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AN EFFICIENT ROUTING STRATEGY ON SPATIAL SCALE-FREE NETWORKS

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Traffic dynamics has drawn much more attention recently, but most current research barely considers the space factor, which is of critical importance in many real traffic systems. In this paper we focus our research on traffic dynamics of a spatial scale-free network with the restriction of bandwidth proportional to links' Euclidean distance, and a new routing strategy is proposed with consideration of both Euclidean distance and betweenness centralities of edges. It is found that compared with the shortest distance path strategy and the minimum betweenness centralities of links strategy, our strategy under some parameters can effectively balance the traffic load and avoid excessive travelling distance which can improve the spatial network capacity and some system behaviors reflecting transportation efficiency, such as average packets traveling time, average packets waiting time and system throughput, traffic load and so on. Besides, though the restriction of bandwidth can trigger congestion, the proposed routing strategy always has the best performance no matter what bandwidth becomes. These results can provide

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insights for research on real networked traffic systems.

Keywords: Networked traffic; Complex network; Routing strategy; Spatial network; Betweenness centralities

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1. Introduction

With the development of the economy, many complex network systems (such as air traffic system, Internet and World Wide Web) play more and more important role in modern society.[1−5] However, their performance has been approaching to their limit because of the increasing serious congestion condition which has also incurred huge financial losses.[6−9] How to relieve congestion on these systems has thus promptly become a hot and intriguing research topic in different areas. Especially, network traffic has attracted much more attention in the past decade due to these complex systems, which can be abstracted into networks.[10−12]

Network traffic models are proposed to mimic the traffic process. Based on some routing protocols, R packets are produced to travel in the network whose sources and destinations are chosen randomly.[13−15] As R increases, the network will become more and more congested. The capacity of a network is defined by a critical value R_c, at which a continuous transition occurs from free flow to congestion.[16−17]

Normally it is expensive to modify the network and thus adopting efficient routing strategies seems to be more practical to improve the traffic efficiency. Therefore, various routing strategies have been studied.[18−21] The random walk strategy has been researched at the very beginning.[22] However, the real traffic behavior is not random but rather purposeful. The shortest path strategy is widely adopted in literatures and real life,[23] but it can easily cause the failure of hub routers with high degree and high betweenness. Yan et al. proposed an efficient routing strategy via redistributing traffic loads from central nodes to other non-central nodes, which can improve the network capacity more than ten times.[24] Ling et al. introduced a global dynamic routing strategy for networks and it is found that the system capacity is almost two times as much as that with the efficient routing strategy.[25] Considering the combination of static structural properties and dynamic traffic conditions together, Xia et al. proposed a hybrid routing strategy.[26] The routing strategies mentioned above need the knowledge of the whole system, which will become impractical if the network size is huge. Consequently, routing strategies using local topological information has been studied. With consideration of local topological information, Wang et al. presented the nearest neighbor searching strategy.[27] Hu et al. proposed a local routing strategy based on the local information on link bandwidth.[28]

However, current works mainly ignore the space factor. In fact, many real networks are embedded into a two- or three-dimensional space with spatial constraints, such as transportation network and wireless communication networks.[29−30] Moreover, it is found that the space factor have important effects on a networks topological properties and consequently on the processes which take place on them.[31] For example, power grids and transportation networks obviously depend on distance, many communication network devices have a short radio range, and most of people have their friends and relatives in their neighborhood. In the important case of the brain system, regions that are spatially closer have a greater probability of being connected than remote regions as longer axons are more costly in terms of material and energy. Another particularly important example of such case is the Internet, which is influenced by a set of routers linked by physical cables with different lengths and latency times.[31]

Moreover, previous studies never consider the relation of each link's bandwidth and Euclidean distance. However, obviously in real systems, the bandwidth of each link is limited to the Euclidean distance, and in most cases, these restrictions contribute to the triggering of congestion.

In this paper, we focus our research on network traffic on a spatial scale-free network in which the bandwidth of the links is assumed to be proportion to links' Euclidean distance. We find that bandwidth can decrease the network capacity, and a routing strategy with consideration of both Euclidean distance and betweenness centralities (BC) of edges under some parameters can effectively improve transportation efficiency by balancing the traffic load and avoid unbearable travelling distance. Simulation results on the spatial network show that compared with the shortest routing strategy and the minimum BC of links strategy, even at the situation of low bandwidth, our routing strategy can considerably improve spatial network capacity and some system behaviors which reflect transportation efficiency, such as average packets traveling time, average packets waiting time and system throughput, traffic load and so on. These results do provide insights for research on real networked traffic systems.

