Seismology is about earthquakes
Earthquakes and rupture

- Focal mechanism
- Source processes
Earthquakes and ground shaking

- Earthquakes => ground shaking
- Earthquake engineering
- Seismic hazard and seismic risk

Taiwan, 1999

Colombia
Earthquakes and waves

- Seismic waves
- Other sources: volcanoes, explosions
- Seismometers
- Propagation

Snapshots of simulated wave propagation in the LA area for the hypothetical SAF earthquake (K. Olsen, UCSB)

The first earthquake recorded on a seismometer!

Typical recording of an earthquake by a modern 3-component seismometer
Seismology is about imaging the Earth

- Tomography
- Reflection - refraction
- Earth’s structure

Seismic reflection profile across the Nankai accretionary prism (Japan)

Anomalies of seismic wave velocities in SE Asia
Stress and strain

- When a force is applied to a material, it deforms: stress induces strain
  - Stress = force per unit area
  - Strain = change in dimension
- For some materials, displacement is reversible = elastic materials
  - Experiments show that displacement is:
    - Proportional to the force and dimension of the solid
    - Inversely proportional to the cross-section
  - One can write: $\Delta h \propto F h/A$
  - Or $\Delta h/h \propto F/A$
  - **Strain is proportional to stress = Hooke’s law**
  - Hooke’s law: good approximation for many Earth’s materials when $\Delta h$ is small

Reference: read Appendix 2, Fowler, and Stein, 2.3.2 through 2.4.5m p.39-62
Stress and strain

• Stress-strain relation:
  – Elastic domain:
    • Stress-strain relation is linear
    • Hooke’s law applies
  – Beyond elastic domain:
    • Initial shape not recovered when stress is removed
    • Plastic deformation
    • Eventually stress > strength of material => failure
      – Failure can occur within the elastic domain = brittle behavior
  • Strain as a function of time under stress:
    – Elastic = no permanent strain
    – Plastic = permanent strain
  • Our goal: find the relation between stress and strain
• Let’s assume:
  – A rectangular prism with 3 sides defining (O,x,y,z)
  – A uniform tension \( N_z \) exerted on 2 sides perpendicular to (O,z)

• When the prism is stretched along (O,z):
  – Change in length is proportional to tension: \( \varepsilon_z = \frac{\Delta h}{h} \propto N_z \)
  – One can show experimentally that:
    \[
    \varepsilon_z = \frac{\Delta h}{h} = \frac{1}{E} N_z
    \]
    – \( E = \text{Young’s modulus} \)
    – Units of stress = km/m\(^2\)
    – Small \( E \) => more elastic

• If the prism is stretched along (O,z), it must shrink along (O,x,y) to conserve mass:
  – One can show experimentally that contraction:
    \[
    \varepsilon_x = \varepsilon_y = -\frac{\nu}{E} N_z
    \]
    – \( \nu = \text{Poisson’s ratio} \) (dimensionless)
Poisson’s ratio

\[ \varepsilon_z = \frac{\Delta h}{h} = \frac{1}{E} N_z \]
\[ \varepsilon_x = \varepsilon_y = -\frac{\nu}{E} N_z \]
\[ \Rightarrow \nu = -\frac{\varepsilon_x}{\varepsilon_z} \]

- **Poisson's ratio** = ratio of transverse to longitudinal normal strain under uniaxial stress, in the direction of stretching force, with:
  - Tensile deformation positive
  - Compressive deformation negative

- All common materials become narrower in cross section when they are stretched \( \Rightarrow \) Poisson’s ratio positive.
Poisson’s ratio

Examples:

- Perfectly incompressible material $\Rightarrow \nu = 0.5$ (no volume change)
- If $\nu > 0.5 \Rightarrow$ volume increase under compression = *dilatant*.
- If $\nu < 0 \Rightarrow$ become thicker when stretched = *auxetic* materials (some polymer foams)
- Rubber: $\nu = 0.5$, Cork: $\nu = 0$ (why is cork used to close glass bottles?)
- Earth’s interior $\nu = 0.24$-$0.27$
- Granite: $\nu = 0.2$-$0.3$
- Carbonate rocks $\nu \sim 0.3$
- Sandstones $\nu \sim 0.2$
- Shale $\nu > 0.3$
- Coal $\nu \sim 0.4$. 
Poisson’s ratio

\[ \nu = \frac{(V_p^2 - 2V_s^2)}{2(V_p^2 - V_s^2)} \]

- If \( V_s = 0 \), \( \nu = 0.5 \):
  - Either a fluid (shear waves do not propagate through fluids)
  - Or material that maintains constant volume regardless of stress = incompressible.
- \( V_s \sim 0 \) is characteristic of a gas reservoir.

A negative Poisson’s ratio change is associated with the top of a gas zone,
Elasticity: Hooke’s law

• The equations derived above can be generalized to the whole rectangular prism, with tractions $N_x$, $N_y$, and $N_z$ applied on its 3 sides (no shear stresses):

\[
\varepsilon_x = \frac{1}{E} \left( N_x - \nu N_y - \nu N_z \right) \\
\varepsilon_y = \frac{1}{E} \left( -\nu N_x + N_y - \nu N_z \right) \\
\varepsilon_z = \frac{1}{E} \left( -\nu N_x - \nu N_y + N_z \right)
\]

• Hooke’s law:
  – Relates stress and strain in elastic materials.
  – The rheology of elastic material is fully described by $E$ and $\nu$.
  – Can be verified experimentally ($E$ and $\nu$ can be measured).
  – Is valid for small deformations only.
Elasticity: deformation by traction

• Assume a uniform plate perpendicular to (O,x), with infinite dimensions along (O,y) and (O,z):
  - Apply a traction $N_x$ parallel to (O,x)
  - No deformation along (O,y) and (O,z), therefore $\varepsilon_y = \varepsilon_z = 0$

