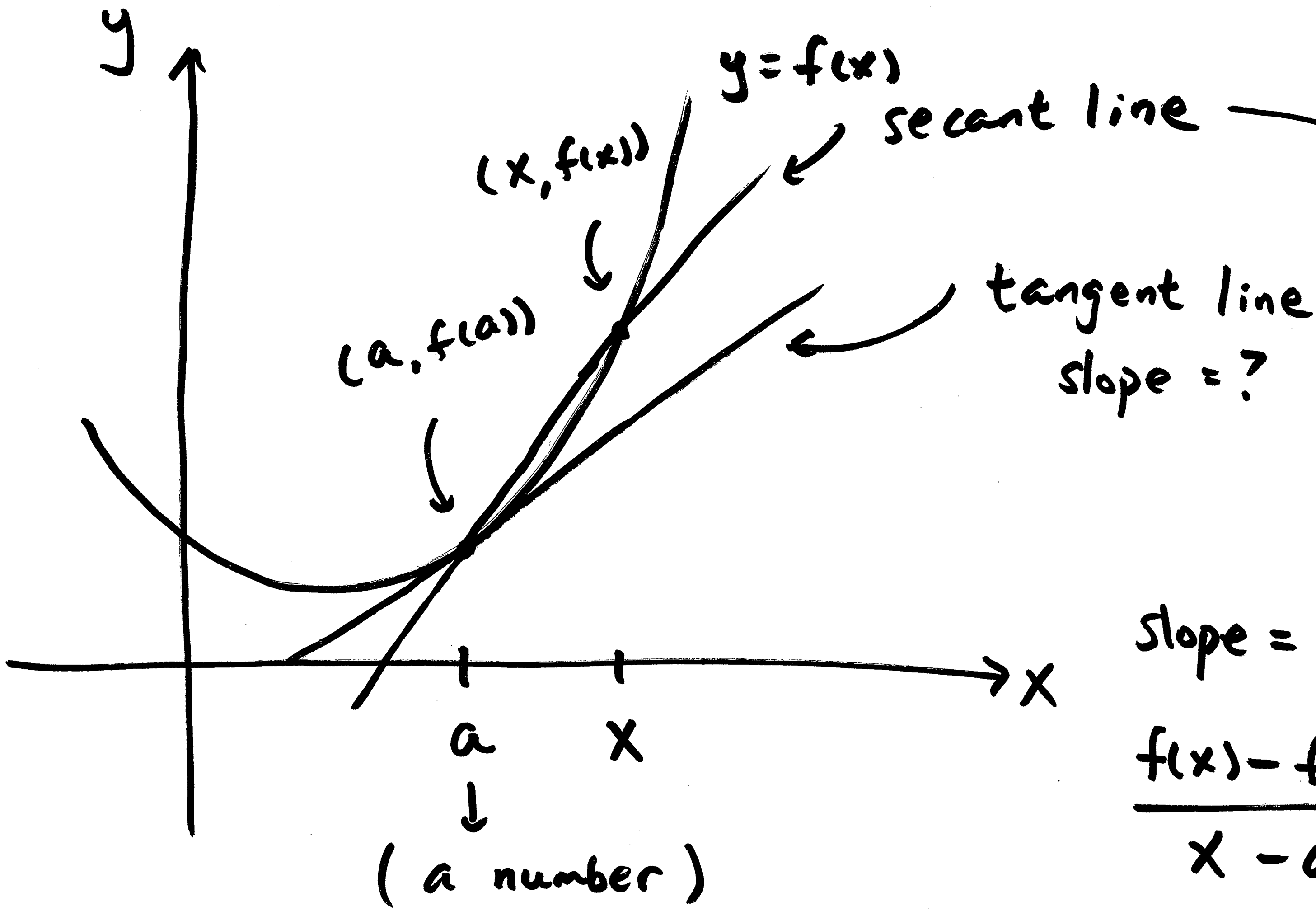
MA 161 EXAM 1

Tuesday 9/23 6:30 pm (1-hr)

Location: CL 50 224

(the BIG lecture hall)

2.7 Derivatives and Rates of Change



slope = $\dfrac{f(x) - f(a)}{x - a}$

secant → tangent as $x \to a$ ⇒ limit!