The paper is organized as follows. In Section 2, a network traffic model is introduced. The simulation results and discussions are given in Section 3. Conclusion is presented in Section 4.

2. Network Traffic Model

In this paper, we adopt a spatial network model proposed by S.S. Manna[32] which essentially elaborates on the preferential attachment model proposed by Albert and Barabasi[5] and has many important ingredients in the formation of various real-world networks. The network grows by systematically introducing one node at a time with randomly chosen coordinates $\{(x, y){:}0 \leq x, y \leq a\}$ with uniform probabilities. In addition, the attachment probability that the new node introduced at time t would be connected to its i-th predecessor ($0 \leq i \leq t$-1) is: $P_i(t) \sim k_i(t)l^{\gamma}$, where l is the minimum integer of the Euclidean distance between the t-th and the i-th node, $k_i(t)$ is the degree of the i-th node at time t and γ is a continuously varying parameter. Then the physical infrastructure of the spatial network is constructed based on the same rules of the well-known Barabsi Albert (BA) scale-free network model. When $\gamma = 0$, it is the usual BA model. Besides, the network is indicated to be scale-free for all values of $\gamma >$ -1, and the degree distribution decays stretched exponentially for the other values of γ. The link length distribution follows a power law: $D(l) \sim l^{\delta}$, where δ is calculated exactly for the whole range of values of γ.[32]

In network traffic models, at each time step, there are R packets generated with sources and destinations being chosen randomly. The packets are delivered according to a certain routing strategy with the velocity of v. Besides, each node has two functions: delivery and storage of packets. The delivery capability of each node is denoted by C. The packet queue length in buffers can be infinite. Assuming that $L_i(t)$ is the number of packets queuing in the buffer of node i at time step t, then the number of packets p_i^t which will be delivered can be denoted as

$$p_i^t = \begin{cases} L_i(t), if L_i(t) < C \\ C, else \end{cases} \tag{1}$$

If the queue length is less than C, then all packets can be delivered. Otherwise, C packets are delivered according to the first-in-first-out strategy and $L_i(t) - C$ packets will be delayed. In addition, we assume that each packet on an edge will be delivered a unit Euclidean distance in one time step. In this paper, bandwidth of each link (B) is also considered and assumed to be proportion to the Euclidean distance (f) which can be described by $B = \lambda f$, where λ is a tunable parameter. Once packets reach their destinations, they will be removed from the system.

Next, the routing strategy adopted in this work will be explained in detail. The shortest path strategy can make packets reach their destination with the shortest Euclidean distance, but it may cause severe congestion at the hub routers with high degree and high betweenness. On the other side, the minimum betweenness centralities of edges routing strategy may relieve traffic load at the hub routers, but it could result in very long travelling distance for nodes. In order to balance the traffic load and travelling distance to improve the transportation efficiency, the routing strategy based on the Euclidean distance and betweenness centralities of edges (EB) will be considered. For any path between nodes s and d denoted as $Path(s \to d) := s \equiv x_1, \cdots, x_i, \cdots, x_n \equiv d$, the effective path between s and d is corresponding to the path that makes the value minimum for the combination of the Euclidean distance and BC of edges, which is defined by

$$L(Path(s \to d) : \alpha, \beta) = \sum_{i=1}^{n-1} f(x_{i+1}, x_i)^{\alpha} g(x_{i+1}, x_i)^{\beta}. \tag{2}$$

where $f(x_{i+1}, x_i) = \|x_{i+1} - x_i\|$, which denotes the Euclidean distance between nodes x_i and x_{i+1}, and $g(x_{i+1}, x_i)$ is the betweenness centralities of the edge from x_i to x_{i+1} which is defined by

$$g(x_{i+1}, x_i) = \sum_{s \neq d} \frac{\delta_{sd}(x_{i+1}, x_i)}{\delta_{sd}} \tag{3}$$

where δ_{sd} is the number of paths with shortest Euclidean distance going from s to d and $\delta_{sd}(x_{i+1}, x_i)$ is the number of shortest Euclidean distance paths going from

s to d and passing through x_i and x_{i+1}. When $\alpha = 1$ and $\beta = 0$, it is defined as the shortest distance path (SDP) routing strategy with a definition different from the traditional shortest path routing strategy used in the most network literatures, because the shortest distance path in this paper is the one with the minimum Euclidean distance. Besides, we can see that the betweenness centrality is defined based on the shortest distance path. When $\alpha = 0$ and $\beta = 1$, it is the minimum betweenness centralities (MBC) of edges routing strategy. Hence, we can conclude that different values of α and β do have a clear influence on the routing strategy, and we will analyze it in detail below.

