A Link Transmission Model for Air Traffic Flow Management

Yi Cao and Dengfeng Sun
Purdue University, West Lafayette, IN 47906-2045

Optimization of the air traffic flow in the National Airspace System is by nature a large-scale problem which entails intensive computation. This paper presents a linear integer programming approach which decreases the computing time by aggregating the aircraft count at a link level based on the Large-capacity Cell Transmission Model. The improved model has fewer state variables and constraints which result in a reduced dimension problem. Taking advantage of dual decomposition method, the nationwide optimization is decomposed into independent subproblems path by path and optimized sequentially. The solution converges asymptotically to the global optimum. The major contribution of the proposed model is that it is about six times faster than Large-capacity Cell Transmission Model while maintaining an equally optimal solution. Another improvement is the modeling of ground delay and air delay. The traffic forecasting capability of the new model is also examined statistically at the sector level and nationwide level respectively. Accuracy statistics indicate that the sector counts are predicted with relative errors less than 20% within 80% of the high altitude sectors in the U.S. airspace, further analysis also suggests that the proposed model is comparable to two earlier Eulerian models.
Nomenclature

$C_s(t) = \text{maximum number of aircraft allowed in sector } s \text{ at time } t$

$d^*(\lambda), d^{k*}(\lambda) = \text{objectives of master problem and } k^{th} \text{ subproblem respectively}$

$f^k(t) = \text{departures into path } k \text{ at time } t$

$h^k_i(t) = \text{the number of ground held flight in link } i \text{ on path } k \text{ at time } t$

$K = \text{number of paths in simulation and optimization}$

$n^k = \text{the number of links on path } k$

$p^k_i(t) = \text{transition probability describing aircraft that move from link } i \text{ to link } i+1 \text{ at time } t$

$q^k_i(t) = \text{equals } \beta^k_i(t)x^k_i(t), \text{optimal number of aircraft transitioning from link } i \text{ to link } i+1$

at time $t$

$Q_{s_i} = \text{set of links that lie inside sector } s_i$

$S = \text{number of sectors in National Airspace System}$

$T = \text{planning time horizon of air traffic prediction and optimization}$

$T_i = \text{length of link } i, \text{scaled in minute}$

$u^k_i(t) = \text{delay control of aircraft in link } i \text{ on path } k \text{ at time } t$

$x^k_i(t), x^k_i(t)' = \text{deterministic component and probabilistic component of aircraft count in link } i \text{ on path } k \text{ at time } t$

$\lambda_s(t) = \text{Lagrange multiplier for sector } s \text{ at time } t$

$\beta^k_i(t) = \text{transmission coefficient, representing the fraction of aircraft that move from link } i$

\text{to link } i+1 \text{ at time } t$

$i = \text{index of link}$

$j = \text{index of cell}$

$k = \text{index of path}$

$s = \text{sector index}$

$s_i = \text{sector that link } i \text{ lies in}$

$t = \text{time step}$
I. Introduction

The goal of optimizing the air traffic in the National Airspace System (NAS) is to achieve overall minimum delays while respecting the en route sector capacity constraints. One of the paradigms addressing such traffic flow management (TFM) problem is the Bertsimas and Stock-Patterson model [1] which uses 0-1 Integer Program (IP) to propagate the individual aircraft trajectory. This approach is categorized as Lagrangian model. Drawback of these models lies in the varying state dimension which depends on the number of aircraft in the system. In nationwide TFM problems where thousands of aircraft are involved during the peak hours, state variables could grow unmanageably. The Eulerian models are introduced to decrease the problem size by aggregating aircraft into flows. Outstanding paradigms include Menon’s Model [2, 3], Linear Dynamic System Model (LDSM) [4, 5], and Partial Differential Equation Model (PDE) [6]. These models have dimensions that are independent of the number of aircraft. However, these models are designed for traffic prediction, and TFM optimization to the NAS has not been attempted, or only restricted to small airspace scenario [6].

An Eulerian-Lagrangian model, known as Large-capacity Cell Transmission Model (CTM(L)), was developed in [7]. This model aggregates aircraft by airways. The traffic flow is modeled using conservation of flow principle. Since the dynamics is in a form of linear discrete-time equation, it is suited for computer implementation. The CTM(L) was initially designed for traffic prediction, then for nationwide traffic optimization [8]. Its dimension is between the Lagrangian models and the Eulerian models, which is fixed but still large-scale in nationwide scenario. [8] introduced dual decomposition method to make the large-scale nationwide TFM problems computationally tractable. However, each subproblem involves about 6,000 variables and 10,000 constraints for a two-hour optimization. In a preliminary study [9] to improve the computational efficiency, the CTM(L) was improved through aggregating the aircraft count at a link level, resulting in a more efficient model named Link Transmission Model (LTM).

As a continuing effort, this paper goes further to explore and analyze the LTM. The goal is to seek its potential in two important aspects of air traffic management (ATM), namely air traffic prediction and optimization. The contributions of this paper are: (1) extension of the model in [9]
to contain both deterministic and probabilistic components, thus generalizing it to accommodate zero and nonzero initial conditions and resulting in an improve prediction accuracy; (2) modeling of the ground delay and air delay; and (3) statistical analysis of the accuracy of traffic forecast and computational efficiency of air traffic optimization in nationwide scenario, which is a large-scale problem.

