Decentralized Control Framework and Stability Analysis for Networked Control Systems

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The combination of decentralized control and networked control where control loops are closed through a network is called Decentralized Networked Control System (DNCS). This paper introduces a general framework that converts a generic decentralized control configuration of non-networked systems to the general setup of a Networked Control Systems (NCS). Two design methods from the literature of decentralized control for non-networked systems were chosen as a base for the design of a controller for the networked systems, the first being an observer-based decentralized control, while the second is the well-known Luenberger combined observer-controller design. The main idea of our design is to formulate the DNCS in the general form and then map the resulting system to the general form of the NCS. First, a method for designing decentralized observer-based controller is discussed. Second, an implementation using a network is analyzed for the two designs. Third, two methods to analyze the stability of the DNCS are also introduced. Fourth, perturbation bounds for stability of the DNCS have been derived. Finally, examples and simulation results are shown and discussed.

1 Introduction

Decentralized control is used when there is a large scale system (LSS) whose sub-systems have interconnections with existing constraints on data transfer between them. Unlike centralized control, the decentralized control can be robust and scalable especially to the systems that are distributed over a large geographical area. The main feature of decentralized control is that it uses only local information to produce control laws [1].

The recent research efforts in the area of control systems have paved the way to better understand and interact with large-scale decentralized modern control systems [4, 6, 23]. To mention a few, large-scale Networked Control Systems (NCS) can be found in many diverse applications, such as: transportation networks, smart-grids, digital communication systems, and robotics. Since communication networks are an essential component of these systems, the analysis of a networked version of decentralized control systems is becoming crucial. The objective of this paper is to introduce a general framework that converts a generic decentralized control configuration of non-networked systems to the general setup of an NCS.

1.1 Decentralized Control

The decentralized control methodology, in many cases, is intended to replace the complex, expensive, and impractical applications of centralized control. A main field of decentralized control is the large-scale interconnected systems. Transportation systems, communication networks, power systems, economic systems, manufacturing processes and many others, are examples where decentralized control is used. The main idea behind designing decentralized controllers is the use of local information to achieve global results.

In this paper we are considering the observer-based decentralized control design for large-scale interconnected systems where the feedback loops are closed through a network. The robust design of the decentralized control strategies has been introduced in [4–6]. In [7], the authors proposed an observer-based control algorithm for linear systems where the design uses low-order linear functional observers. The individual subsystem states are estimated in [8, 9] by using an observer where the separation principal needs information exchange between subsystems in order to be utilized. Observer-based control design for non-linear systems is introduced in [10–13]. The key feature of the design proposed in [10] is that the separation principle of the linear systems case holds in their design for the non-linear system.

1.2 Networked Control Systems

The digital and computation progress spur the development of distributed control systems. These modern systems which include sensors and actuators that are controlled via a centralized or decentralized controllers, are connected by using a shared communication medium. This type of real-time networks are called networked control systems (NCS) [14].

NCS applications can be found in passenger cars, trucks and buses, aircraft and aerospace electronics, factory automation, industrial machine control, medical equipment, mobile sensor networks and many more [17]. However the NCS can potentially increase system reliability, reduce weight, space, power and wiring requirements, there are constraints that limit the applications. Generally, these limitations arise from multiple-packet transmission, data packet dropouts and finite bandwidth that is, only one node can access the shared medium at a time. Conventional control theories having ideal assumptions, such as synchronization of the control or non-delayed sensing and actuation, have to be reevaluated to take the network effects in account before they are applied to NCS. Basically, the primary objective of NCS analysis and design is to efficiently use the finite bus capacity while maintaining good closed-loop control system performance [16].

1.3 Decentralized Networked Control Systems

It is noteworthy to mention that NCSs and decentralized control applications do often overlap, which adds to the significance of studying and analyzing Decentralized Networked Control Systems (DNCS). Generally, decentralized control is used when there is a large scale system (LSS) whose sub-systems have interconnections with existing constraints on data transfer between them. The problem of decentralized control can be viewed as designing local controllers for subsystems comprising a given system. Decentralized control is especially viable for systems whose subsystems are separated geographically. Unlike centralized control, the decentralized control can be robust and scalable especially to the systems that are distributed over a large geographical area. The main feature of decentralized control is that it uses only local information to produce control laws.

It is very common to see systems which include sensors, actuators and controllers are connected through a shared communication medium. Some advantages of connecting the system components via network compared to traditional point-to-point control systems are modularity, flexibility of the system design, and simplicity of implementation such as reduced system wiring and configuration tools. Considering the benefits of decentralized control and the fact that modern control systems are increasingly becoming networked control systems, the area of decentralized networked control systems (DNCS) has recently emerged [23].

Figure 1 shows the overall structure of a Decentralized Networked Control System (DNCS) model. In this system example, we have three dynamical systems modeling the plant behavior:

$$\begin{aligned} \dot{x}_i &= f_i(x_i, u_i, w_i, t) \\ y_i &= h_i(x_i, u_i, w_i, t), \qquad \forall i = 1,2,3, \end{aligned}$$

and N controller dynamical systems:

$$\begin{aligned} \dot{z}_i &= g_i(z_i, r_i, v_i, t), \\ p_i &= q_i(z_i, r_i, v_i, t), \qquad \forall i = 1,\dots,N, \end{aligned}$$

where x_i and z_i are the states of the plant and controller, u_i, r_i are the plant and controllers' inputs, w_i and v_i are the possible disturbances, noises, or attacks against the system. Sensors $(\alpha_i^{(j)})$ and actuators $(s_i^{(j)})$ form networks that are inherently connected to the plant. Through sensors, the plants' outputs are sent to the controllers via the network, and the controllers' commands are sent back to the actuators, through the network as well.

1.4 Research Gaps, Paper Preliminaries, Contributions and Organization

As mentioned in the abstract and introduction, the objective of this paper is to introduce a general frameowrk that converts a generic decentralized control configuration of non-networked systems to the general setup of an NCS. To our knowledge, there is no similar framework in the recent DNCS literature to the one we are proposing. In this paper, we are addressing this research gap with the formulation of the framework. In order to introduce the proposed framework, a decentralized control design scheme of non-networked systems is chosen. We consider the observer-based control design in [7]. The authors considered the case when there is no communication network between the system's components. In this paper, we analyze the case where the control loops of the conventional decentralized controlled system are closed through a network. We adopt a design of the observed-based controller for the DNCS and then analyze the stability of the networked closed loop system. Two approaches to model the network effect are chosen to analyze the stability of the DNCS.

The contributions in the paper are as follows:

1. Development of a general framework that converts a generic decentralized control configuration of non-networked systems to the general setup of NCS
2. Applying the general framework for two different designs of decentralized control
3. Analysis of the closed-loop system stability of the DNCS through two approaches for the two designs