3. Simulation Results and Discussion

We set the network parameters as $N = 1225$, $m = m_0 = 2$, γ = -0.5, $C = 5$, $a = 200$ and $v = 1$. The queue buffer on each node is assumed to be unlimited, and the total simulation time T is set to be 10,000. Firstly, λ is set to be large enough to check the traffic behavior under different routing strategies without the influence of bandwidth. To be accurate, the simulation results are averaged by 30 individual runs on 30 BA networks with the same network parameters.

Firstly, the order parameter[33]

$$\eta(R) = lim_{t\to\infty} \frac{C}{R}\frac{< \Delta N_p >}{\Delta t}. \tag{4}$$

is introduced to describe the transitions of traffic flow in the network, where in Eq.(3) $\Delta N_p = N_p(t + \Delta t) - N_p(t)$, and $< \cdots >$ represents the average over time windows of width Δt, and $N_p(t)$ denotes the number of packets in the network at time t. With the increase of packet generation rate R, there will be a critical value of R_c which characterizes the phase transition from free flow to congestion. When $R < R_c$, $\Delta N_p = 0$, and $\eta(R) = 0$, it indicates that the network is in the free flow state, while for $R > R_c$, $\eta(R) = 0$ is larger than zero, which indicates the system is in the congestion state.

To investigate the combined effect of the two parameters on the proposed routing strategy, R_c under different values of α and β are shown in Fig.1. It is found that there exists an optimal island in the parameter space (α, β) where R_c reaches the highest value, indicating that the cooperation can be promoted by both the Euclidean distance and BC of edges. For example, when $\alpha = 0.5$, R_c will reach the peak value for $\beta = 0.2$. Besides, for simplicity, the strategy with the optimal combination, in this case $\alpha = 1.1$ and $\beta = 0.2$, is named as the EDBC routing s-trategy. Next, the simulation results of the SDP routing strategy, the MBC routing strategy and the EDBC routing strategy which reflect transportation efficiency will be described.

Firstly, the simulation result of the critical packet generating rate R_c under the three routing strategies is examined. It can be seen in Fig.2 (a) that R_c of the EDBC routing strategy outperforms those of the other two strategies. It is 28 for

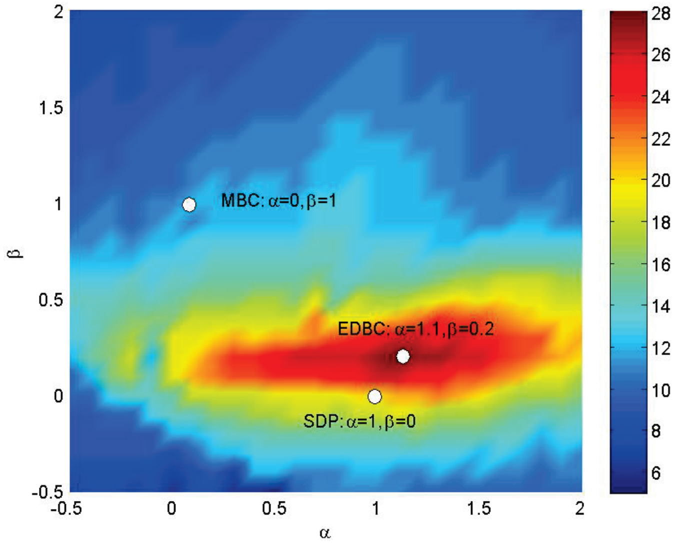

Fig. 1. The distribution of R_C at different values of α and β.

the EDBC routing strategy, but 11 and 19 for the MBC routing strategy and the SDP routing strategy respectively.

We also investigated the effects of the average node degree $< k >$ and network size N on the traffic capacity of a network. Fig.2 (b) shows that R_c increases almost linearly with the average degree $< k >$. The rank of the network capacities is EDBC routing $>$ SDP routing $>$ MBC routing. In Fig.2 (c), R_c also increases slightly with the network size. Again, with the same average degree or network size, the networks capacity under the EDBC routing strategy is the largest and that under MBC is the smallest.