The rest of this paper is structured as follows. Section II first introduces the geographic structure of en route air traffic and reviews the CTM(L), then development of LTM dynamics and the modeling of control strategies are addressed. Section III describes the formulation of a TFM optimization problem based on the LTM, and algorithm used to solve the large-scale optimization problem is outlined. Section IV presents the numerical results, where prediction accuracy is statistically analyzed and the computational efficiency is discussed. Finally, concluding remarks are presented in Section V.

II.  Development of the Link Transmission Model

A.  Airspace Structure for the NAS

The entire airspace in United States is vertically divided into three layers in terms of altitude (low, high and super-high). Each layer is partitioned into smaller regions called sectors in which air traffic controllers ensure that the aircraft within them are safely separated. In order to simplify the problem, this paper deals with traffic in the high altitude sectors only, because traffic flows in en route airspace are largely two-dimensional and have relatively stable transmission pattern [10]. The high altitude sectors (FL230~FL330) defined in the Future ATM Concept Evaluation Tool (FACET) [11] are used to represent the airspace of the NAS in this study.

B.  Brief Review of the Large-capacity Cell Transmission Model

The CTM(L) is constructed as a multi-commodity networkS [12] across the NA. Figure 1 illustrates the concept of CTM(L). A Path is an airway connecting the origin and destination airports. Each path comprises a series of Links, each lying inside a sector. Each link is further subdivided into a number of Cells. The overall number of cells account for the length of a path which is obtained by averaging the flight time of historical trajectories (scaled in minute) [7]. The state variable of
CTM(L) is the aggregate aircraft count in a cell. The sector count is obtained by summing all cell counts within a sector. The CTM(L) is an airway-level model. Therefore, its overall dimension is fixed and determined by the number of paths identified in the NAS.

Basically, the dynamics of CTM(L) obeys conservation of flow principle. It is assumed that an aircraft heads for its destination with constant air speed such that it smoothly moves to the next cell at the next time instant if no delay is applied. In addition, aircraft (regardless different aircraft types) take the same flight time to pass the same path. The dynamics is given by:

\[ x_{k,i}^j(t+1) = x_{j-1}^{k,i}(t) - u_{j-1}^{k,i}(t) + u_j^{k,i}(t), \]
\[ x_0^{k,0}(t+1) = f^k(t) + u_0^{k,0}(t), \]

where \( x_{j}^{k,i}(t) \) is the aggregate aircraft count for the \( j \)th cell of link \( i \) on path \( k \). \( u_j^{k,i}(t) \) is the control variable representing the number of aircraft being held in that cell. In air traffic prediction, \( u_j^{k,i}(t) \) is not applicable and absent from Eq. (1), then traffic flow simply moves forward cell by cell.

A computational barrier associated with CTM(L) is that a nationwide TFM formulation incurs large numbers of state variables. For example, for a path from east coast to west coast averaging about five hours flight time, there are about 300 cells which introduce 300 state variables and 300 control variables. Moreover, the number of variables also depends on the planning time horizon. As a result, for a two-hour optimization with 1-minute interval, \( 2 \times 300 \times 120 \) variables are defined for
this single path. In practice, thousands of paths are identified throughout the NAS, then billions of variables and constraints are involved which is a great challenge for both hardware and software. [8] used dual decomposition method to decouple the path network. The large-scale problem is decomposed into small subproblems path by path, each subproblem can be solved independently by the optimization tool.

C. Link Transmission Model

1. Derivation of the Dynamics

It is intuitive that the state variable could be reduced if the aircraft count is aggregated at a link level, as shown in Fig. 2.

![Illustration of the Link Transmission Model](image)

**Fig. 2 Illustration of the Link Transmission Model**

Since the conservation of flow that governs the flow dynamics has not been changed, the modification leads to a crucial distinction between LTM and CTM(L), that is, only a subset of the aircraft in an upstream link will move to the downstream link at the next time instant. This is due to the fact that many links have length that are longer than 1-minute interval. To modify the dynamics of CTM(L), a transmission coefficient $\beta^k_{t}(t)$ is introduced into the dynamics:

$$x^k_{i}(t + 1) = \beta^k_{t-1}(t)x^k_{i-1}(t) - u^k_{t-1}(k) + (1 - \beta^k_{t}(t))x^k_{i}(t) + u^k_{t}(t)$$

$$x^k_{0}(t + 1) = f^k(t) + (1 - \beta^k_{0}(t))x^k_{0}(t) + u^k_{0}(t).$$

From Eq. (2), the aircraft count in a link consists of two parts. Part $A$ represents the fraction that comes from the upstream link. Part $B$ represents the fraction that remains in the same link. As the transmission coefficient is capable of controlling the flow rate, the control variable $u^k_{t}(t)$ is not necessary. The dynamics is simplified as follows:

$$x^k_{i}(t + 1) = \frac{\beta^k_{t-1}(t)x^k_{i-1}(t)}{A} + \frac{(1 - \beta^k_{t}(t))x^k_{i}(t)}{B}$$

