Mathematics and Scientific Representation

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I. Summary

This book aims to investigate the philosophical consequences of the central role of mathematics in contemporary science. This is a perennial question for scientists, mathematicians, historians and philosophers, but much of the traditional discussion is hampered by a poorly framed worry or a selection of a few puzzling examples. The book will pursue the issue with a newly developed version of the following central questions: for each scientific representation, what does the mathematics contribute, how does the mathematics make this contribution and what does this contribution presuppose? I argue that there are five different kinds of contributions and structure my discussion around examples that fall naturally into these five kinds. The main conclusion of the book is that mathematics makes an epistemic contribution to the success of our scientific representations. Epistemic contributions include aiding in the confirmation of the accuracy of a given representation through prediction and experimentation. But they extend further into considerations of calibrating the content of a given representation to the evidence available, making an otherwise irresolvable problem tractable, and offering novel insights into the nature of physical systems. As part of the success of science is the fact that we take our evidence to confirm the accuracy of our best scientific representations, it is here that the mathematical character of these representations makes its decisive mark.

II. Outline

1. Introduction

The problem is introduced and the general solution in epistemic terms is contrasted with other viable solutions. Five different sorts of contributions are distinguished and illustrated in a preliminary way: (i) concrete causal, (ii) abstract acausal, (iii) abstract varying, (iv) relations of scale, (v) constitutive frameworks.

2. Content and Confirmation

A proposal for how mathematics contributes to the content of a given scientific representation is made. This leads to a discussion of the related issues of modeling and idealization. The second half of the chapter concerns the confirmation of representations. A discussion of standard approaches to confirmation theory and how they can accommodate the proposed role for mathematics suggests an unappreciated problem for confirmation theory. Special attention is given to the difference between an intrinsic and extrinsic role for mathematics in fixing the content of a representation. I argue that different approaches to confirmation theory face different challenges with respect to intrinsic and extrinsic mathematics.

3. Causes

The central contrast here is between a concrete, causal representation of a system and an abstract, acausal representation of that system. Acausal representations typically result from causal representations through mathematical transformations, so here we have one important kind of mathematical contribution. Central examples are equilibrium representations of systems. I explore to what extent these representations are more easily confirmed and argue that we can trace these features to the mathematics.

4. Varying Representations

The main axis of the discussion of this chapter is the difference between a representation with a fixed physical interpretation and a family of representations which result from varying the physical interpretation of some of the mathematics. The resulting family is united by mathematical similarities and it is often these similarities which make the members of the family more attractive. I argue that this is another crucial contribution from mathematics that contributes to the confirmation of scientific representations.

5. Scale Matters

Mathematics may link representations at different scales for magnitudes such as time, space and energy. Sometimes a series of representations can be found that are mathematically similar across a wide range of temporal or spatial scales. These self-similar representations give important insights into the nature of the systems represented. Another distinct way in which scale is important is when two different representations are directed at a system at different scales, e.g. microscale vs. macroscale. I explore the parallels between these sorts of families of representations and what was discussed in chapters 3 and 4. New issues arise for scales, especially the mathematical techniques of scaling and perturbation theory. These prove to be especially important for a proper understanding of idealizations which involve limits and infinite quantities.

6. Constitutive Frameworks

Philosophers such as Carnap, Kuhn and Michael Friedman have argued for a special role for constitutive frameworks in allowing scientific knowledge, and mathematics often plays a prominent role in setting up these frameworks. In this chapter I determine whether or not there is a special contribution for mathematics here and conclude that these framework principles provide an agent with determinate beliefs in the range of physical possibilities that can obtain for a given kind of system. Again, then, the link between mathematics and science is made via confirmation, but in a distinct way than what we saw in chapters 3-5. A central issue turns out to be the epistemic status of the framework principles themselves. I propose that a priori sources of justification need to be available to clarify this sort of contribution, but defer the discussion of these sources until ch. 12.

7. Failures

To provide a useful contrast with all the ways in which mathematics can help science, I here provide a discussion and potential diagnosis of when mathematics stands in the way of scientific progress. An important strand of the cases discussed is the danger of excessive reliance on mathematics without any underlying well-motivated physical interpretation. I argue that the very interpretative flexibility of a given mathematical formalism that was exploited to the benefit of the confirmation of a representation in earlier examples can also pose a danger. What results when things go badly is a mathematical representation whose limitations are not recognized until empirical and practical failures emerge. This poses a problem for certain kinds of scientific realism.

8. Discovery

Starting with Wigner's famous "Unreasonable Effectiveness" paper, there has been much discussion of the surprising or miraculous results of relying on mathematics to generate new scientific discoveries. I critically evaluate this argument with special reference to Mark Steiner's book *The Applicability of Mathematics as a Philosophical Problem*. I conclude that mathematics has no special or mysterious part to play in scientific discovery and that this negative point is consistent with my positive point about mathematics and scientific confirmation.

9. Indispensability and Explanation

Most of the discussion in philosophy of mathematics concerned with applicability is occupied with the 'indispensability argument' found in Quine, Putnam and, more recently, Colyvan. The argument is that the indispensable or essential role of mathematics in our best science constitutes some kind of empirical argument for mathematical platonism, i.e. the view that mathematics is about abstract objects. I return to this argument in light of the framework developed in previous chapters and conclude that it is not convincing. One problem is that the contributions that I have delineated require that mathematics have some support prior to being used in science. A more significant problem is that there are several viable non-platonist interpretations of mathematics that do a good job at accounting for how our mathematical scientific representations work. These points are pursued for both the traditional indispensability argument and more recent versions which emphasize the explanatory role of mathematics. This leads into an investigation of Batterman's views on asymptotic explanation and the special way in which mathematics can contribute in such cases.

10. Fictionalism

An increasingly popular way to reject platonism about mathematics is called fictionalism, and in this chapter I discuss whether or not a fictionalist can make sense of the contributions made by mathematics that I have enumerated in chapters 3-6. Focusing on the views of Yablo, Balaguer, Leng and Rayo, I conclude that fictionalists have yet to overcome what I call the 'export problem' of explaining when fictional claims can be taken to represent something genuine about the world. As this is the central issue in evaluating applied mathematics, I argue that fictionalism is not a viable interpretation of mathematics. So, even though the role of mathematics in science is not sufficient to support platonism, as I argued in chapter 9, it is able to rule out fictionalism.

11. Facades

This chapter marks an extended engagement with Mark Wilson's book *Wandering Significance: An Essay on Conceptual Behavior*. Most of this book is occupied with the lessons about concepts and representations that should be drawn from the use of mathematics in scientific representation. Several aspects of Wilson's account seem to be in tension with the views I develop earlier in the book, and I aim to reconstruct these arguments and see how damaging they really are for my account. The main assumption that I make in earlier chapters and which Wilson challenges is that scientists are able to refer to physical properties prior to developing their mathematical representations. I argue that even if Wilson is right on this point, the epistemic picture of the contributions from mathematics that I develop can be sustained.

12. Conclusion: Pure Mathematics

This concluding chapter will summarize the resulting conception of pure mathematics. I argue that we cannot conclude much about the proper interpretation of mathematics from its role in science beyond the failure of fictionalism and some kind of non-scientific confirmation of some core mathematical beliefs. These minimal conclusions are then seen to be consistent with some forms of mathematical structuralism, although they do not mandate this interpretation. They also seem to be consistent with views of a priori justification which emphasize the possession conditions of our concepts, as with Peacocke's moderate rationalism. Still, I argue that an adequate epistemology for mathematics has not yet been achieved along these lines.