• Distribution of elevations: two preferred elevations => fundamental difference between ocean and continents
• Continents ~ granite (2.67), oceans ~ basalts (3.3) => idea that Earth’s elevations are supported hydrostatically
Isostasy

- Problem: In spite of the additional terrain volume, mountains are associated with negative Bouguer anomalies…
- **Airy** (1854): Mountains have a crustal root that compensates for the relief
- **Pratt** (1855): Density varies laterally (e.g. lateral variations of temperature or composition)
- In both models, mountains “float” on denser mantle in equilibrium = isostatic equilibrium, or **isostasy**
- Isostasy condition: the weight of columns of rock, at some depth called the depth of compensation, is everywhere equal.
Isostasy

- Isostasy condition: weight of columns of rock at depth of compensation is everywhere equal.

- Airy model:
  - $\rho_c (h_c + h_r + h_m) = \rho_c h_c + \rho_m h_r$
  - $h_r = h_m \left( \frac{\rho_c}{\rho_m - \rho_c} \right)$
  - Since $\rho_m > \rho_c \Rightarrow$ root thicker than mountain elevation

- Pratt model:
  - $\rho_c (h_c + h_m) = \rho_o h_c$
  - $h_m = h_c \left( \frac{\rho_o - \rho_c}{\rho_c} \right)$
  - Assume an homogeneous plate of density $\rho$: $\rho_o = \rho_c \Rightarrow h_m = 0$
  - If $\rho$ decreases locally (e.g. heating from below), $h_m$ increases $\Rightarrow$ positive topography
Isostasy

http://atlas.geo.cornell.edu/education/student/isostasy.html
Think about this…

• Why are mountains high?
• Why do mountains have roots?
• How to produce a root?
• What are the effects of erosion?
• Are mountain roots permanent features?
• How to produce a positive topography in isostatic equilibrium assuming an Airy model? A Pratt model?
• What could disturb isostatic equilibrium?
• Can you think of another kind of equilibrium (besides isostatic equilibrium) that would be called “dynamic equilibrium”? Describe the processes that dynamic equilibrium may involve.
Isostasy and buoyancy forces

Initial state, isostatic equilibrium
Weight of column to compensation depth = \( \rho_l g h_{lith} + \rho_a g h_{root} \)

A load is applied (= action)
⇒ A restoring force develops (= reaction)
⇒ It is driven by density contrasts = buoyancy force

Isostatic equilibrium is reached again when:
Acting force = Restoring force
Weight of column to compensation depth = \( \rho_l g (h_{lith} + h_{load}) \)

Buoyancy force = (weight of column after) - (weight of column before)
\[
= \rho_l g (h_{lith} + h_{load}) - \rho_l g h_{lith} - \rho_a g h_{root} \\
= (\rho_l h_{load} - \rho_a h_{root}) g
\]
Isostasy and buoyancy forces

- Buoyancy force: vertical, opposes load
- In reality, buoyancy force results in additional horizontal force:
  - Load (Fl) acts downward on lithospheric column
  - Restoring buoyancy (Fb) acts upward on lithospheric column
  - As a result, lithospheric column experiences vertical compression (green arrows)
  - Which is associated with horizontal extensional (blue arrows)
- As a result, state of horizontal stress:
  - Extensional at the center of the elevated region => drives “gravitational collapse”
  - Compressional at its edges => drives shortening
Buoyancy forces in action?

- Alps
- Tibet
Isostasy and flexure

Local compensation (Airy-type here):

- Isostasy = crust is in a static equilibrium
  - Airy model => topographic loading is compensated by buoyancy forces acting on the surface of equilibrium, resulting from lateral variations in crustal thickness
  - Pratt model => topographic loading is compensated by buoyancy forces that are produced due to lateral density variations within the crust

Regional compensation:

- In reality, the weight of a load is also resisted by the strength of the lithosphere.
- If the load is large enough compared to that strength, the (elastic) lithosphere bends downwards = flexure
Flexure