To better understand why the EDBC routing strategy can improve the network capacity, we investigate the edge load distribution $n(e)$ and the node load distribution $n(k)$ under the three routing strategies in the congestion state, as shown in Fig.3. Due to the spatial distance factor of edges, there are packets travelling along each edge at each time step. Edge load distribution $n(e)$ is defined by $n(e) = \sum_{t=1}^{T}(\sum_{i=1}^{max_e}(x_{ti}(e))/max_e)/T$, where $x_{ti}(e)$ is the number of packets at an edge with BC of e at time step t, and max_e is the number of edges with BC of e. Fig.3 (a) shows that under the shortest path routing strategy $n(e)$ greatly increases as BC grows. That is because that in the spatial network, edges with larger BC are more central and they are more likely to bear heavy traffic load. On the other hand, the MBC routing strategy causes heavy traffic load at the edges with the small value of BC via making the packets choose the path with the small value of BC. Whereas, considering both the Euclidean distance and BC of edges, the EDBC routing strategy can effectively balance traffic load among different links. The network capacity may be improved by making the effective use of all edges in the network to deliver

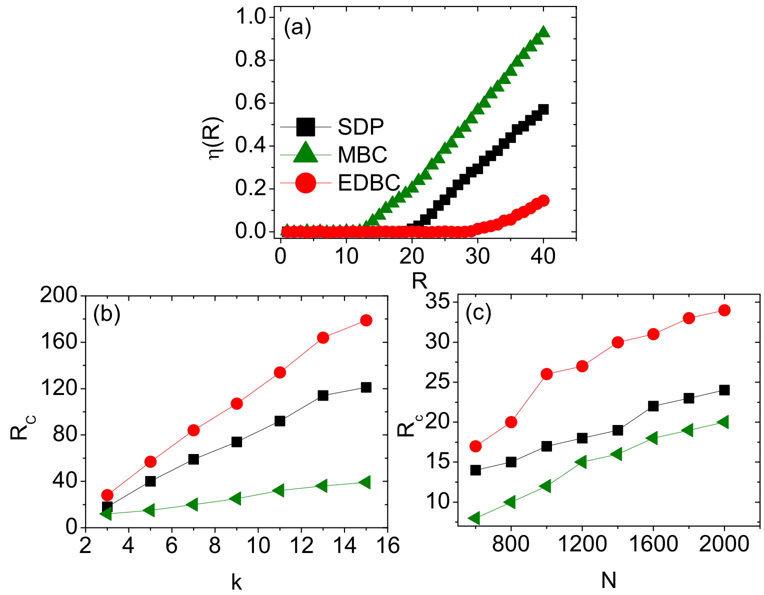

Fig. 2. (Color online) (a) The relationship between the order parameter $\eta(R)$ and the packet generating rate R under the routing strategies with different parameters. (b) Network capacity R_c vs average degree $< k >$ with the same network size of $N = 1225$. (c) Network capacity R_c vs network size N with the same average degree of $< k >= 3$.

packets. Fig.3 (b) also demonstrates our conclusion by comparing the load distribution $n(k)$ under the routing strategies. It can be shown that in the congestion state the larger the degree of nodes is, the more crowded the nodes are. Besides, the load of nodes under the EDBC routing strategy is usually the lightest compared with other routing strategies. In addition, Fig.3 (c) can further explain our conclusion by describing the relationship between the queue length and the number of nodes with the same queue length of r under the three routing strategies in the congestion state. Q_r indicates the number of nodes with the same queue length of r from time step t_0 to T, and is defined by

$$Q_r = \sum_{j=t_0}^{T} \sum_{i=1}^{N} (\delta_{rij}), \tag{5}$$

where $\delta_{rij} = \begin{cases} 1, if\, queue\, length\, of\, node\, i\, is\, r\, at\, time\, j \\ 0, else \end{cases}$.

Fig.3 (c) shows that the maximum queue length caused by the EDBC routing strategy is the least, close to 11,000, compared with 120,000 and 80,000 caused by the SDP routing strategy and the MBC routing strategy respectively, which are much higher. Besides, we can conclude that both the SDP routing strategy and the

MBC routing strategy could aggravate traffic load and result in serious congestion at some nodes, but the EDBC routing strategy can preferably balance traffic load and make the queue length among different nodes more even.