$$x^k_{0}(t + 1) = f^k(t) + (1 - \beta^k_{0}(t))x^k_{0}(t).$$

From Eq. (3), the aircraft count in a link consists of two parts. Part $A$ represents the fraction that comes from the upstream link. Part $B$ represents the fraction that remains in the same link. As the transmission coefficient is capable of controlling the flow rate, the control variable $u^k_{t}(t)$ is not necessary. The dynamics is simplified as follows:

$$x^k_{i}(t + 1) = \beta^k_{t-1}(t)x^k_{i-1}(t) + [1 - \beta^k_{t}(t)]x^k_{i}(t)$$

$$x^k_{0}(t + 1) = f^k(t) + [1 - \beta^k_{0}(t)]x^k_{0}(t).$$
The prediction accuracy of the model depends on two factors: the accuracy of the link length, and the transmission coefficient. The link length is computed from historical traffic data. For zero initial state (no flight is present in the links at \( t = 0 \)), the transmission coefficient can be determined as follows:

\[
\begin{align*}
\text{link } 0 & \quad \beta_k(0) = \frac{f^k(t-T_0)}{f^k(t-T_0) + f^k(t-T_0+1) + \cdots + f^k(t-T_n)} \quad t \geq T_0 \\
& \quad \beta_k(0) = 0 \quad t < T_0,
\end{align*}
\]

(4)

\[
\begin{align*}
\text{link } i & \quad \beta_i(t) = \frac{f^k(t-T_0-T_1-\cdots-T_i)}{x_i^k(t)} \quad t \geq T_0 + \cdots + T_i \\\n& \quad \beta_i(t) = 0 \quad t < T_0 + \cdots + T_i.
\end{align*}
\]

(5)

Eqns. (4), (5) are derived from the conservation of flow principle. Eq. (4) indicates that aircraft entering path \( k \) will leave the first link after dwelling in it for \( T_0 \). The transmission coefficients of subsequent links could be deduced recursively from Eq. (3). Therefore, computation of transmission coefficient is updated online when predicting the traffic.

In practical application, the nonzero initial state should not be overlooked, that is, airborne flights are present in the links at \( t = 0 \). Since the positions of these airborne flights are uncertain at the initial state, they can not be described by Eqns. (4), (5). The transmission of these flows is estimated using a probabilistic method, which is inspired by LDSM [4, 13]. Combining the nonzero initial state, the dynamics is extended to the following form:

\[
X^k(t+1) = X_1^k(t+1) + X_2^k(t+1)
\]

(6)

where

\[
\begin{align*}
X_1^k(t+1) &= A_1^k(t)X_1^k(t) + B^k f^k(t), \\
X_2^k(t+1) &= A_2^k(t)X_2^k(t),
\end{align*}
\]

\[
X_1^k(t) = [x_0^k(t), x_1^k(t), \ldots, x_n^k(t)]^T, \quad X_1^k(0) = 0,
\]

\[
X_2^k(t) = [x_0^k(t)', x_1^k(t)', \ldots, x_n^k(t)']^T, \quad X_2^k(0) \neq 0,
\]

\[
A_1^k(t) = \begin{bmatrix}
(1 - \beta_0^k(t)) \\
\beta_0^k(t) & (1 - \beta_1^k(t)) \\
& \ddots \\
& & \beta_{n-1}^k(t) & (1 - \beta_n^k(t))
\end{bmatrix}
\]

\[
B^k = \begin{bmatrix}
1 \\
0 \\
\vdots \\
0
\end{bmatrix}.
\]
Eq. (6) indicates that the aircraft count in a link comprises two parts: the deterministic component \( X_k^1(t) \) and the probabilistic component \( X_k^2(t) \), which is explained in details as follows.

\( X_k^1(t) \) is the aggregation of states on path \( k \), whose initial state is zero. It only accounts flights taking off after \( t = 0 \). Given a series of input \( f^k(t) \), \( X_k^1(t) \) and \( A_1^k(t) \) will be updated in a deterministic fashion at each time instant according to Eqns. (3), (4), (5).

\( X_k^2(t) \) accounts for the nonzero initial state. It counts flights that are already airborne at \( t = 0 \). Since it is uncertain when the aircraft will move to the next link, this part of traffic must be estimated. A transition probability matrix is defined as:

\[
A_2^k(t) = \begin{bmatrix}
(1 - p_0^k(t)) & p_0^k(t) & (1 - p_1^k(t)) & \cdots & p_n^k(t) & (1 - p_{n+1}^k(t)) \\
\end{bmatrix},
\]

(7)

where

\[
p_i^k(t) = \frac{\bar{\lambda}_i(t)}{\bar{x}_i(t)}.
\]

\( \bar{\lambda}_i(t) \) is the average number of aircraft moving from link \( i \) to link \( i + 1 \). \( \bar{x}_i(t) \) is the average aircraft count in link \( i \). \( p_i^k(t) \) describes the transmission in history of link \( i \) at time \( t \). This information come from mining the historical traffic data. Generally, historical data encapsulate the traffic impacted by the weather. This paper does not consider weather influence and assumes that each link has a relatively steady transmission pattern in a day. In order to weaken the impact of weather, historical data of multiple days are used. Figure 3 shows the transition probabilities of three randomly selected links in four days (May 1, 3, 6, 10, in 2005). By averaging over ten days data, \( p_i^k(t) \) is able to capture the transmission pattern of the link. Exclusion of weather may be relaxed in future by using Corridor Integrated Weather System (CIWS) data, transmission patterns in different weather are categorized, this work requires further analysis though. In contrast to \( \beta_i^k(t) \), the extraction of \( p_i^k(t) \) requires filtering a large amount of historical data; thereby it is done offline.