- Earth’s lithosphere can be approximated as a thin elastic plate:

\[ w(x) = \text{deflection} \]
\[ x = \text{distance} \]
\[ q(x) = \text{vertical force per unit length (load)} \]
\[ F = \text{constant horizontal force per unit length} \]
Flexure

- Causes for flexure of oceanic lithosphere:
  - Seamount loading
  - Oceanic plateau loading
  - Sediment loading
  - Plate bending entering a subduction zone

- Causes for loading of continental lithosphere:
  - Sediment loading
  - Icesheets
  - Thrust sheets
**Flexure**

- Bending of elastic plate as a function of distance $x$ is given by:

$$q(x) = D \frac{d^4w}{dx^4} + F \frac{d^2w}{dx^2} + R$$

- $D$ is the flexural rigidity of the plate, defined by:

$$D = \frac{Eh^3}{12(1-\sigma^2)}$$

($= \text{force couple required to bend a rigid structure to a unit curvature}$)

- Example values for $T_e$ and $D$:
  - Appalachians: $T_e = 105 \text{ km}$, $D = 10600 \times 10^{21} \text{ Nm}$
  - Appenines: $T_e = 11.5 \text{ km}$, $D = 14 \times 10^{21} \text{ Nm}$
Flexure

- Deformation of oceanic lithosphere under vertical load => depression + water fills depression => isostatic equilibrium perturbed: restring buoyancy force?

\[
\begin{align*}
\text{Weight per unit area of column:} & \quad \rho_w gh_w + \rho_m g(h + w) \\
\text{Weight per unit area of column:} & \quad \rho_w g(h_w + w) + \rho_m gh
\end{align*}
\]

- Net hydrostatic force is the difference = weight after - weight before:

\[(\rho_m - \rho_w)gw\]
Flexure

- Further assumptions:
  - No horizontal force => $F = 0$
  - Line load:
    - At $x=0$, load = $q_o$
    - At $x \neq 0$, load = 0
- For $x \neq 0$, the flexure equation becomes:
  \[ D \frac{d^4w}{dx^4} + (\rho_m - \rho_w)gw = 0 \]

- With a solution for $x>0$:
  \[ w = \frac{q_o \alpha^3}{8D} e^{-\frac{x}{\alpha}}(\cos \frac{x}{\alpha} + \sin \frac{x}{\alpha}) \]

- Important parameters and length scales in this solution:
  - $\alpha$ = flexural parameter
  - $2\pi\alpha$ = flexural wavelength
  - $x_o = 3\pi\alpha / 4$ = distance to the first zero crossing.

\[ \alpha = \left[ \frac{4D}{(\rho_m - \rho_w)g} \right]^{\frac{1}{4}} \]
Flexure, infinite plate, line load

\[ w = \frac{q_o \alpha^3}{8D} e^{-\frac{x}{\alpha}} (\cos \frac{x}{\alpha} + \sin \frac{x}{\alpha}) \]

- Zero crossing
- Flexural forebulge

**Figure 3-29** Deflection of the elastic lithosphere under a line load.

**Figure 3-30** Half of the theoretical deflection profile for a floating elastic plate supporting a line load.

e.g., oceanic island chain
Plate flexure under line load: e.g., Hawai'i

Bathymetry and free-air gravity anomaly along a N-S line centered on the Hawaiian island of Ohau => Combination of 3 effects:

1. Topography => short wavelengths gravity signal
2. Weight of the volcano => flexure of the plate => long-wavelength negative anomaly
3. Uprising asthenospheric plume => very long-wavelength positive anomaly
Plate flexure under line load: e.g., Hawai

After removal of the very long wavelength (mantle plume) signal:

(a) Best-fitting flexural models using conventional, two-dimensional techniques. The dotted curve is the response of a 25-km-thick elastic plate while the solid curve is the response of a variable thickness plate where the thickness ranges from 35 km away from the load to 25 km beneath the load.