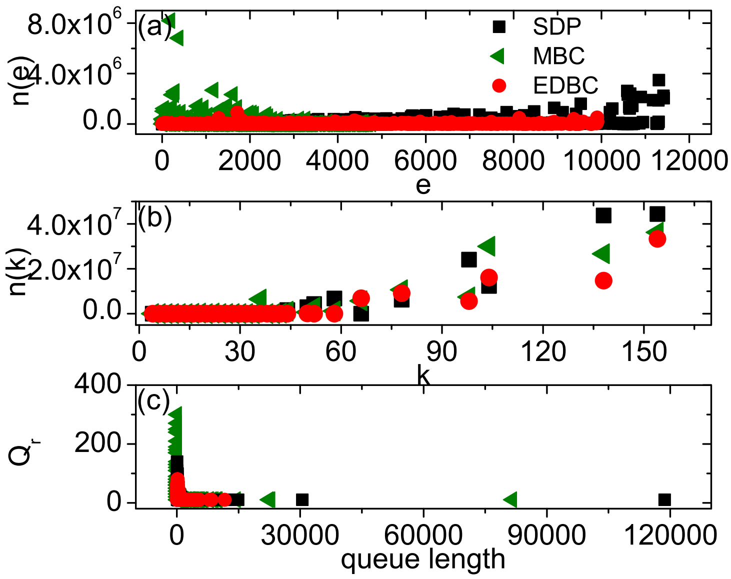

Fig. 3. (Color online) (a) The relationship between the average packet number and the betweenness centralities of edges under the three routing strategies when $R = 35$. (b) The relationship between the average packet number and the degree under the three routing strategies when $R = 35$. (c) The relationship between the queue length and the number of nodes with the same queue length under the three routing strategies when $R = 35$.

The average travelling time $< T >$ which represents the traffic speed is a critical feature for traffic systems. For example, for the air traffic system, reducing the time cost is an important problem and thus it is an effective measure to decrease the average travelling time of flights. Fig.4 (a) shows the relationship between $< T >$ and the packet generating rate R under the different routing strategies. Here $< T >$ can be denoted as $< T >= \sum_{i=1}^{N_{arrive}} t_i/N_{arrive}$, where t_i is the travelling time of the arrived packet i, and N_{arrive} is the number of arrived packets. Obviously, the smaller $< T >$ is, the faster the traffic speed is. As Fig.4 (a) shows, in the free-flow state, packets can be freely delivered. However, $< T >$ under the different routing strategies behaves much differently. $< T >$ under the SDP routing strategy is the smallest with a value of 302, and that under the EDBC routing strategy is 331. However, the value of $< T >$ under the MBC routing strategy reaches up to 956, which is much higher than that of other two routing strategies. The SDP routing strategy makes packets choose the shortest path to destination. Hence, it has the least average travelling time. On the contrary, in order to avoid the crowded links, the MBC routing strategy prompts packets to select the paths with the minimum betweenness centralities of edges, which is less crowded, but can cause the much large travelling distance. In the congestion state, $< T >$ dramatically increases when $R > R_c$, for packets have to wait in the buffer because of the limited delivery capability. Besides, $< T >$ under the MBC routing strategy is still the largest, however, that under the EDBC routing strategy becomes the smallest: e.g., when $R = 40$, $< T >$ under the MBC routing strategy and the SDP routing strategy is 1,358 and 683 respectively, but only 442 under the EDBC routing strategy. Perhaps it is because that the SDP routing strategy causes extreme congestion and costs packets much more time to wait, but the EDBC routing strategy relieves congestion and makes packets travel faster.

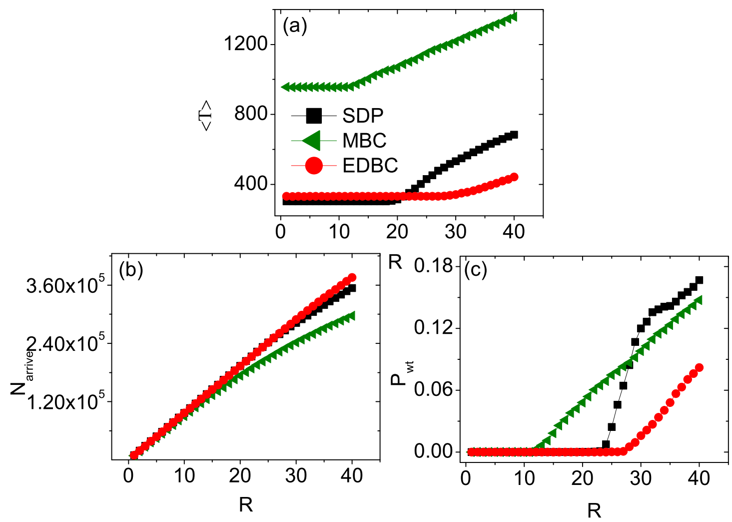

Fig. 4. (Color online) (a) The relationship between the average traveling time and the packet generating rate R under the three routing strategies. (b) The relationship between the number of arrived packets N_{arrive} and the packet generating rate R under the three routing strategies. (c) The relationship between the rate of waiting time to travelling time P_{wt} and the packet generating rate R under the three routing strategies.