Note that \( X_k^2(t) \) is a transient component. For path \( k \) with length \( T_k \), \( X_k^2(t) \) will diminish to 0 in \( T_k \) time steps at most. Then its dynamics has deterministic component \( X_k^1(t) \) left only.
2. Modeling the Ground Delay and the Air Delay

The ground delay and air delay are two important control means that have been intensively studied [1, 14–17]. LTM takes these strategies into consideration. In TFM optimization problem, the transmission coefficient $\beta_k(t)$ is no longer a known parameter but a control means that controls the flow rate of link $i$. A special link ($\text{link } 0, T_0 = 1$) is appended to the start of each path to represent the airport. The control imposed on $\text{link } 0$ amounts to ground delay. When departure $f_k(t)$ enter $\text{link } 0$, the aircraft are considered to join in a departure queue. Some aircraft may not leave the airport at the next time instant if the traffic is suffering from congestion. A subset of these queued aircrafts will be held until the traffic permits more takeoff. The number of held aircraft can
be calculated recursively by Eq. (8).

\[ h_k^0(t + 1) = h_k^0(t) + f_k^1(t - 1) - \beta_k^0(t + 1)x_0^k(t + 1). \] (8)

By summing the aircraft held in link 0 of paths sharing the same origin airport, the ground delay of a interested airport can be determined.

Controls imposed on the other links are considered to be air delay. Optimizing \( \beta_k^i(t) \) amounts to controlling the flow rate of link \( i \). The air delay can be calculated by Eq. (9).

\[ h_k^i(t + 1) = h_k^i(t) + \beta_k^{i-1}(t - T_i)x_{i-1}^k(t - T_i) - \beta_k^i(t + 1)x_k^0(t + 1). \] (9)

Theoretically, the air delay is as effective as the ground delay. However, the latter is more preferred in ATM operations due to safety and cost [18]. A heuristic metric to assess the cost of control actions is established in this paper:

\[ f_{cost} = W_{gnd} \sum_{t=0}^{T} \sum_{k=0}^{K} n_k \sum_{i=0}^{c_k^i} x_k^i(t), \] (10)

where \( W_{gnd} \) and \( W_{airborne} \) are costs for the ground delay and air delay respectively. Generally, \( W_{airborne} > W_{gnd} \). The LTM is able to leverage the cost by assigning different parameters to set up the optimization problem, which will be shown in Section IV.B.

III. Optimization Formulation for the TFM Problem Based on LTM

A. Optimization Problem Formulation

This section presents the formulation of TFM optimization problem and introduces the dual decomposition method. In this paper, a simple metric is used to formulate the objective function, which is the weighted sum of all state variables:

\[ \min \sum_{t=0}^{T} \sum_{k=0}^{K} \sum_{i=1}^{c_k^i} x_k^i(t), \] (11)

where \( c_k^i \) is a weight imposed on link \( i \). This metric reflects the minimum total flight time in the planning time horizon. The minimum flight time problem is subject to the following constraints:

1) Initial conditions

\[ x_i^k(0) = 0, \] (12)

\[ x_0^k(0) = f_k^1(0). \]
Initially, the deterministic component $X_1^k(t)$ is zero except for the first link. The probabilistic component $X_2^k(t)$ is not taken into consideration for simplicity. In other words, pre-departure flights are not subject to optimization but only treated as the “background” flights. These flights are simulated by LTM. This relaxation can be fixed in future by trivially generating a trajectory for each pre-departure flight, but this is a regression toward Lagrangian model and would increase the size of the whole problem.

2) LTM dynamics

$$
\begin{align*}
    x_k^i(t + 1) &= \beta_{i-1}^k(t)x_{i-1}^k(t) + [1 - \beta_{i-1}^k(t)]x_i^k(t) \\
    x_0^k(t + 1) &= f^k(t) + [1 - \beta_0^k(t)]x_0^k(t)
\end{align*}
$$

(13)

$$
0 \leq \beta_i^k(t) \leq 1
$$

3) Sector capacity constraint

$$
0 \leq \sum_{(i,k) \in Q_s} x_k^i(t) \leq C_s(t),
$$

(14)

which restricts the number of aircraft in a sector to be below the allowed maximum number of aircraft. In the implementation of this paper, the “background” flights are estimated using probabilistic method, which may change over time. For simplicity, the $C_s(t)$ is obtained by subtracting the mean “background” flights from the nominal sector capacity.

4) Conservation of flow

$$
\sum_{t=0}^{T} \beta_0^k(t)x_0^k(t) = \sum_{t=0}^{T} \beta_{i-1}^k(t)x_i^k(t) = \sum_{t=0}^{T} f^k(t),
$$

(15)

This constraint guarantees that all flights pass their paths in the planning time horizon. In other words, no aircraft will be held in the air at the end of the planning time horizon. This constraint is important in that it drives the aircraft moving toward their destinations.