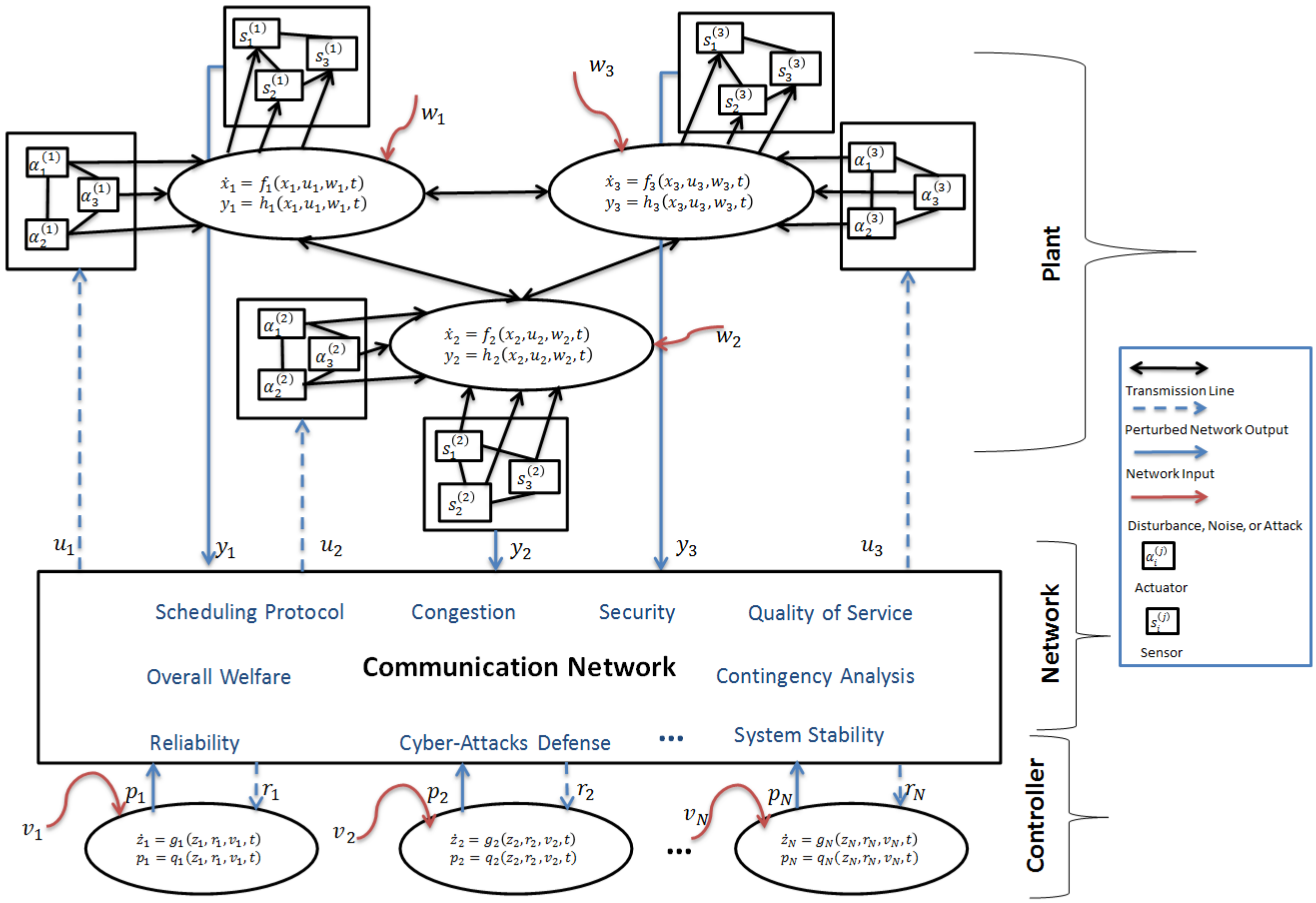

Fig. 1. Example of an NCS Configuration.

4. Derivation of the perturbation bounds of the networked system.

The remainder of this paper can be summarized as follows. Section II is dedicated to the problem formulation. Section III addresses the stability analysis and the perturbation bounds. In section IV we introduce two examples and show the simulation results. Conclusions and summary of the paper are given in the last section.

2 Problem Formulation

To validate and test the proposed formulation for decentralized networked control systems, and to highlight the applicability of the DNCS framework, we choose two designs of decentralized control. We introduce the two designs in two different parts. In the first part, we apply a controller design that is based on the observer-based decentralized control for multi-agent systems from [7], while in the second part we consider the well-known Luenberger combined controller-observer design from [24].

Part 1: Observer-based Decentralized Control

In this part of the problem formulation, we first introduce a decentralized observer-based control method from the literature of non-networked systems as a controller design for the networked systems. Then, we map the closed-loop non-networked system formulation to the equivalent configuration in networked dynamical systems. The last step in our problem formulation of this part is augmenting the state of the network induced error with the state of the closed-loop system. This will facilitate in applying the stability analysis tools from the NCS literature.

2.1 Observer Based Control Design Formulation

In this paper, we are considering the observer based control design from [7]. We have a large-scale system where the plant dynamics are described as follow:

$$\begin{cases} \dot{x} = Ax + \sum_{i=1}^{N} B_i u_i \\ y_i = C_i x, \; i = 1, 2, \dots, N \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector of the plant of the large-scale system, $u_i \in \mathbb{R}^{m_i}$ is the input vector of the i$^{\text{th}}$ subsystem and $y_i \in \mathbb{R}^{p_i}$ is the output vector of the i$^{\text{th}}$ subsystem. $A \in \mathbb{R}^{n\times n}, B_i \in \mathbb{R}^{n\times m_i}$, and $C_i \in \mathbb{R}^{p_i \times n}$ are all real constant matrices. Let

$$\begin{cases} u = \left[u_1^\top \; \dots \; u_N^\top\right]^\top, y = \left[y_1^\top \; \dots \; y_N^\top\right]^\top \\ B = \left[B_1 \; \dots \; B_N\right], C = \left[C_1^\top \; \dots \; C_N^\top\right]^\top. \end{cases}$$

Then the plant can be written in the following compact form:

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u_p \\ y &= C_p x_p. \end{aligned}$$

We assume the following as in [7]:

Assumption 1. *The triplet (A_p, B_p, C_p) is controllable and observable.*

Assumption 2. *The triplets (A_p, B_i, C_i) are stable if there exist decentralized fixed modes that are associated with the triplets.*

Assumption 3. *There exists a complete decentralized structure of the information of each subsystem (i.e., only the local output and control law of each subsystem are available).*

Assumption 4. *Global state feedback control exists such that $u = -Fx$, where $F \in \mathbb{R}^{m\times n}$.*

The global state feedback control gain F can be obtained by using any standard state feedback control method. Partitioning the global controller u, we get,

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix} = -\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \end{bmatrix} x.$$

In [7], the authors proposed the following decentralized controller:

$$u_i = -F_i x \approx -(K_i L_i + W_i C_i)x \approx -K_i z_i - W_i y_i,$$

where $z_i \in \mathbb{R}^{o_i}$ is an estimate of the weighted plant state (z_i tracks $L_i x$) that has the following dynamics:

$$\dot{z}_i = E_i z_i + L_i B_i u_i + G_i y_i, \tag{2}$$

where

$$E_i \in \mathbb{R}^{o_i\times o_i}, L_i \in \mathbb{R}^{o_i\times n}, K_i \in \mathbb{R}^{m_i\times o_i}, W_i \in \mathbb{R}^{m_i\times p_i}$$

$$\text{and } G_i \in \mathbb{R}^{o_i\times p_i}$$

are real matrices that represent the controller design parameters [7], and

$$F_i \approx K_i L_i + W_i C_i. \tag{3}$$

The observation error vector is defined as:

$$e_{o_i} = z_i - L_i x, \qquad i = 1, 2, \ldots, N.$$

Therefore, the observation error dynamics are:

$$\dot{e}_{o_i} = \dot{z}_i - L_i \dot{x}.$$

After some simple manipulations, we obtain the following equation:

$$\dot{e}_{o_i} = E_i e_{o_i} + (G_i C_i - L_i A + E_i L_i)\, x - L_i B_{r_i} u_r. \tag{4}$$

