

Propelling a Paramecium: Recitation

Unicellular organisms such as bacteria and protists are small objects that often live in dense fluids (water, not air). Making sense of the movement of small things through water seems different from the movement of everyday objects in air (e.g. balls, rocks, cars on ramps).

Consider the following conversation between two biology students:

Liz: I work in a microbiology lab, and in our experiments we are often manipulating the motion of objects suspended in a fluid. In such a fluid, to get objects to move at a constant speed, we have to apply a force that is proportional to the *velocity* of the object.

Jack: That can't be right. In physics, I learned that force is equal to the mass of the object times the *acceleration* of the object.

- a. In your group, first make sure you understand each perspective. What is Liz saying about the relationship between speed and force? What is Jack saying?

In the remainder of this problem we'll use the example of a paramecium swimming through water to further understand whether the perspectives of these two students are consistent or contradictory.

Paramecia move by pushing their cilia (little hairs on their surface) through the fluid. The fluid (of course) pushes back on them by Newton's third law. We will call this force of the fluid on the cilia of the paramecium "the applied force", F^{app} (since it wouldn't happen if the paramecium didn't try to move its cilia). This is the force that moves the paramecium forward. The paramecium also feels a viscous force, whose magnitude has the form $F^{viscous} = \beta Rv$, proportional to the velocity and the effective radius of the object and in the opposite direction. (The "effective radius" of an object depends somewhat on its shape and its size, but for an object that is not too stretched out, we can take it to be some average radius of the object.)



- b. Draw a diagram that shows the forces acting on the paramecium as it moves.
- c. If the paramecium is moving at a constant velocity v_T , what is the applied force? Write an equation that could give a value for F^{app} . How will this value depend on the shape and size of the paramecium (i.e. its mass (m) and effective radius (R))?
- d. Solve the equation that you found in (c) for v_T . How does v_T depend on the applied force? How would v_T change if the paramecium pushed harder on the fluid?
- e. It's not very realistic for the paramecium to start at constant velocity. Suppose the paramecium is starting from rest and starts to move, and comes fairly quickly to its constant velocity, v_T . Sketch graphs of x , v , a , F^{net} , F^{app} , and $F^{viscous}$, and explain what is happening to each variable and why. You can assume that the applied force is constant, meaning that the paramecium is pushing on the fluid with a constant force. (You might find it helpful to consider what would happen if the paramecium applied a different force, and draw more than one curve on your graph.)
- f. Now, return to the original question. Are the perspectives of Liz and Jack contradictory? Consistent? If they are consistent, explain how they can be reconciled. If they are contradictory, explain the inconsistencies. (Regardless of the position you take, please address both the arguments of Liz and Jack.)