5) Minimum dwell time constraint

$$
\sum_{t=0}^{T_0 + T_1 + \ldots + T_i - 1} \beta_i^k(t)x_i^k(t) = 0,
$$

(16)
\[
\sum_{t=0+T_1+\cdots+T_i}^{T^*} \beta^k_i(t)x^k_i(t) \leq \sum_{t=0+T_1+\cdots+T_i-1}^{T^*-T_i} \beta^k_{i-1}(t)x^k_{i-1}(t),
\]
(17)

\[
T^* \in \{T_0 + T_1 \cdots + T_i, \ldots, T\}
\]

This constraint enforces a flight stay in a link for a minimum time (the link length), which amounts to guaranteeing that the traffic flow will not move “too fast”. Detailed discussion about this constraint can be found in [19].

6) \textit{Integer constraint}

\[
x^k_i(t) \in \mathbb{Z}_+, \quad \beta^k_i(t)x^k_i(t) \in \mathbb{Z}_+
\]
(18)

\textbf{Remark:} (1) the range of subscripts or superscripts above are defined here, \( t \in \{0, \ldots, T\}, i \in \{1, \ldots, n^k\}, k \in \{0, \ldots, K\} \). (2) \( \beta^k_i(t) \) is the control to be optimized. Note that \( \beta^k_i(t) \) always appears together with \( x^k_i(t) \), using substitution \( q^k_i(t) = \beta^k_i(t)x^k_i(t) \) will yield a linear program. This does not increase the size of the problem, since \( q^k_i(t) \) simply takes the place of \( \beta^k_i(t) \).

B. The Dual Decomposition Algorithm based on the LTM

An algorithm based on CTM(L) using dual decomposition method is presented in [8]. Its mathematical foundation can be borrowed to derive the LTM version with few modifications. This subsection only outlines the derivation of the objective function of subproblems, which is the key difference:

\[
\sum_{t=0}^{T} e^k x^k(t) + \sum_{t=0}^{T} \sum_{i=0}^{n^k} \lambda_{s_i}(t)Z^k_{s_i}(t)
\]
(19)

\[
= \sum_{t=0}^{T} \sum_{i=0}^{n^k} e^k \sum_{j=0}^{n_i} x^k_{j,i}(t) + \sum_{t=0}^{T} \sum_{i=0}^{n^k} \lambda_{s_i}(t) \sum_{j=0}^{n_i} x^k_{j,i}(t)
\]

\[
= \sum_{t=0}^{T} \sum_{i=0}^{n^k} (e^k_i + \lambda_{s_i}(t)) \sum_{j=0}^{n_i} x^k_{j,i}(t)
\]

\[
= \sum_{t=0}^{T} \sum_{i=0}^{n^k} (e^k_i + \lambda_{s_i}(t))x^k_i(t)
\]

where \( Z^k_{s_i}(t) \) is defined as the sum of all cells on path \( k \) in sector \( s_i \). \( n_i \) is the number of cells in link \( i \). \( e^k \) is the weight imposed on path \( k \). \( \sum_{j=0}^{n_i} x^k_{j,i}(t) \) is the aggregate aircraft count in link \( i \), denoted as \( x^k_i(t) \) in LTM. The dual decomposition algorithm based on LTM is given in Table 1.
Basically, each subproblem is a smaller problem being optimized independently. \( \lambda_s(t) \) is updated when all subproblems are solved. The master problem \( d^*(\lambda) \) functions as a convergence judgment. the algorithm approaches the global optimum in an iterative fashion.

Table 1 Dual Decomposition Algorithm based on LTM

| Step 1: solve subproblems one by one (for path \( k \)):

\[
d^k(\lambda) = \min_{t=0}^{T} \sum_{i=0}^{N} [c_i(t) + \lambda_s(t)] x_i(t),
\]

s.t. \[
X_i^0(0) = B f^k(0),
\]

\[
X_i^k(t+1) = A_i(t) X_i^k(t) + B f^k(t),
\]

\[
0 \leq x_i^k(k) \leq C_s(t),
\]

\[
\sum_{t=0}^{T} \beta_0^k(t)x_i^k(t) = \sum_{t=0}^{T} \beta_n^k(t)x_n^k(t) = \sum_{t=0}^{T} f^k(t),
\]

\[
\sum_{t=0}^{T} \beta_i^k(t)x_i^k(t) = 0,
\]

\[
\sum_{t=T_0+T_1+\cdots+T_i}^{T} \beta_i^k(t)x_i^k(t) \leq \sum_{t=T_0+T_1+\cdots+T_{i-1}}^{T} \beta_i^{k-1}(t)x_i^{k-1}(t),
\]

where \( T^* \in \{T_0 + T_1 \cdots + T_i, \ldots, T\} \)

| Step 2: update master problem:

\[
d^*(\lambda) = \max \{-\sum_{t=0}^{T} \sum_{s_i=1}^{S} \lambda_s(t) C_s(t) + \sum_{k=0}^{K} d^k(\lambda)\},
\]

If \( d^*(\lambda) \) converge or \( i = \text{max iteration} \):

output \( x_i^{k,i}(t) \), stop.

else update:

\[
g_s(t) = -(\sum_{(i,k) \in Q_s} x_i^k(t) - C_s(t)),
\]

\[
\lambda_s(t) := (\lambda_s(t) - \alpha_i g_s(t))_+.
\]

Go to Step 1.

where

- \( s_i \in \{0, \cdots, S\}, t \in \{0, \ldots, T\}, k \in \{0, \ldots, K\} \).