B_{r_i} is a partition of B, $B = \begin{bmatrix} B_i & B_{r_i} \end{bmatrix}$, where $B_{r_i} \in \mathbb{R}^{n\times(m-m_i)}$ is the input matrix for $u_r(t)$ which contains $(N-1)$ input vectors of the remaining $(N-1)$ subsystems. With this particular partition of the input matrix B, the dynamics of the plant states are:

$$\dot{x} = Ax + B_i u_i + B_{r_i} u_r, \qquad i = 1, 2, \ldots, N.$$

Choosing E_i to be asymptotically stable, (2) can be viewed as a decentralized linear observer if L_i and G_i fulfill the following set of constraints:

$$L_i B_{r_i} = \boldsymbol{O} \tag{5}$$

$$K_i L_i + W_i C_i = F_i \tag{6}$$

$$G_i C_i - L_i A + E_i L_i = \boldsymbol{O}\ , \tag{7}$$

To compute the four unknowns (K_i, L_i, W_i, G_i), we are using a simpler approach other than the one proposed in [7]. Our approach is as follows. From (5), we can find L_i:

$$L_i = \left(Null(B_{r_i}^\top) \right)^\top.$$

Note that L_i is not unique. To find K_i, W_i, G_i we use the Kronecker product properties. From (6), we get:

$$(L_i^\top \otimes I_{m_i})\mathrm{vec}(K_i) + (C_i^\top \otimes I_{m_i})\mathrm{vec}(W_i) = \mathrm{vec}(F_i),$$

then,

$$\begin{bmatrix} L_i^\top \otimes I_{m_i} & C_i^\top \otimes I_{m_i} \end{bmatrix} \begin{bmatrix} \mathrm{vec}(K_i) \\ \mathrm{vec}(W_i) \end{bmatrix} = \mathrm{vec}(F_i). \tag{8}$$

Since we chose E_i and computed L_i, (7) has only one unknown which is G_i. Let $L_i A - E_i L_i = V_i$. Now we have

$$G_i C_i = V_i.$$

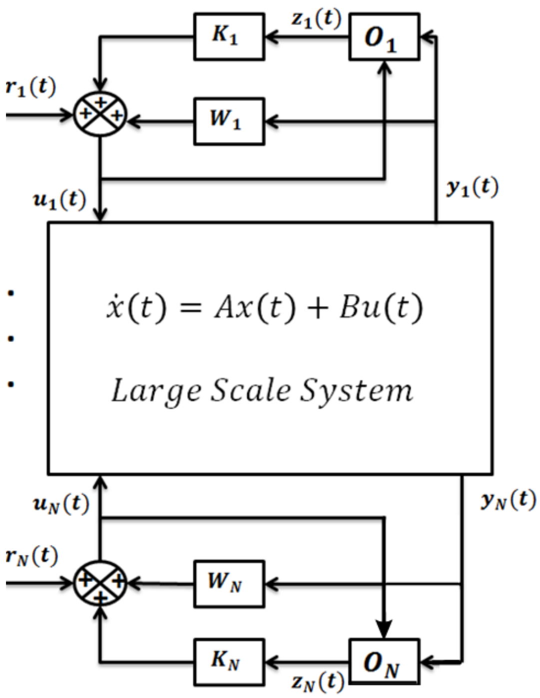

Fig. 2. Observer-Based Control Design Scheme.

Using the Kronecker product properties again we get:

$$\left(C_i^\top \otimes I_{o_i}\right) \mathrm{vec}(G_i) = \mathrm{vec}(V_i). \tag{9}$$

Combining (8) and (9), we get:

$$\underbrace{\begin{bmatrix} L_i^\top \otimes I_{m_i} & C_i^\top \otimes I_{m_i} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{O} & C_i^\top \otimes I_{o_i} \end{bmatrix}}_{\Psi} \begin{bmatrix} \mathrm{vec}(K_i) \\ \mathrm{vec}(W_i) \\ \mathrm{vec}(G_i) \end{bmatrix} \tag{10}$$

$$= \begin{bmatrix} \mathrm{vec}(F_i) \\ \mathrm{vec}(V_i) \end{bmatrix}.$$

Solving (10), we get:

$$\begin{bmatrix} \mathrm{vec}(K_i) \\ \mathrm{vec}(W_i) \\ \mathrm{vec}(G_i) \end{bmatrix} = \Psi^\dagger \begin{bmatrix} \mathrm{vec}(F_i) \\ \mathrm{vec}(V_i) \end{bmatrix},$$

where $\Psi^\dagger$ is the pseudo-inverse for Ψ.

After solving for the system design unknowns, we now have all the design parameters. Figure 2 shows the large-scale closed-loop system where the observer-based control design is applied in the feedback loops of each subsystem.

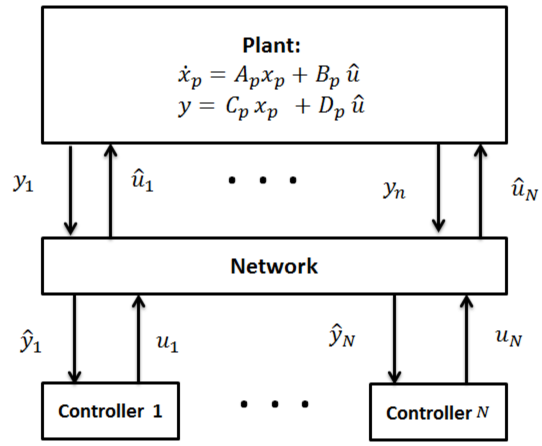

Fig. 3. DNCS State-Space Configuration (2).

2.2 Mapping the DNCS to the NCS Setup

The general setup of a DNCS is shown in Figure 3. The state-space representation for the plant is:

$$\begin{cases} \dot{x}_p = A_p x_p + B_p \hat{u} \\ y = C_p x_p + D_p \hat{u}, \end{cases} \tag{11}$$

where

$$\begin{cases} B_p = \left[B_{p_1} . . B_{p_N}\right], C_p = \left[C_{p_1}^\top \; \dots \; C_{p_N}^\top\right]^\top \\ y = \left[y_1^\top \; \dots \; y_N^\top\right]^\top, \hat{u} = \left[\hat{u}_1^\top \; \dots \; \hat{u}_N^\top\right]^\top. \end{cases}$$

The controller state-space representation is given by:

$$\begin{cases} \dot{x}_c = A_c x_c + B_c \hat{y} \\ u = C_c x_c + D_c \hat{y}, \end{cases} \tag{12}$$

where,

$$\begin{cases} B_c = \left[B_{c_1} . . B_{c_N}\right], C_c = \left[C_{c_1}^\top \; \dots \; C_{c_N}^\top\right]^\top \\ u = \left[u_1^\top \; \dots \; u_N^\top\right]^\top, y = \left[\hat{y}_1^\top \; \dots \; \hat{y}_N^\top\right]^\top. \end{cases}$$

To analyze the stability of the overall system under the proposed observer-based decentralized control design, we convert the DNCS setup to the general setup of the NCS, as shown in Figure 4. The delayed versions of u and y are defined as: $\hat{u} = u - e_{nu}$ and $\hat{y} = y - e_{ny}$, where e_{nu} and e_{ny} are the delay error due to the presence of the network.

Now we map the decentralized controller to the typical NCS form of the controller.

$$\begin{aligned} \dot{z}_i &= E_i z_i + L_i B_i u_i + G_i \hat{y}_i \\ &= E_i z_i + L_i B_i(-K_i z_i - W_i \hat{y}_i) + G_i \hat{y}_i \\ &= (E_i - L_i B_i K_i) z_i + (G_i - L_i B_i W_i) \hat{y}_i. \end{aligned}$$