(b) Gravity predictions based on the models in Figure 6a. Within the uncertainties of the data, both models provide reasonable fits, although the constant thickness model fits slightly better

(Wessel et al., 1993).
Flexure, 1/2 plate, line load

- Case of a 1/2 plate under a line load:
  Same principle, but slightly different solution:

\[ w = \frac{q_o \alpha^3}{4D} e^{-x/\alpha} \cos \frac{x}{\alpha} \]

- Examples: subduction, fold and thrust belt

Figure 3-31 Deflection of a broken elastic lithosphere under a line load.

Figure 3-32 The deflection of the elastic lithosphere under an end load.
Flexure - subduction

- In addition to load of overriding plate:
  - Sediments
  - Non-elastic response
Flexure ⇔ Isostasy

• Recall that: \[ D \frac{d^4w}{dx^4} + H \frac{d^2w}{dx^2} + (\rho_m - \rho_c)gw = V(x) \]

• Let’s assume that:
  – Plate is very thin, or has ~zero strength or zero flexural rigidity => D = 0
  – No horizontal forces acting => H = 0
  – Load is due to topography h(x) => V(x) = \rho_c gh(x)

• Then the flexure equation reduces to:

\[ (\rho_m - \rho_c)gw = \rho_c gh(x) \]

\[ w = \frac{\rho_c h(x)}{(\rho_m - \rho_c)} \]

• Which is … Airy isostasy! (w here is hr in our Airy equation)

• Airy isostasy is a special case of flexure when D\rightarrow0
A special case of flexure and isostasy...

11,000 years ago, large parts of N. Europe and N. America were covered by ice sheets up to 3 km thick.

Ice sheets melted rapidly ~10,000 years ago as a result of global climate change.
Glacial Isostatic Adjustment

• Ice sheets act as a load, causing:
  – Downward flexure of the elastic lithosphere
  – Outward flow in the mantle

• As ice sheets melt, the removal of the load results in:
  – Upward flexure ( = “rebound”) of the (elastic) lithosphere
  – Inward flow in the mantle

• Glacial Isostatic Adjustment (GIA) =
  – Elastic response of the lithosphere (instantaneous)
  +
  – Viscous response of the mantle (time delayed)

• GIA is still active today…
• Morphological and gravity observations in Scandinavia:
  – Total uplift ~ 275 m
  – Current uplift: up to ~ 1 cm/yr
  – Negative Bouguer anomaly (mass deficit because the lithosphere is still rising)
In North America…
GIA is happening today…

- GIA tells us about the viscosity of the mantle…

$\chi^2$ misfit per degree of freedom between GPS-derived crustal velocities as a function of $v_{um}$ (ordinate scale) and $v_{lm}$ (abscissa scale) for the (A) radial, (B) horizontal, and (C) full 3D velocity components, respectively. The lithospheric thickness of the Earth models was fixed to 120 km.

(Milne et al., 2001)
A link with sea-level change

Rates of sea level change determined from tide gauge records at 20 sites in Fennoscandia, corrected for regional geoid variations due to GIA versus GPS-determined radial velocities at the same (or nearby) sites.

The lower diagonal line is the result for a zero sea-level change.

The upper diagonal line is the best estimate through the data, and it yields 2.1±0.3 mm/year of regionally coherent sea surface rise.

(Milne et al., 2001)
What have we learned?

- **Isostasy:**
  - State of hydrostatic equilibrium where the weight of columns of rock, at some depth called the depth of compensation, is everywhere equal.
  - Isostasy can be achieved by:
    - Varying crustal density laterally (Pratt model)
    - Varying crustal thickness laterally (Airy model)

- **Load applied to (elastic) lithosphere (= infinite or 1/2 infinite plate):**
  - Resisted by buoyancy forces (cf. isostasy) + strength of plate (flexural rigidity)
  - Results in depression + peripheral uplift (= flexural bulge)
  - Amplitude and wavelength depends on strength of plate and density contrast

- **Special case of fast removal of a load = glacial isostatic adjustment:**
  - Combination of elastic response + non elastic (=> time dependent) response of the mantle
  - Results in time-delayed rebound, still active today