PHYS 233 Recitation:

How long are your capillaries?

Each student in the group should write their name at the top of the whiteboard. Your group needs to complete all five parts to get full credit.

1. In the Recitation Room you will see a garden hose that is 25 feet long. A section from an identical hose is on your table. There is also a meter stick. Your first question: **How much water can the whole hose hold?**

A. Estimate this volume in liters (1 liter = 1000 cm^3). Feel free to search for any conversion factors or formulas you might need.
B. Also, estimate the uncertainty in your predicted volume. Identify the main sources of uncertainty.

Show your calculations on your whiteboard. Consider using a format like this:

1. Volume = xxxx Liters	Uncertainty +/- yyyy Liters	Main source of uncertainty is….

2. A typical human body is said to contain 5 liters of blood. **How long would this hose have to be to contain all 5 liters?**

Someday nanomedicine may develop nanorobots that can be programmed to navigate through all of the many parts of the circulatory system. Today we will consider the general question, what is the total length of the circulatory system? That is, if you strung together in a single line every artery, arteriole, capillary, vein, etc., how long would this be? We will tackle this problem like physicists: First create a highly **simplified model**, then apply some **general principles** to the model.

Simplified model: Just consider the capillaries, since they probably contribute the most to the length.

General principle: Just like the garden hose above, if you know the volume of fluid and you know the radius of the tube, you can solve for the length.

---

3. Make a reasonable estimate for how much blood resides in capillaries at any time. This will be some number greater than zero and less than 5 liters. Write down your number. Define the quantity α (alpha) to be the ratio of the amount of blood in the capillaries to the 5 liter total. **What is your alpha?**

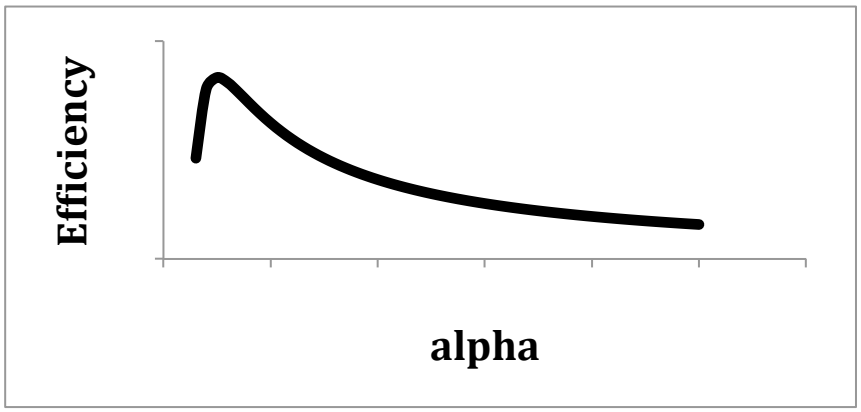
Also, give some justification, some reason why you did not choose a much larger or much smaller number.

4. Assume capillaries have an average diameter of 8 microns (radius = 4 microns =$4x10^{-6}$ m = $4x10^{-4}$ cm). **How long would this one capillary have to be to contain the volume of blood you chose in part 3?** Compare your answer to some known physical length, such as the length of a football field, the distance from here to Chicago, etc.

5. Some future doctor of nanomedicine derives a model for the optimal value of alpha, the fraction of the total amount of blood that should reside in capillaries, based on the idea that either too many or too few capillaries would be inefficient for delivering oxygen to all the cells in the body. This model produces a formula for the efficiency as a function of alpha:

$$\text{Efficiency } E = 0.2\alpha^{-1} - 0.01\alpha^{-2}$$

A generic plot of this function is shown below. Note that the efficiency is a maximum for some value of alpha.



Using the formula above, **determine the value of alpha that gives the greatest efficiency**.

There are at least two ways to do this. First, remember your calculus, which says that the slope of a function is zero where it is a maximum. You can take the derivative of this function with respect to alpha, set the result equal to zero, then solve for alpha.

Alternatively, you can put this function in Excel, plot it for values of alpha between zero and one, and by inspection find the value of alpha that gives the largest efficiency.