The rate of waiting time to travelling time P_{wt} is another critical feature for traffic systems to describe traffic efficiency and thus is an important index to depict user satisfaction. The less P_{wt} is, the higher the user satisfaction is. For example, it may be tolerable for an airplane to be delayed by 10 minutes in its 2 hours travel. However, it might be unacceptable if an airplane took a short flight. P_{wt} can be denoted as

$$P_{wt} = \frac{1}{N_{arrive}} \sum_{i=1}^{N_{arrive}} \frac{w_i}{t_i}. \tag{6}$$

where w_i is the waiting time of packet i, and t_i is its total travel time. In Fig.4 (b), one can see that in the free-flow state, P_{wt} under the different routing strategies are the same, and with the increment of R, it increases obviously. Besides, in the congestion state, P_{wt} under the EDBC routing strategy keeps the minimum value. At the beginning, P_{wt} under the SDP routing strategy is much smaller than that of the MBC routing strategy. However, as R increases, P_{wt} under the SDP routing strategy grows sharply. We can conclude that the network is most congested under the SDP routing strategy, and packets are severely delayed in the congested state.

The system throughput N_{arrive} is the index denoting the total number of packets that are delivered to their terminals in a fixed time span. It indicates the delivery capability of the whole network. Fig.4 (c) shows the relationship between N_{arrive} and the packet generating rate R. In the free-flow state, all packets can successfully arrive at their destinations, and $N_{arrive}(R) \approx T \times R$. However, in the congestion state, not all packets can arrive their destination and thus $N_{arrive}(R) < T \times R$. It can be shown that the value of N_{arrive} under the EDBC routing strategy is the largest compared with those under the two others.

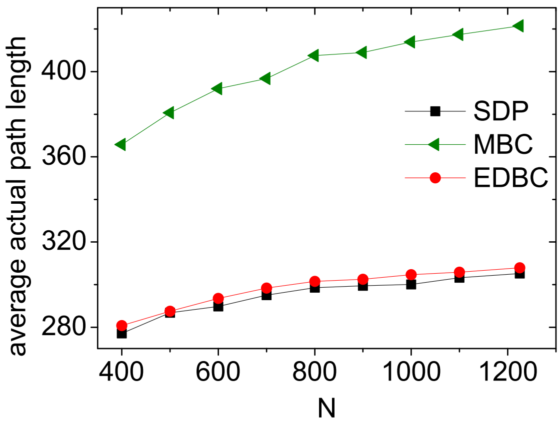

Fig. 5. (Color online) The relationship between the average actual path length and the network size N under the three routing strategies.

We also show the results of the average actual path length versus the network size under the three routing strategies in Fig.5. One can see that as the N grows the average actual path length under the three routing strategies increases. Moreover, the average actual path length under the EDBC strategy is slightly higher than that of the SDP strategy, but much smaller than that under the MBC strategy: e.g., for $N = 1225$, the average actual path length under the MBC strategy is 421, but only 307 and 305 under the EDBC strategy and the SDP strategy respectively. Though the average actual path length under the EDBC strategy is not the least, R_c under the EDBC strategy is the highest. Such loss may be worthwhile when a network requires large R_c.

In the previous discussions, C is a constant value. However, in many situations C is related to the degree of nodes k.[34−35] Next we consider the case when $C = k$ to test the robustness of the routing strategy. Fig.6 shows the results of the order parameter, average packets traveling time, average packets waiting time, and system throughput under the three strategies. It can be shown that the EDBC routing strategy outperforms the others in almost all aspects.