- \( g_s(t) \) is the subgradient of dual function.

- \( \lambda_s(t) \) is the Lagrange multiplier, \((\cdot)_+\) denotes the non-negative part of a number.

- \( \alpha_i = \frac{1}{i+1} \) is the subgradient step and \( i \) is the iteration index.
This section presents two practical applications of the LTM: NAS-wide air traffic prediction and optimization. In the first application, the accuracy of air traffic prediction is statistically examined. In the second application, the LTM-based optimization of traffic flow is compared with CTM(L)-based optimization.

The models are programmed with C++ as a single thread program on a 2.8 GHz INTEL i7 CPU, 16G RAM DELL workstation running LINUX. The optimization tool used is CPLEX11.0 [20]. Historical nonmilitary flight data are acquired from ASDI/ETMS [21], which provides latitude, longitude, altitude and time information of all airborne aircraft in the United States. A full month ASDI data in May, 2005 are used to compute the length of links and the transition probability. These parameters are used to predict the traffic in September, 2005. In the following subsections, the ASDI/ETMS record serves as a baseline for the prediction and optimization.

A. Air Traffic Flow Prediction

1. Sector Level

This subsection demonstrates the predictive capability of the LTM at a sector level. A single sector is analyzed first, then the LTM is compared with other Eulerian approaches by examining the traffic predictions of ten sectors in the Oakland Center. NAS-wide traffic prediction will be presented in the next subsection. A full day (September 1, 2005) filed departures $f^k(t)$ is extracted from the ASDI/ETMS data, and fed into the model.

Figure 4 shows the predicted sector count in ZOA14 (Oakland Center). In a 24-hour horizon, the predicted sector count is largely consistent with the ASDI record despite small deviation in a short time period. Detailed statistics is shown in Figure 5. Figure 5(a) is the prediction error, which is defined as the difference between the ASDI record $x^k_i(t)$ and the predicted value $\hat{x}^k_i(t)$:

$$e_{si}(t) = \sum_{i \in Q_{si}} x^k_i(t) - \sum_{i \in Q_{si}} \hat{x}^k_i(t), \quad s_i \in \{1, 2, \ldots, 485\}.$$  

From Fig. 5(a), it can be seen that the maximum error is three. The error is small during the nighttime hours and increases as the traffic rises. Figure 5(b) presents the error distribution. Majority of the errors concentrate around zero. Note that the magnitude of sector count changes a lot during
the day, the errors should be assessed in terms of the absolute sector count. Fig. 5(c) shows the relative error which is defined as the ratio of the absolute error to the ASDI record:

\[ r_{si}(t) = \frac{|c_{si}(t)|}{\sum_{i \in Q_{si}} x^i(t)} \] \[ s_i \in \{1, 2, \ldots, 485\}. \]

Figure 5(d) presents the relative error distribution. The result suggests that over 70% of the relative errors are smaller than 30%. In other words, the prediction has achieved an accuracy of more than 70% within 70% of the time throughout the day in ZOA14. Note that there are nearly 9% of the relative errors that are greater than 100%, which is due to the small absolute sector count (usually one or two). From Fig. 5(c), majority of the big relative errors occur during the nighttime hours when there are few aircraft. This observation also implies that the LTM may perform better in sectors having heavy traffic.

[22] made a thorough comparison between three important Eulerian models. Based on its work, the PDE and modified Menon Model are chosen to compare with LTM. Prediction errors of ten high altitude sectors in Oakland Center are shown in Fig 6. The LTM is comparable to the PDE but inferior to the Modified Menon Model. This is partially due to the resolution of a model. Although three models have distinct dynamics, their accuracies rely on parameters extracted from historical data. The LTM and PDE use finest resolution, i.e. link [6], even a small error inclines to yield
Fig. 5 Statistics of sector count prediction in sector ZOA14

bigger bias when computing the transmission between sector boundaries. In contrast, the modified Menon model uses latitude-longitude tessellation [2], which computes the transmission pattern at a larger geographic scale and consequently results in smaller bias.

Fig. 6 Mean error and Standard deviation of predicted sector counts in Oakland Center
This subsection evaluates the overall performance of LTM throughout the NAS, the statistics of 485 high altitude sectors are analyzed. Figure 7(a) presents the relative errors of the 485 sectors, which are defined as the ratio of mean errors to the average aircraft count in a sector during that day:

\[
\bar{r}_{s_i} = \frac{\sum_{t=0}^{T} |e_{s_i}(t)|}{\sum_{t=0}^{T} \sum_{i \in Q_{s_i}} x^i_k(t)}, \quad s_i = \{1, 2, \ldots, 485\}
\]

Figure 7(b) shows the relative error distribution. Over 80% of the sectors have achieved an accuracy of relative error less than 20%. Figure 7(c) and 7(d) shows the standard deviation of relative errors. Over 80% of the sectors have deviation less than two aircraft. In order to demonstrate prediction accuracy, the prediction is also applied to different days spanned in September 2005. Table 2 shows the relative aircraft count error distributions. The second column suggests that LTM is able to maintain an accuracy of relative error less than 20% in more than 80% of the sectors.