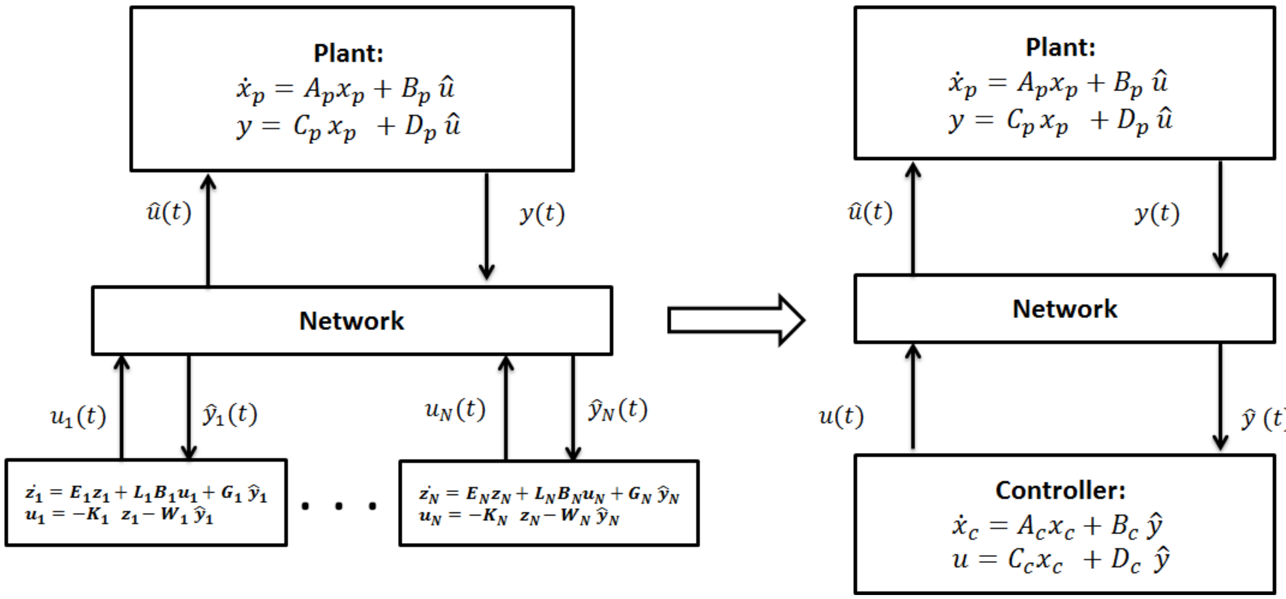

Fig. 4. Mapping DNCS to the Typical NCS Setup.

Let $x_c = z$, where $z = \begin{bmatrix} z_1^\top & z_2^\top & \dots & z_N^\top \end{bmatrix}^\top$, and introduce the following compact matrix notation:

$$\begin{cases} E = \mathrm{diag}(E_1, E_2, \dots, E_N), \\ K = \mathrm{diag}(K_1, K_2, \dots, K_N), \\ L = \begin{bmatrix} L_1^\top & L_2^\top & \dots & L_N^\top \end{bmatrix}^\top, \\ B_p = \begin{bmatrix} B_1 & B_2 & \dots & B_N \end{bmatrix}, \\ G = \mathrm{diag}(G_1, G_2, \dots, G_N), \\ W = \mathrm{diag}(W_1, W_2, \dots, W_N). \end{cases}$$

Therefore, we now have a compact form of the controller's dynamics:

$$\begin{cases} \dot{z} = (E - LBK)z + (G - LBW)\hat{y} \\ u = (-K)z + (-W)\hat{y} \end{cases} \tag{13}$$

Knowing that $\hat{y} = y - e_{ny}$, we can map (13) to the standard NCS state-space form of the controller from (12):

$$\dot{x}_c = A_c x_c + B_c \hat{y}, \qquad \hat{y} = C_p x_p - e_{ny},$$

then,

$$\dot{x}_c = A_c x_c + B_c C_p x_p - B_c e_{ny}, \tag{14}$$

where

$$\begin{cases} A_c = E - LB_pK \ , \ B_c = G - LB_pW \\ C_c = -K \qquad\quad\ , \ D_c = -W. \end{cases}$$

The plant state dynamics can be represented as:

$$\dot{x}_p = A_p x_p + B_p u - B_p e_{nu}.$$

The controller's output u can be written as:

$$\begin{aligned} u &= -Kz - W\hat{y} \\ &= -Kx_c - W(y - e_{ny}) \\ &= -Kx_c - WC_p x_p + We_{ny}. \end{aligned}$$

Recall that $\hat{u} = u - e_{nu}$ and by substituting u in the plant state-space dynamics equation, we get:

$$\dot{x}_p = (A_p - B_p W C_p)x_p - B_p K x_c + B_p W e_{ny} - B_p e_{nu}. \tag{15}$$

2.3 Network Effect Augmentation with the System's States.

In this section we first find the dynamics of the network-induced error. After finding an expression for the networked-induced error, we then augment the error dynamics with the general state of the closed-loop system. The network-induced error is defined as: $e_n = \begin{bmatrix} e_{n_y}^\top & e_{n_u}^\top \end{bmatrix}^\top$. Note that in our system $D_p = \boldsymbol{O}$, thus $y = C_p x_p$. Recall that $\hat{y} = y - e_{ny}$. In addition,

$$u = C_c x_c + D_c \hat{y}. \tag{16}$$

The networked-induced error can be written as:

$$e_n = \begin{bmatrix} e_{n_y} \\ e_{n_u} \end{bmatrix} = \begin{bmatrix} y - \hat{y} \\ u - \hat{u} \end{bmatrix} = \begin{bmatrix} C_p x_p - \hat{y} \\ C_c x_c + D_c \hat{y} - \hat{u} \end{bmatrix}.$$

Note that $\hat{y}$ and $\hat{u}$ are both piece-wise constant functions, thus: $\dot{\hat{y}} = \mathbf{0}$, and $\dot{\hat{u}} = \mathbf{0}$. Then,

$$\dot{e}_n = \begin{bmatrix} C_p \dot{x}_p \\ C_c \dot{x}_c \end{bmatrix} = \begin{bmatrix} C_p A_p x_p + C_p B_p u - C_p B_p e_{n_u} \\ C_c A_c x_c + C_c B_c C_p x_p - C_c B_c e_{n_y} \end{bmatrix}.$$

$$\dot{e}_n = \begin{bmatrix} (C_pA_p + C_pB_pD_cC_p)x_p + C_pB_pC_cx_c - C_pB_pD_ce_{n_y} - C_pB_pe_{n_u} \\ C_cB_cC_px_p + C_cA_cx_c - C_cB_ce_{n_y} \end{bmatrix}. \tag{17}$$

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_c \\ \dot{e}_{ny} \\ \dot{e}_{nu} \end{bmatrix} = \underbrace{\begin{bmatrix} A_p + B_pD_cC_p & B_pC_c & -B_pD_c & -B_p \\ B_cC_p & A_c & -B_c & \boldsymbol{O} \\ (C_pA_p + C_pB_pD_cC_p) & C_pB_pC_c & -C_pB_pD_c & -C_pB_p \\ C_cB_cC_p & C_cA_c & -C_cB_c & \boldsymbol{O} \end{bmatrix}}_{\hat{A}} \begin{bmatrix} x_p \\ x_c \\ e_{ny} \\ e_{nu} \end{bmatrix}. \tag{18}$$

Substituting (16) into the error dynamics we have (17).

Let x be the overall state of the closed loop system: $x = \begin{bmatrix} x_p^\top & x_c^\top \end{bmatrix}^\top$. Let w be the general state vector that includes the network-induced error vector: $w = \begin{bmatrix} x^\top & e_n^\top \end{bmatrix}^\top$. From (14)-(17), we can formulate the general state dynamics of the system as in (18).

Equation (18) combines the nominal closed-loop system and the perturbation that represents the network effect.

Part 2: Combined Observer-Controller Design

In this part of the problem formulation, and to validate the proposed framework and test the applicability of the DNCS scheme, we consider another controller design. In this formulation, we consider the well-known Luenberger combined controller-observer design method. We consider the system as described in Figure 5, where the plant state and the control are distributed over N subsystems.

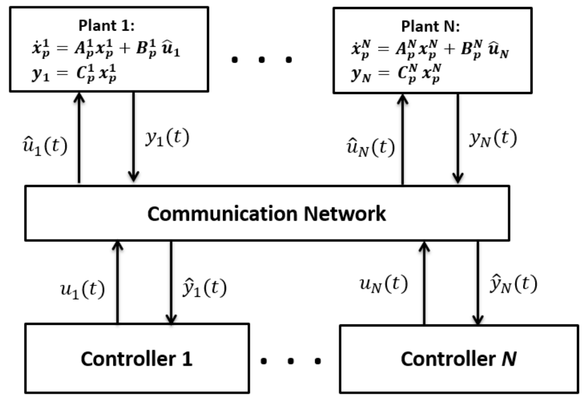

Fig. 5. Distributed State of the Plant and Distributed Control.