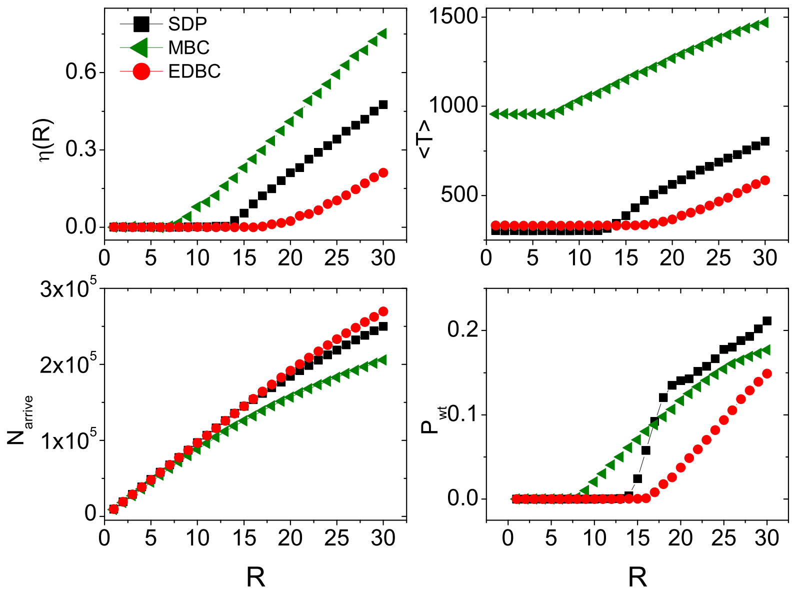

Fig. 6. (Color online) The relationship between $\eta(R)$ (a), $< T >$ (b), N_{arrive} (c), P_{wt} (d) and R when $N = 1225$, $m = m_0 = 2$, $\gamma = -0.5$ and $C = k$ under the three routing strategies.

Next we will investigate the network capacity and other traffic behaviors under different routing strategies with the bandwidth restriction. Firstly, the results of λ versus $< T >$ under the three routing strategies when $R = 30$ are shown in Fig.7. For simplicity, the optimal values of α and β of the proposed routing strategy are always assumed to be 1.1 and 0.2. We can find that in Fig.7(a) as λ decreases R_c under the three routing strategies decreases. No matter what λ is, the rank of the network capacities is still EDBC routing $>$ SDP routing $>$ MBC routing. Besides, in Fig.7(b) we can see that as λ increases, $< T >$ under the three routing strategies sharply deceases, and then it will quickly reach the minimum. It is also shown that $< T >$ under the EDBC strategy is the least, and that under the MBC strategy is the largest. Hence, we can conclude that the bandwidth restriction can

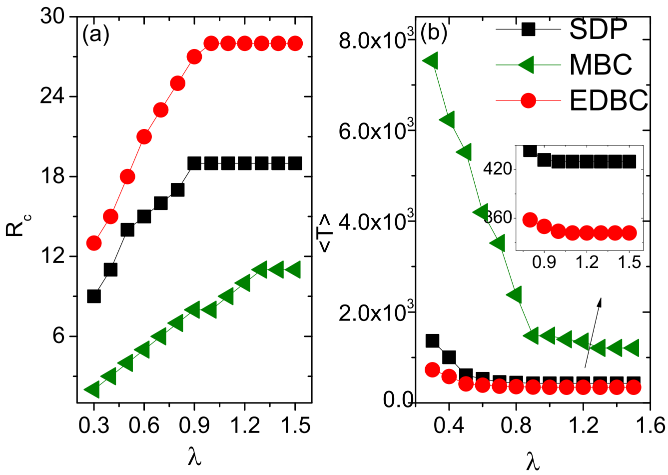

Fig. 7. (Color online) (a)The relationship between λ and R_c under the three routing strategies when $N = 1225$, $m = m_0 = 2$, $\gamma = -0.5$, $\alpha = 1.1$, $\beta = 0.2$ and $C = 5$. (b)The relationship between λ and $< T >$ under the three routing strategies when $N = 1225$, $m = m_0 = 2$, $\gamma = -0.5$, $\alpha = 1.1$, $\beta = 0.2$, $R = 30$ and $C = 5$.

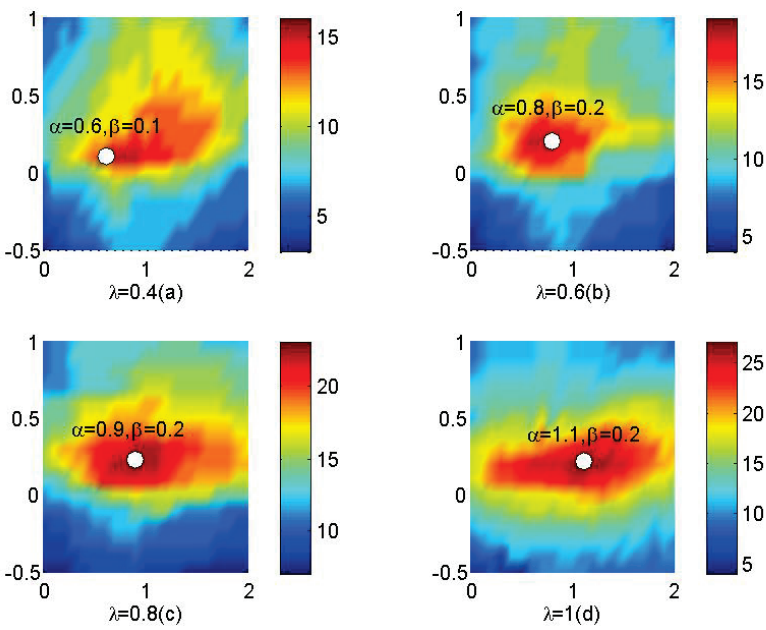

Fig. 8. (Color online) The distribution of R_C at different values of α and β when $\lambda = 0.4$(a), $\lambda = 0.6$(b), $\lambda = 0.8$(c), and $\lambda = 1$(d).