Fig. 7 statistics of sector count prediction errors of 485 high altitude sectors
Table 2 Relative error distribution of air traffic predictions of selected dates in Sep. 2005

<table>
<thead>
<tr>
<th>Date</th>
<th>Relative error of aircraft count</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>≤ 20%</td>
</tr>
<tr>
<td>01/09/2005</td>
<td>81.03%</td>
</tr>
<tr>
<td>05/09/2005</td>
<td>84.44%</td>
</tr>
<tr>
<td>10/09/2005</td>
<td>81.54%</td>
</tr>
<tr>
<td>15/09/2005</td>
<td>83.64%</td>
</tr>
<tr>
<td>20/09/2005</td>
<td>85.36%</td>
</tr>
<tr>
<td>25/09/2005</td>
<td>85.18%</td>
</tr>
<tr>
<td>30/09/2005</td>
<td>78.79%</td>
</tr>
</tbody>
</table>

B. Air Traffic Flow Optimization

Compared to CTM(L), the key advantage of LTM is the reduced-dimension resulting in reduced computing time in air traffic optimization. This subsection first uses a simple Two-cross-paths scenario to validate the optimization algorithm and program. Then an two-hour NAS-wide TFM optimization is conducted and analyzed.

1. Algorithm and Program Verification

Fig 8(a) shows two paths intersecting in ZDC19 (high altitude), each path consisting of a few links. These two paths are coupled by the sector capacity constraints in ZDC15, ZDC19 and ZDC58. A two-hour traffic is extracted from ASDI data, and the sector capacities are deliberately set to seven which is below the peak traffic. Since the dimension of the optimization problem is low, a direct optimization is feasible without employing the dual decomposition method. The commonly used MATLAB function `LINPROG()` is used to produce a direct optimization which serves as a baseline, as shown in Fig 8(b). Both LTM and CTM(L) yield optimized traffic that are close to the direct optimization by `LINPROG()`. Due to the lack of preknowledge of convergence, a maximum iteration is used as the termination criteria in the algorithm in Table 1. Fig 8(c) shows the solutions generated by LTM using different number of iteration. 50-iteration and 100-iteration are the most close to the optimum. Further analysis also indicates that the master problem has little change
between 50-iteration and 100-iteration, hence 100-iterations does not show superior optimality than 50-iteration. However, the computing time linearly increases. Given the balance between optimality and efficiency, a maximum 50 iterations is used as the termination criteria. Although neither LTM nor CTM(L) reaches the rigorous optimum, the algorithm has been verified to yield a near-optimal solution in reasonable time.

![Diagram](image)

Fig. 8 A simple Two-cross-paths scenario

2. **NAS-wide TFM optimization problem**

This subsection applies the algorithm to the NAS-wide TFM problem. The input data are the extracted departures between 5:00 and 7:00 PM on September 1, 2005, “background” flights that
are already airborne before 5:00 PM are excluded from the simulation. 2326 paths and 3054 flights are involved. Given that ASDI traffic is under control in reality. only 70% of the nominal sector capacity is used in order to create a “busy” traffic. For this reason, the sector capacities are modified Monitor Alert Parameters (MAPs) [23]. This approach may not be totally accurate but provides a convenient means to examine the control capability of the algorithm.

Optimized traffic of two contiguous sectors is presented in Fig. 9 (ZNY75, ZNY42, New York Center). Three facts are observed from Fig. 9: (1) Both LTM and CTM(L) keep the sector counts below the sector capacity. (2) Respecting the sector capacity constraint incurs delay as a trade-off. Although the filed traffic is within two-hour planning time horizon, optimized aircraft are delayed to alleviate the congestion and the planning time horizon is prolonged beyond two hours (three hours in the simulation). (3) Both models leverage the traffic of the sectors. Figure 9(a) shows a heavy traffic in which the aircraft are restricted to enter ZNY42 in order to meet the capacity constraint. Figure 9(b) shows an opposite case: the aircraft are delayed in ZNY75 in order to alleviate the congestions in its contiguous sectors. Hence, the traffic of sectors are balanced and utilization of airspace is maximized.

![Fig. 9 Comparisons of air traffic flow optimization](image)

Figure 10 compares the parameters between LTM and CTM(L) with respect to the dual decomposition algorithm. Eq. (19) makes it clear that the objectives of the subproblems in both models are equal despite the different definition of state variables. Figure 10(a) and (b) show that the total flight time and summation of the Lagrangian multipliers for both models are almost identical. Hence, in terms of optimization, the LTM yields an equally optimal solution as CTM(L).