We have the following system

$$\begin{cases} \dot{x}_i = A_ix_i + B_i\hat{u}_i \\ y_i = C_ix_i. \end{cases} \tag{19}$$

Let,

$$\begin{cases} B_p = \begin{bmatrix} B_1 .. B_N \end{bmatrix}, C_p = \begin{bmatrix} C_1^\top & \dots & C_N^\top \end{bmatrix}^\top \\ y = \begin{bmatrix} y_1^\top & \dots & y_N^\top \end{bmatrix}^\top, \hat{u} = \begin{bmatrix} \hat{u}_1^\top & \dots & \hat{u}_N^\top \end{bmatrix}^\top \\ x_p = \begin{bmatrix} x_p^{(1)\top} & \dots & x_p^{(N)\top} \end{bmatrix}^\top, \\ A_p = \mathrm{diag}(A_1, \dots, A_N). \end{cases}$$

We can now formulate the global dynamics of the plant in a compact form, consisting of N subsystems:

$$\begin{cases} \dot{x}_p = A_px_p + B_p\hat{u} \\ y = C_px_p. \end{cases} \tag{20}$$

Since we are adapting the Luenberger combined controller-observer design, and given that we have N different controllers with respective feedback control and observer gains (i.e., K_{L_i}, L_{L_i}), we can derive a compact state-space representation of the global combined controller-observer state:

$$\begin{cases} \dot{x}_c = A_px_c + B_pu + L_L(\hat{y} - \tilde{y}) \\ u = -K_Lx_c, \end{cases} \tag{21}$$

where

$$\begin{cases} x_c = \begin{bmatrix} \tilde{x}_1^\top & \dots & \tilde{x}_N^\top \end{bmatrix}^\top, \tilde{y} = \begin{bmatrix} \tilde{y}_1^\top & \dots & \tilde{y}_N^\top \end{bmatrix}^\top \\ \hat{y} = \begin{bmatrix} \hat{y}_1^\top & \dots & \hat{y}_N^\top \end{bmatrix}^\top, K_L = \mathrm{diag}(K_{L_1}, \dots, K_{L_N}). \\ L_L = \mathrm{diag}(L_{L_1}, \dots, L_{L_N}). \end{cases}$$

Writing the estimated output in a compact form ($y = Cx_c$), and with simple manipulations to the combined controller-observer system dynamics equation, we can write a standard compact form for the controller of the networked closed-loop system:

$$\begin{cases} \dot{x}_c = A_cx_c + B_c\hat{y} \\ \hat{y} = u = C_cx_c, \end{cases} \tag{22}$$

where

$$A_c = A_p - B_pK_L - L_LC_p, B_c = L_L, C_c = -K_L. \tag{23}$$

Recall that $\hat{y} = y - e_{n_y} = C_p x_p - e_{n_y}$ and $\hat{u} = u - e_{n_u} = -K_L x_c - e_{n_u}$. Hence, substituting the terms in (22) and the previous two equations, in (20) and (21), we get:

$$\dot{x}_p = A_p x_p + B_p C_c x_c - B_p e_{n_u}, \tag{24}$$

and

$$\dot{x}_c = B_c C_p x_p + A_c x_c - B_c e_{n_y}. \tag{25}$$

Recall that the networked-induced error dynamics is defined as:

$$e_n = \begin{bmatrix} e_{n_y} \\ e_{n_u} \end{bmatrix} = \begin{bmatrix} y - \hat{y} \\ u - \hat{u} \end{bmatrix} = \begin{bmatrix} C_p x_p - \hat{y} \\ C_c x_c - \hat{u} \end{bmatrix}.$$

Since $\hat{y}$ and $\hat{u}$ are both piece-wise constant functions, thus: $\dot{\hat{y}} = \mathbf{0}$, and $\dot{\hat{u}} = \mathbf{0}$. Then,

$$\dot{e}_n = \begin{bmatrix} C_p \dot{x}_p \\ C_c \dot{x}_c \end{bmatrix} = \begin{bmatrix} C_p A_p x_p + C_p B_p C_c x_c - C_p B_p e_{n_u} \\ C_c B_c C_p x_p + C_c A_c x_c - C_c B_c e_{n_y} \end{bmatrix}. \tag{26}$$

Following the same methodology for the observer-based decentralized control in Part 1, we can augment the general state dynamics of the Luenberger combined controller-observer design, in addition to the networked-induced error state:

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_c \\ \dot{e}_{n_y} \\ \dot{e}_{n_u} \end{bmatrix} = \underbrace{\begin{bmatrix} A_p & B_p C_c & \boldsymbol{O} & -B_p \\ B_c C_p & A_c & -B_c & \boldsymbol{O} \\ C_p A_p & C_p B_p C_c & \boldsymbol{O} & -C_p B_p \\ C_c B_c C_p & C_c A_c & -C_c B_c & \boldsymbol{O} \end{bmatrix}}_{\hat{A}} \begin{bmatrix} x_p \\ x_c \\ e_{n_y} \\ e_{n_u} \end{bmatrix}. \tag{27}$$

Equation (27) combines the nominal closed-loop system and the perturbation that represents the network effect for the Luenberger combined controller-observer design.

3 Stability Analysis

In this section, we analyze the stability of the DNCS. To analyze the stability of the system, we consider two different approaches. In the two approaches, we separate the nominal system and the perturbation using two different methods. This is followed by deriving perturbation bounds for both methods.

3.1 The First Approach

Let x be the overall state of the closed loop system: $x = [x_p^\top \, x_c^\top]^\top$. Let w be the general state vector that augments the state of closed-loop system and the network-induced error vector. Hence, $w = [x^\top \, e_n^\top]^\top$. Based on the general state dynamics in (18) and (27), the nominal closed-loop system can be found when the network effect is null. Therefore, we can separate the nominal system and the perturbation in (18) and (27) and derive the following perturbation-separated representations.

The observer-based decentralized control can be written as in the following perturbation-separated formulation (28), whereas the combined Luenberger observer-controller design can be formulated as in (29), where S represents the dynamics of the nominal closed-loop system and ΔS represents the perturbation in the system dynamics. For stability analysis purposes, we introduce the matrix ΔC which is used to guarantee that $(S + \Delta C)$ is Hurwitz. We can now write the general system dynamics as:

$$\dot{w} = (S + \Delta C)w + (\Delta S - \Delta C)w = S_c w + \Delta S_c w. \tag{30}$$

Theorem 1. *For the DNCS in (11) and (12) and for any $Q = Q^\top \succ \boldsymbol{O}$, if the solution to the Lyapunov matrix equation*

$$S_c^\top P + P S_c^\top = -2Q, \; Q = I$$

is $P = P^\top \succ \boldsymbol{O}$, and if the norm of the perturbation matrix (ΔS_c) is upper bounded by:

$$\|\Delta S_c\| \le \frac{1}{\lambda_{max}(P)}$$

then the DNCS is globally asymptotically stable.