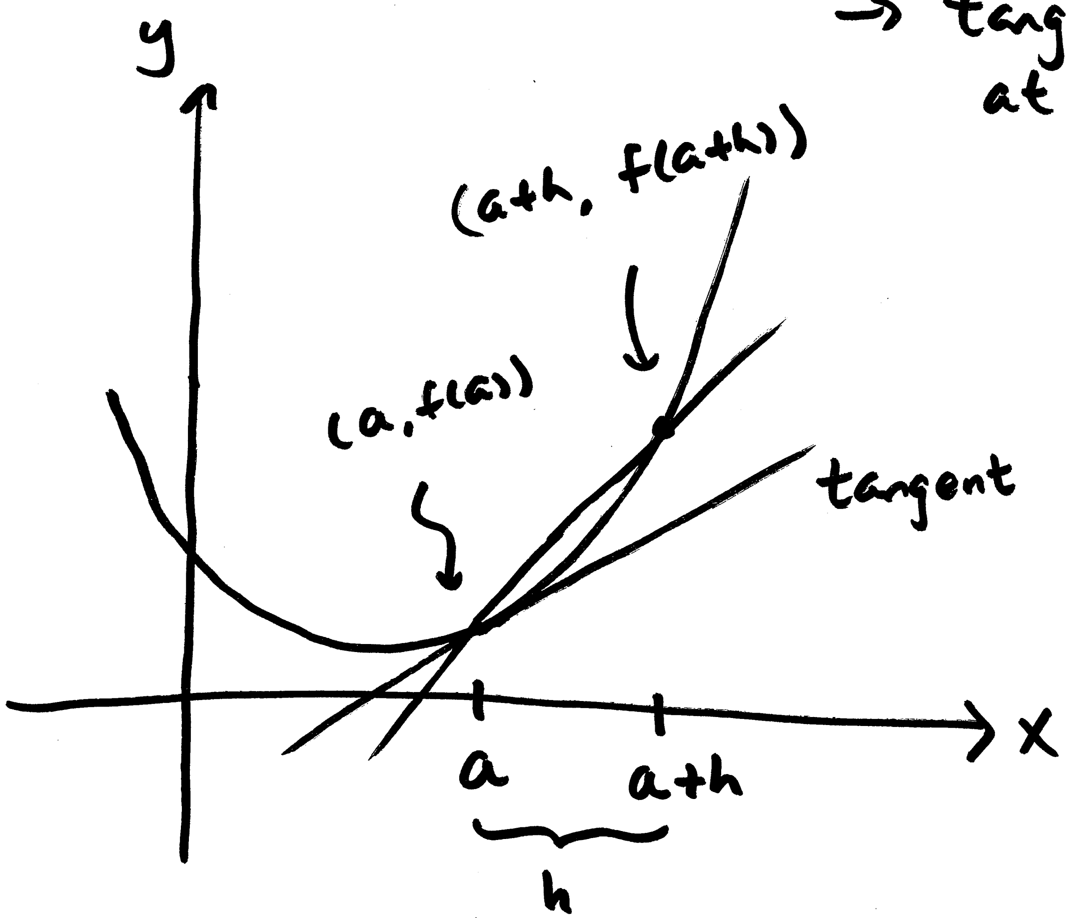
secant → tangent

"prime"

$$\boxed{\lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \underbrace{f'(a)}_{\text{"f prime of a"}}}$$

derivative

→ tangent line slope at $x = a$



$$\frac{f(a+h)-f(a)}{\cancel{a}+h-\cancel{a}}$$

$$\boxed{f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}}$$

example: find slope of tangent line to

$$f(x) = 8x - x^2 \quad \text{at} \quad (\underline{\underline{1}}, 7)$$

first form: $$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

here, $a = 1$

$$f'(1) = \lim_{x \to 1} \frac{\overbrace{(8x - x^2)}^{f(x)} - \overbrace{7}^{f(a)}}{x - 1}$$

$$= \lim_{x \to 1} \frac{-(x^2 - 8x + 7)}{(x-1)} = \lim_{x \to 1} \frac{-(x-7)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= -(1-7) = \boxed{6}$$ slope of tangent line at $x = 1$

the other form: $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$$f(x) = 8x - x^2 \qquad a = 1$$

$$f'(1) = \lim_{h \to 0} \frac{\overbrace{8(1+h) - (1+h)^2}^{f(1+h)} - \overbrace{7}^{f(1)}}{h}$$

$$= \lim_{h \to 0} \frac{8 + 8h - (1 + 2h + h^2) - 7}{h}$$

$$= \lim_{h \to 0} \frac{8 + 8h - 1 - 2h - h^2 - 7}{h} = \lim_{h \to 0} \frac{-h^2 + 6h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-h + 6)}{\cancel{h}} = \boxed{6}$$

Second form more useful if we leave
a as a (no number specified).

$f(x) = 8x - x^2$ @ $x = a$

<u>Form 1</u>:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{8x - x^2 - \overbrace{\boxed{[(8a - a^2)]}}^{f(a)}}{x - a}$$

if $f(x)$ complicated, hard to
find a factor of $x - a$
from numerator

recall: secant line slope → <u>average</u> rate of change

tangent line slope → <u>instantaneous</u> rate of change

(or just "rate of change")

<u>example</u>: Height of an object thrown up in the air is $y = 50t - 16t^2$ ft, second

Find: velocity when $t = 2$

how fast was it when it hits the ground?

given height/distance, want velocity

so we want y' at $t=2$

use 2nd form, leave $t=a$ (since need to evaluate at two different times)

$$y = 50t - 16t^2 \qquad y'(a) = \lim_{h\to 0} \frac{y(a+h) - y(a)}{h}$$

$$\begin{aligned} y(a+h) &= 50(a+h) - 16(a+h)^2 \\ &= 50a + 50h - 16(a^2 + 2ah + h^2) \\ &= 50a + 50h - 16a^2 - 32ah - 16h^2 \end{aligned}$$

$$y(a) = 50a - 16a^2$$

$$y(a+h) - y(a) = 50h - 32ah - 16h^2$$

$$\frac{y(a+h)-y(a)}{h} = \frac{\cancel{h}(50 - 32a - 16h)}{\cancel{h}}$$

$$y'(a) = \lim_{h \to 0} (50 - 32a - 16h) = \boxed{50 - 32a = v(a)}$$

$$v(2) = 50 - 64 = -14 \quad \text{ft/s} \qquad \text{speed} = 14 \text{ ft/s}$$

downward

need time to impact ground, then $v(a)$ at that time

$$y = 50t - 16t^2 \qquad \text{ground: } y = 0$$

$$0 = 50t - 16t^2$$

$$= 2t(25 - 8t)$$

$$t = 0, \quad \boxed{t = \frac{25}{8}}$$

$$v\left(\frac{25}{8}\right) = 50 - 32\left(\frac{25}{8}\right)$$

$$= -50 \quad \text{ft/s}$$