The computational efficiency of LTM improves remarkably. The computing time almost linearly
Fig. 10 Comparisons of parameters in the dual decomposition algorithm

increases as iteration increases (see Fig. 11(a)). The LTM takes about 60 minutes to finish 50 iterations whereas the CTM(L) takes 340 minutes, in other words, the LTM is about six times faster than CTM(L). There are two reasons for this. (1) The LTM reduces the number of state variables. Suppose a path has \( m \) links and \( n \) cells, \( 2T \times (n - m) \) state variables are reduced (control variables are also taken into account) by using LTM. Generally, \( n \) is far bigger than \( m \). For instance, a path connecting the airport LAX (Los Angeles, CA) and JFK (New York, NY) takes roughly five and a half hours (\( n = 330 \) minutes), but it merely contains about 20 links. For the two-hour optimization in which \( T = 120 \), the reduction is quite significant, namely \( 2 \times 120 \times (330 - 20) = 74400 \). (2) The number of constraints is proportional to the number of state variables and the planning time horizon. To formulate the constraints described in Section III, the CTM(L) requires about \( (2 + 2n + m) \times T \) constraints for a path whereas the LTM only involves \( (2 + 3m) \times T \) constraints. The difference is \( (n - m) \times 2T \). The models are formulated as a linear Integer Program, SIMPLEX method is used to to solve the problem. Given that there are fewer variables and constraints for the LTM, the SIMPLEX has fewer vertices to search [24]. As shown in Fig. 11(b), the LTM generally uses a sixth SIMPLEX iterations that are needed by CTM(L) to solve a subproblem. Therefore, the LTM reduces the size of the subproblems and save computing time. Generally, the longer are the paths and the planning time horizon, the bigger is the computational savings.

Without loss of generality, another five full day traffic is used to verified the computational efficiency. Table 3 shows the statistics. The second column is the computing time ratio. The third column presents the relative difference between the total flight time. It is clear that LTM is almost six times faster than CTM(L) with small objective function (i.e. total flight time) difference.
Therefore the LTM produces equally optimal solutions as CTM(L) dose.

C. Assess the Control Cost

Effort are also made to further explore other capabilities of LTM in this study. Equation (13) suggests that aggregate aircraft count $x^k_i(t)$ in a link is a function of control $\beta^k_i(t)x^k_i(t)$. Changing the weight that is imposed on a link will influence the delay control in that link. The control cost defined in Eq. (10) varies accordingly. For this reason, it is possible to leverage the ground delay and the air delay by tuning the weights imposed on link 0 and other en route links. Figure 12 presents the control cost against the ratio of the weights, which is a monotonically decreasing curve. The cost $W_{\text{ground}}$ and $W_{\text{airborne}}$ are tentatively assigned in this study but can be adjusted to reflect actual cost in practical applications. As the air delay incur more cost than the ground delay, it is desirable to decrease the air delay and “translate” it into the ground delay. This is accomplished by

\begin{table}[h]
\centering
\caption{Computing time saving statistics}
\begin{tabular}{lcc}
\hline
Data & Computing time ratio & Average relative difference of total flight time \\
\hline 
(CTM(L)/LTM) & & \\
09/01/2005 & 5.9184 & 4.31\% \\
09/10/2005 & 6.4675 & 5.43\% \\
09/15/2005 & 5.362 & 6.63\% \\
09/20/2005 & 6.222 & 5.92\% \\
09/30/2005 & 5.752 & 1.97\% \\
\hline
\end{tabular}
\end{table}
increasing the weights of the en route links. For the case where no air delay is allowed, the inequality in Eq. (17) should be changed to equality, which amounts to enforcing that all aircraft pass through en route links without being delayed. The control strategy examined in this paper is preliminary, further attempt can be make to study other strategy, such as the rate control, which can be achieved by assigning different weights on links passing congested sector, or may even defining the weight as a function of time.

V. Conclusion

This paper examines a Link Transmission Model developed based on the Large-capacity Cell Transmission Model. Detailed analysis indicates that LTM can be five to six times faster than CTM(L) in nationwide air traffic flow optimization while maintaining an equally optimal solution. The LTM also achieves a relative prediction errors that are less than 20% within 80% of the high altitude sectors in the NAS. The LTM incorporates ground delay and air delay into the model. A heuristic cost analysis has shown that imposing higher weights on the en route links results in less overall control cost. This is consistent to the common recognition that ground delay is more preferred than the air delay due to safety and cost saving. LTM is an Eulerian-Lagrangian model providing a means for TFM evaluation at both sector-level and airway-level (i.e. path-level in this paper). It contributes to speedup of the evaluation process by reducing the runtime.

The framework presented also suggests further computational efficiency improvement. Optimization in this paper uses a single thread to solve subproblem sequentially. Since the air traffic is constructed as a network of paths, the optimization can be decomposed path by path via the dual decomposition method. Taking advantage of modern multi-core computer, the computational
efficiency can be further improved by solving subproblems in parallel.

Acknowledgments

The authors are thankful to Banavar Sridhar and Shon Grabbe (NASA Ames) for their suggestion regarding aggregate air traffic modeling, traffic flow management and their ongoing support for our research. The authors are also grateful to Kapil Sheth (NASA Ames) for his help with FACET and ASDI/ETMS data. The authors would like to acknowledge Gano Chatterji (University of California Santa Cruz) for his initial suggestion on studies for resolution, accuracy and computational efficiency of different aggregate models.

References


doi:10.2514/1.31717


doi: 10.1109/JPROC.2008.2006141


doi: 10.1287/opre.42.2.249


doi: 10.1287/trsc.32.3.268


doi: 10.2514/1.40300