Proof. Since S_c is stable, then for $Q = I$, the solution to the Lyapunov matrix equation:

$$S_c^\top P + P S_c^\top = -2Q, \; Q = I$$

is symmetric positive definite. Using the following candidate Lyapunov function, $V = \frac{1}{2} w^\top P w$. Then,

$$\dot{V} = w^\top P \dot{w} = w^\top P S_c w + w^\top P \Delta S_c w.$$

Notice that

$$\begin{aligned} w^\top P S_c w &= \frac{1}{2} w^\top S_c^\top P w + \frac{1}{2} w^\top P S_c w \\ &= \frac{1}{2} w^\top \left(S_c^\top P + P S_c \right) w = -\|w\|^2. \end{aligned}$$

In addition, we have:

$$\begin{aligned} w^\top P \Delta S_c w \le \|P \Delta S_c\| \|w\|^2 &= \|P\| \|\Delta S_c\| \|w\|^2 \\ &= \lambda_{max}(P) \|\Delta S_c\| \|w\|^2. \end{aligned}$$

$$\dot{w}=\begin{bmatrix}\dot{x}_p\\\dot{x}_c\\\dot{e}_{n_y}\\\dot{e}_{n_u}\end{bmatrix}=\left\{\underbrace{\begin{bmatrix}A_p+B_pD_cC_p & B_pC_c & \boldsymbol{O} & \boldsymbol{O}\\ B_cC_p & A_c & \boldsymbol{O} & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O}\end{bmatrix}}_{S}+\Delta C\right\}\begin{bmatrix}x_p\\x_c\\e_{n_y}\\e_{n_u}\end{bmatrix}$$

$$+\left\{\underbrace{\begin{bmatrix}\boldsymbol{O} & \boldsymbol{O} & -B_pD_c & -B_p\\ \boldsymbol{O} & \boldsymbol{O} & -B_c & \boldsymbol{O}\\ (C_pA_p+C_pB_pD_cC_p) & C_pB_pC_c & -C_pB_pD_c & -C_pB_p\\ C_cB_cC_p & C_cA_c & -C_cB_c & \boldsymbol{O}\end{bmatrix}}_{\Delta S}-\Delta C\right\}\begin{bmatrix}x_p\\x_c\\e_{n_y}\\e_{n_u}\end{bmatrix}. \tag{28}$$

$$\dot{w}=\begin{bmatrix}\dot{x}_p\\\dot{x}_c\\\dot{e}_{n_y}\\\dot{e}_{n_u}\end{bmatrix}=\left\{\underbrace{\begin{bmatrix}A_p & B_pC_c & \boldsymbol{O} & \boldsymbol{O}\\ B_cC_p & A_c & \boldsymbol{O} & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O}\end{bmatrix}}_{S}+\Delta C\right\}\begin{bmatrix}x_p\\x_c\\e_{n_y}\\e_{n_u}\end{bmatrix}+\left\{\underbrace{\begin{bmatrix}\boldsymbol{O} & \boldsymbol{O} & \boldsymbol{O} & -B_p\\ \boldsymbol{O} & \boldsymbol{O} & -B_c & \boldsymbol{O}\\ C_pA_p & C_pB_pC_c & \boldsymbol{O} & -C_pB_p\\ C_cB_cC_p & C_cA_c & -C_cB_c & \boldsymbol{O}\end{bmatrix}}_{\Delta S}-\Delta C\right\}\begin{bmatrix}x_p\\x_c\\e_{n_y}\\e_{n_u}\end{bmatrix}. \tag{29}$$

Hence,

$$\begin{aligned}\dot{V} &\leq -\|w\|^2+\lambda_{max}(P)\,\|\Delta S_c\|\|w\|^2\\ &= -\left(1-\lambda_{max}(P)\,\|\Delta S_c\|\right)\|w\|^2.\end{aligned}$$

For a valid Lyapunov candidate function, we should have $\dot{V}<0$, thus:

$$\|\Delta S_c\|\leq\frac{1}{\lambda_{max}(P)}.$$

3.2 The Second Approach

In this approach we partition the augmented states in (18) as follows:

$$\dot{w}(t)=\hat{A}w(t)=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}w(t).$$

The state dynamics of the networked closed-loop system can be represented as: $\dot{x}(t)=A_{11}x(t)+A_{12}e_n(t)$, where

$$A_{11}=\begin{bmatrix}A_p+B_pD_cC_p & B_pC_c\\ B_cC_p & A_c\end{bmatrix},$$

and

$$A_{12}=\begin{bmatrix}-B_pD_c & -B_p\\ -B_c & \boldsymbol{O}\end{bmatrix}.$$

Consider the time interval between transmissions: $t\in[t_i,t_{i+1}]$ where $i=0,1,2,\ldots$, we get:

$$\hat{y}(t)=y(t_i)=C_px_p(t_i)$$

and

$$\hat{u}(t)=u(t_i)=C_cx_c(t_i)+D_cy(t_i)=C_cx_c(t_i)+D_cC_px_p(t_i).$$

Let $g(t,x)=A_{12}e_n(t)$, then the system dynamics equation can be written as:

$$\dot{x}(t)=A_{11}x(t)+g(t,x), \tag{31}$$

where $g(t,x)$ is the perturbation caused by the network. Let $e_x(t)=x(t)-x(t_i)$, then we can write the perturbation term as:

$$\begin{aligned} g(t,x) &= A_{12}e_n(t) = A_{12}\underbrace{\begin{bmatrix} C_p & \boldsymbol{O} \\ D_cC_p & C_c \end{bmatrix}}_{D}\left[x(t)-x(t_i)\right] \\ &= D\left[x(t)-x(t_i)\right] = De_x(t). \end{aligned}$$

Since the non-networked system is stable, then there exists a matrix $P = P^\top \succ \boldsymbol{O}$ such that the solution to the Lyapunov matrix equation:

$$A_{11}^\top P + PA_{11} = -Q$$

is symmetric positive definite ($P = P^\top \succ \boldsymbol{O}$). Let $\lambda_1 = \lambda_{min}(P)$ and $\lambda_2 = \lambda_{max}(P)$. In [22], Zhang et al. mentioned that an NCS is stable if the maximum allowable transfer interval (MATI) τ_m is upper bounded by:

$$\tau_m < \frac{\lambda_{min}(Q)}{16\lambda_2\sqrt{\frac{\lambda_2}{\lambda_1}}\|A\|^2\left(1+\sqrt{\frac{\lambda_2}{\lambda_1}}\right)\sum_{i=1}^{p} i}.$$

Based on this τ_m upper bound and treating $g(t,x)$ as a vanishing perturbation as in [20], we can introduce a bound to the perturbation that guarantees the stability of DNCS.

Theorem 2. *For the perturbed general state of the system in (30), if the origin is a globally exponentially stable point of the non-networked system, and if τ_m satisfies:*

$$1 - ||D||\ ||A_{11}+D||^{-1}(e^{||A_{11}+D||\tau_m} - 1) > 0,$$

and the perturbation is upper bounded by

$$||e_x(t)|| \le \gamma||x(t)||,$$

where

$$\gamma = \frac{||A_{11}||\ ||A_{11}+D||^{-1}(e^{||A_{11}+D||\tau_m}-1)e^{||A_{11}+D||\tau_m}}{1-||D||\ ||A_{11}+D||^{-1}(e^{||A_{11}+D||\tau_m}-1)},$$

then the origin is a globally exponentially stable equilibrium point of the DNCS.

Proof. The proof of the above theorem is very similar to the proof of Walsh et al. in [18].

4 Simulation Results

This section is dedicated to discuss our results from simulating the behavior of the proposed design of the DNCS. We first discuss two methods that we used to find a bound for the maximum allowable transfer interval τ_m. The first method considers the network effect as a perturbation as in Theorems 1 and 2. We used the MATI bound for the computation of the sufficiency condition of stability to the DNCS. This bound is used for stability analysis in general NCS systems. From the simulation results, we note that it is very conservative bound for a sufficiency condition of stability.

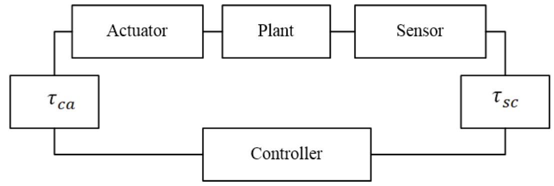

Fig. 6. The Network Effect Modeled as Pure Time Delay.

In the second method we used a less conservative bound from the literature. In [19], they derive the MATI bound by treating the network effect as a pure time delay. Figure 6 shows a high level description for a network modeled as a time delay.