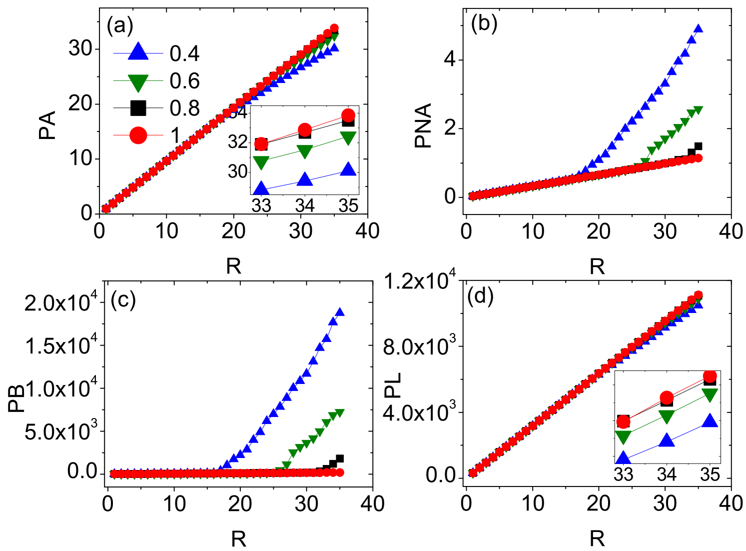

Fig. 9. (Color online) The relationship between the average number of packages arrived at their destinations (a), the average number of packages not arrived at their destinations (b), the average number of packages in the buffer (c), the average number of packages in links(d), and R for different λ under the EDBC routing strategy when $N = 1225$, $m = m_0 = 2$, $\gamma = -0.5$, and $C = 5$.

aggravate congestion, and the smaller λ is, the more serious the congestion will become. Besides, the EDBC routing strategy has the best performance compared with the other strategies even at the situation of very low bandwidth.

Then we study the effect of bandwidth on the network capacity under the proposed routing strategy at different α and β. The distribution of R_C at different combination of α and β for four cases is shown in Fig.8. We can see that the optimal combination of α and β corresponding to the maximum capacity changes for different λ. Moreover, as λ increases α increases and β almost keep the same. It can be concluded that not only bandwidth causes the variation of the optimal local routing coefficient α_c,[28] but also it can induce the variation of the optimal parameters of the proposed global routing strategy.

Then, to better understand why bandwidth can trigger network congestion, we investigate the relationship between the average number of four types of package and R for different λ over T time step under the EDBC routing strategy in Fig.9. Fig.9(a) shows the simulation result of the average number of packages arrived at their destinations PA. We can conclude that as λ increases, more packages can reach their destinations. It can be seen from Fig.9(b) that the average number of packages not arrived at their destinations PNA sharply grows as bandwidth decreases. Besides, the simulation results of the average number of packages in buffers PB is described in Fig.9(c). And we can see that the smaller λ is, the more packages will wait in buffers. Moreover, the average number of packages in links PL is obviously proportional to bandwidth and it can be seen from Fig.9(d). In the end, we can conclude that as λ decreases, more packages cannot be delivered to links, so they are delayed in buffers and finally left in the network which aggravates congestion.

4. Conclusion

In this paper, traffic dynamics on a spatial scale-free network has been considered with the restriction of bandwidth proportional to links' Euclidean distance. Besides, an efficient routing strategy that considers both the Euclidean distance and betweenness centralities of edges has been proposed. It is found that the network capacity is affected by the bandwidth restriction, and compared with the shortest distance path routing strategy and minimum betweenness centralities of links strategy, our strategy with certain parameters can effectively balance the traffic load and avoid severe travelling distance no matter what bandwidth is. It can also effectively improve the spatial network capacity and system behaviors which reflect transportation efficiency, such as the average packets traveling time, average packets waiting time, system throughput, traffic load and so on. Simulation results have demonstrated the performance of the proposed routing strategy. These results provide insights for research on real networked traffic systems.

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