infty} \frac{x-1}{x^2+5} \quad \begin{matrix} \to -\infty \\ \to \infty \end{matrix}$$

$$= \lim_{x\to-\infty} \frac{\frac{x-1}{x^2}}{\frac{x^2+5}{x^2}} = \lim_{x\to-\infty} \frac{\underbrace{\frac{1}{x}}_{\to 0} - \underbrace{\frac{1}{x^2}}_{\to 0}}{1 + \underbrace{\frac{5}{x^2}}_{\to 0}}$$

$$= 0$$

example:

$$\lim_{x\to-\infty} \frac{x^2+5}{x-1}$$

$x^2+5 \to$ very large pos. #

$x-1 \to$ ~~slightly~~ smaller large neg. #

$= -\infty$ (DNE)

if numerator and denom. have same <u>degree</u>

limit is ratio of coefficients of largest terms

if num. has <u>higher</u> degree

limit is $\pm \infty$

if num. has <u>lower</u> degree

limit is 0

<u>example</u>

$$\lim_{x\to\infty}\left(\underbrace{\sqrt{4x^2+x}}_{\text{large \#}}\ -\underbrace{2x}_{\text{large \#}}\right)$$

$$= \lim_{x\to\infty}\left(\frac{\sqrt{4x^2+x}\ -2x}{1}\cdot\frac{\sqrt{4x^2+x}\ +2x}{\sqrt{4x^2+x}\ +2x}\right)$$

$$= \lim_{x\to\infty}\frac{(4x^2+x)-(2x)^2}{\sqrt{4x^2+x}\ +\ 2x}$$

large #

$$= \lim_{x\to\infty}\frac{x}{\underbrace{\sqrt{4x^2+x}\ +2x}_{\text{large \#}}}$$

when $x\to\infty$ denom. has degree 1

$$= \lim_{x\to\infty} \frac{\frac{x}{x}}{\frac{\sqrt{4x^2+x} + 2x}{x}}$$

$$\frac{\sqrt{4x^2+x}}{x} = \frac{\sqrt{4x^2+x}}{\sqrt{x^2}}$$

$$= \sqrt{\frac{4x^2+x}{x^2}}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{\frac{4x^2+x}{x^2}} + 2} = \frac{1}{4}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{4+\frac{1}{x}} + 2}$$