With this modeling, the plant and controller dynamics can be rewritten as:

$$\begin{aligned} \dot{x_c}(t) &= A_cx_c(t) + B_cC_px_p(t-\tau_{sc}) \\ \dot{x_p}(t) &= A_px_p(t) + B_pD_cC_px_p(t-\tau_{sc}-\tau_{ca}) \\ &\quad + B_pC_cx_c(t-\tau_{ca}). \end{aligned}$$

The main idea behind finding a bound on the maximum allowable transfer interval (MATI) or τ_m is to model the delayed state as a taylor series expansion:

$$x(t-\tau) = \sum_{k=0}^{\infty}(-1)^n\frac{\tau^n}{n!}x^{(n)}(t).$$

In [19], they applied the following approximation:

$$x(t-\tau) \approx x(t) - \tau\dot{x}(t),$$

which leads to a significantly less conservative bound on τ_m as follows:

$$\tau_m < \frac{1}{||B_p[WC_p\ ,K]||}. \tag{32}$$

4.1 Numerical Example

In this section we introduce three numerical examples to analyze the behavior of the proposed design of the observer-based controller and the Luenberger combined observer-controller for the DNCS. We also discuss the perturbation bounds that we derived in Theorems 1 and 2.

4.1.1 Example 1 – Observer-Based Decentralized Control of Mobile Robot

The following system appears in [21]:

$$A=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\2&2&0&0\end{bmatrix},B_1=\begin{bmatrix}0&1\\0&0\\1&0.2\\0&0.5\end{bmatrix},$$

$$B_2=\begin{bmatrix}0&0\\0&1\\-1&-2\\0&0.5\end{bmatrix},C_1=\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix},$$

$$C_2=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}.$$

It is an unstable system with two controllers. In the controller design, only the local information are available. Computing the design parameters, we get the following decentralized control laws:

$$u_1=-\begin{bmatrix}0.022&-2.587\\0.896&-2.326\end{bmatrix}z_1-\begin{bmatrix}0.032&0.822&1.801\\1.280&0.303&2.557\end{bmatrix}\hat{y}_1,$$

$$u_2=-\begin{bmatrix}-0.271&0.307\\0.106&-1.467\end{bmatrix}z_2-\begin{bmatrix}-0.543&-0.822&-0.277\\1.072&-0.744&1.641\end{bmatrix}\hat{y}_2.$$

From the simulation results we note that the system becomes unstable for $\tau_m > 0.20065$ sec, as shown in Figure 7. When we compute the bound of MATI using Theorem 2, we get $\tau_m = 1.2580e^{-7}$ sec ,which is very conservative to guarantee the stable behavior of the DNCS as shown in Figure 8. On the other hand, when we use (32), we get $\tau_m = 0.1922$ sec, which is very close to the above bound of stability ($\tau_m < 0.20065$ sec).

In Theorem 1, the sufficiency condition of stability is $\|\Delta S_c\| \le \frac{1}{\lambda_{max}(P)}$. From the simulation results, $\|\Delta S_c\| = 20.2418$ and $\frac{1}{\lambda_{max}(P)} = 0.4341$. We can see that the system is stable even with larger value of the norm of the perturbation which means that the perturbation bound of Theorem 1 is conservative as a sufficiency condition for stability. In Theorem 2, the sufficiency condition of stability is $\gamma < \frac{1}{2\lambda_2}$. From the simulation results, $\gamma = 1.1064e^{-5}$ and $\frac{1}{2\lambda_2} = 0.2171$. Unlike the bound of Theorem 1, the perturbation bound of Theorem 2 is satisfied since the MATI bound in Theorem 2 that we used is very conservative as we mentioned before ($\tau_m = 1.2580e^{-7}$ sec).

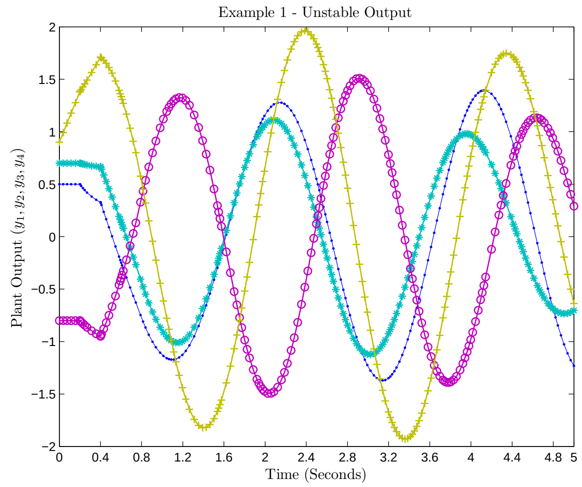

Fig. 7. Unstable Behavior of the System in Example 1 (for $\tau_m > 0.20065$ sec).

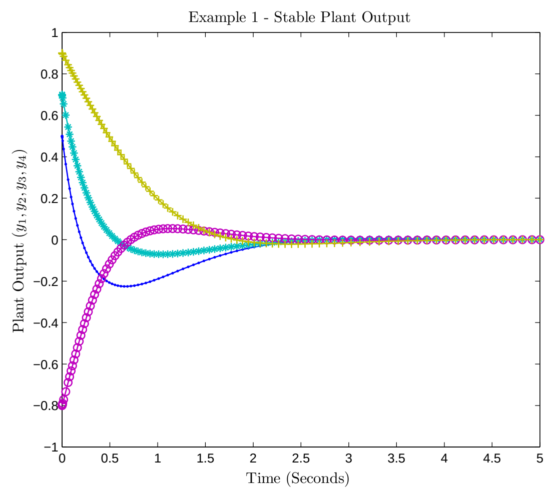

Fig. 8. Stable Behavior of the System of Example 1 (conservative bound, $\tau_m = 1.2580e^{-7}$ sec).

4.1.2 Example 2 – Observer-Based Decentralized Control Numerical Example

The following system appears in [7]:

$$A=\begin{bmatrix}-3&0&-0.6&1.5&-0.30\\-0.3&-6&0&0.6&1.5\\-1.2&1.5&-9&0.3&-3\\-2.25&-0.6&-2.4&2&0\\-0.6&1.5&-1.5&1.5&3.75\end{bmatrix},B_1=\begin{bmatrix}1\\0\\0.5\\1\\-1\end{bmatrix},$$

$$B_2=\begin{bmatrix}0.2\\-0.1\\1\\-2\\0.3\end{bmatrix},C_1=\begin{bmatrix}1&0.2&-0.3&1&2\\1&0&0&0&-0.5\end{bmatrix},$$

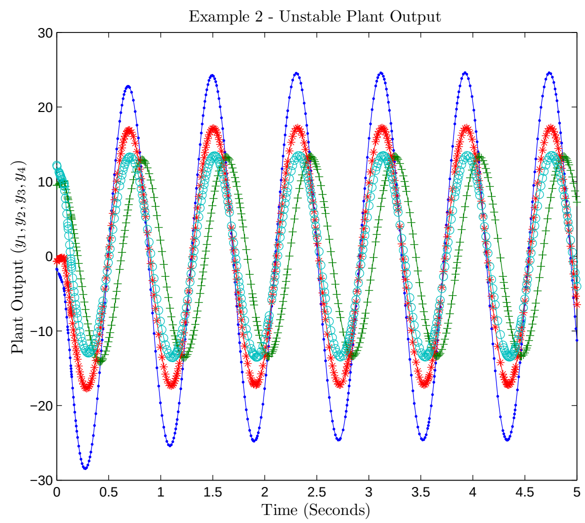

Fig. 9. Unstable Behavior of the System in Example 2 (for $\tau_m > 0.035324$ sec).

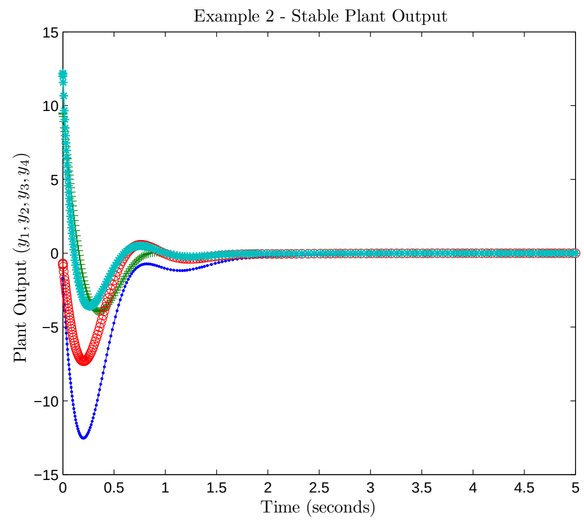

Fig. 10. Stable Behavior of the System in Example 2 (conservative bound, $\tau_m = 5.2753e^{-9}$ sec).

$$C_2 = \begin{bmatrix} 0.5 & 0 & 0.1 & 0.7 & 0.9 \\ 0.6 & 0.4 & -0.4 & 0.5 & 0 \end{bmatrix}.$$

It is also an unstable system with two controllers. After computing the design parameters for the observer-based controller as in [7] (the global state feedback control gain matrix F is computed using the Continuous Algebraic Riccati Equation), we get the following decentralized control laws:

$$u_1 = -\begin{bmatrix} -1.36 & -1.36 & 5.15 \end{bmatrix} z_1 - \begin{bmatrix} -2.81 & 4.72 \end{bmatrix} \hat{y}_1,$$

$$u_2 = -\begin{bmatrix} 9.19 & 9.19 & -24.27 \end{bmatrix} z_2 - \begin{bmatrix} -0.96 & -5.67 \end{bmatrix} \hat{y}_2.$$

From the simulation results we note that the system becomes unstable for $\tau_m > 0.035324$ sec, as shown in Figure 9. Computing the bound of MATI by considering the Theorem 2, we find that $\tau_m = 5.2753e^{-9}$ sec, which is extremely conservative to guarantee the stable behavior of the DNCS as shown in Figure 9. Nonetheless, when we use (32), we get $\tau_m = 0.0399$ sec, which is very close to the above bound of stability ($\tau_m < 0.0353247$ sec).

From Theorem 1, the sufficiency condition of stability is $\|\Delta S_c\| \leq \frac{1}{\lambda_{max}(P)}$. From the simulation results, $\|\Delta S_c\| = 99.3109$ and $\frac{1}{\lambda_{max}(P)} = 0.4216$. Again, we can see that the system is stable even with larger value of the norm of the perturbation which means that the perturbation bound of Theorem 1 is conservative as a sufficiency condition for stability.

In Theorem 2, the sufficiency condition of stability is $\gamma < \frac{1}{2\lambda_2}$. From the simulation results, $\gamma = 5.1640e^{-6}$ and $\frac{1}{2\lambda_2} = 0.2108$. This example also shows that the perturbation bound of Theorem 2 is satisfied, which is because the fact that in Theorem 2 the MATI bound that we used is very conservative as we mentioned before ($\tau_m < 5.2753e^{-9}$ sec), and that can be seen in Figure 10.

4.1.3 Example 3 – Luenberger Combined Observer-Controller Numerical Example

The following system is numerical example that considers second controller design implementation for the DNCS. The state-space matrices are given by:

$$A = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}, B_1 = \begin{bmatrix} -1 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$\text{and } C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

The plant is initially unstable with two controllers, while the non-networked controlled system is stable. After computing the design parameters for the Luenberger controller, we get the following decentralized control laws ($\hat{x}$ is the estimated state here, since we're using the Luenberger observer):

$$u_1 = -\begin{bmatrix} -2.62 & 0.16 \end{bmatrix} \hat{x}_1, \& u_2 = -\begin{bmatrix} -0.162 & 4.57 \end{bmatrix} \hat{x}_2.$$

From the simulation results, we note that the system becomes unstable for $\tau_m > 0.1074$ sec as shown in the lower subfigure in Figure 11. Computing the bound of MATI by applying Theorem 2, we find that $\tau_m = 4.1660e^{-8}$ sec, which is a very conservative bound to guarantee the stable behavior of the DNCS as shown in Figure 11. Nonetheless, when we use (32), we get $\tau_m = 0.2158$ sec. From Theorem 1, the sufficiency condition of stability is $\|\Delta S_c\| \leq \frac{1}{\lambda_{max}(P)}$. From the

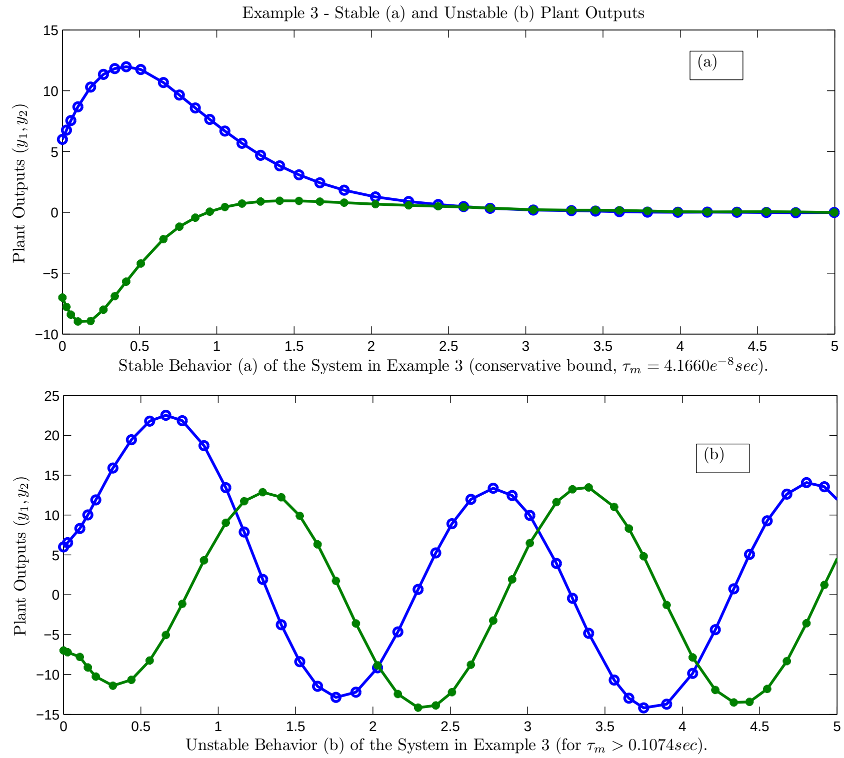

Fig. 11. Example 3 Stability Analysis.

simulation results, $\|\Delta S_c\| = 48.6996$ and $\frac{1}{\lambda_{max}(P)} = 0.2270$. Again, we can see that the system is stable even with larger value of the norm of the perturbation which means that the perturbation bound of Theorem 1 is conservative as a sufficiency condition for stability (same as in the first two examples).

In Theorem 2, the sufficiency condition of stability is $\gamma < \frac{1}{2\lambda_2}$. From the simulation results, $\gamma = 2.3724e^{-6}$ and $\frac{1}{2\lambda_2} = 0.1135$. This example also shows that the perturbation bound of Theorem 2 is satisfied, which is due to the fact that in Theorem 2 the MATI bound that we used is very conservative as we mentioned before ($\tau_m < 4.1660e^{-8}$ sec), and that can be seen in Figure 11.

5 Conclusions

This paper introduces a general framework that converts a generic decentralized control configuration of non-networked systems to the general setup of a Networked Control System. Two design methods from the literature of decentralized control for non-networked systems were chosen as a base for the design of a controller for the networked systems, the first being an observer-based decentralized control, while the second is the well-known Luenberger combined observer-controller design. The main idea of our design is to formulate the DNCS in the general form and then map the resulting system to the general form of the NCS. The network effect has been treated as a perturbation. Two methods to analyze the stability of the DNCS system are introduced and analyzed for the two control designs. Perturbation bounds for stability of the DNCS systems have been derived. The maximum allowable transfer interval (MATI) is computed based on two different methods in the literature. The simulation results showed that if we used a conservative method to compute MATI, we get a less conservative results for the perturbation bound and vise versa. In the future, the results of this paper can be used to analyze the effect of the scheduling protocol on the MATI which is a critical factor in analyzing the stability of DNCSs.

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