• Hooke’s law:

$$
\varepsilon_x = \frac{1}{E} \left( N_x - \nu N_y - \nu N_z \right) \\
\varepsilon_y = \frac{1}{E} \left( -\nu N_x + N_y - \nu N_z \right) \\
\varepsilon_z = \frac{1}{E} \left( -\nu N_x - \nu N_y + N_z \right)
$$

$$
\Rightarrow N_y = N_z = \frac{\nu}{1-\nu} N_x
$$

$$
\varepsilon_x = \frac{1}{E} \left( \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \right) N_x = \frac{N_x}{\alpha}
$$

with $\alpha = E \frac{(1-\nu)}{(1+\nu)(1-2\nu)}$

• $\alpha = \text{modulus of extension}$
Elasticity: deformation by shear

- Assume a uniform plate perpendicular to (O,x), with infinite dimensions along (O,y) and (O,z)
  - Apply a traction \( T_y \) parallel to (O,y) on each side of the plate (= shear stress)
  - The thickness \( l \) of the plate remains constant
- The 2 sides of the plate are sliding w.r.t. each other by the amount \( \Delta l \), angle \( \phi \).

- Shear strain: \( \gamma = \frac{\Delta l}{l} \)
- Ratio of shear stress to shear strain: \( \mu = \frac{T_y}{\gamma} \)
  
  \( \mu = \text{shear modulus} \) or modulus of rigidity

- Using Hooke’s law, one can show that: \( \mu = \frac{E}{2(1 + \nu)} \)
Elastic moduli

• Also called elastic constants (for a given material under given P,T)
• Define the properties of material under elastic strain
• Elastic moduli:
  – Poisson's ratio: $\nu$ (describes lateral deformation under longitudinal load)
  – Young's modulus: $E$ (relation between tensile or compressive strain and stress)
  – Bulk modulus: $K$ (change in volume under hydrostatic pressure)
  – Lame constants:
    • Shear (= rigidity) modulus: $\mu$ (relation between torque and shear strain)
    • $\lambda$
  – Modulus of extension = $\alpha$

Box 2.3-1 Relations between moduli

\[
\begin{align*}
\nu &= \frac{\lambda}{2(\lambda + \mu)} = \frac{\lambda}{3K - \lambda} = \frac{E}{2\mu} = \frac{3K - 2\mu}{2(3K + \mu)} = \frac{3K - E}{6K} \\
E &= \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu} = \frac{\lambda(1 + \nu)(1 - 2\nu)}{\nu} = \frac{9K(K - \lambda)}{3\lambda} = 2\mu(1 + \nu) \\
&= \frac{9K\mu}{3K + \mu} = 3K(1 - 2\nu) \\
K &= \lambda + \frac{2}{3}\mu = \frac{\lambda(1 + \nu)}{3\nu} = \frac{2\mu(1 + \nu)}{3(1 - 2\nu)} = \frac{\mu E}{3(3\mu - E)} = \frac{E}{3(1 - 2\nu)} \\
\lambda &= \frac{2\mu\nu}{1 - 2\nu} = \frac{\mu(E - 2\mu)}{3\mu - E} = K - \frac{2}{3}\mu = \frac{E\nu}{(1 + \nu)(1 - 2\nu)} \\
&= \frac{3K\nu}{1 + \nu} = \frac{3K(3K - E)}{9K - E} \\
\mu &= \frac{\lambda(1 - 2\nu)}{2\nu} = \frac{3}{2}(K - \lambda) = \frac{E}{2(1 + \nu)} = \frac{3K(1 - 2\nu)}{2(1 + \nu)} = \frac{3KE}{9K - E}.
\end{align*}
\]

\[a = \lambda + 2\mu\]
Putting it all together: the wave equation

Case of a longitudinal (=compressional) plane wave

Let’s assume an infinite homogeneous body.

A tension \( N_x \) is applied (parallel to \((O,x)\)), resulting in a displacement \( u \) along the \((O,x)\) axis.

Let’s zoom on a very thin plate within this body, between \( x \) and \( x+dx \), parallel to \((O,x)\):

- At \( x \):
  - The displacement due to \( N_x \) is \( u \)

- At \( x+dx \):
  - The displacement is \( u + \left( \frac{\partial u}{\partial x} \right) dx \)
  - The tension is: \( N_x + \frac{\partial N_x}{\partial x} dx \)
  - The plate is stretched by: \( \delta x = \frac{\partial u}{\partial x} \)
  - The relation between strain and stress (see above) gives: \( N_x = \alpha \frac{\partial u}{\partial x} \)

Therefore the tension at \( x+dx \) can be written: \( N_x + \alpha \frac{\partial^2 u}{\partial x^2} dx \) (= restoring force)
Putting it all together: the wave equation

Case of a longitudinal (= compressional) plane wave

Let’s apply the fundamental equation of dynamics: \( \Sigma F = ma \)

- The forces exerted on that very thin plate are:
  - The tension at \( x : N_x \)
  - The \textit{restoring} tension at \( x+dx : N_x + \alpha \frac{\partial^2 u}{\partial x^2} dx \)

- We neglect the gravitational attraction

- The mass of the plate is density \((\rho) \times \text{volume}\)

- Therefore:

\[
-N_x + N_x + \alpha \frac{\partial^2 u}{\partial x^2} dx = ma
\]

\[
\Rightarrow \alpha \frac{\partial^2 u}{\partial x^2} dx = (\rho dx) \frac{\partial^2 u}{\partial t^2}
\]

\[
\Rightarrow \frac{\partial^2 u}{\partial x^2} - \frac{1}{V_p^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad \text{with} \quad V_p = \sqrt{\frac{\alpha}{\rho}}
\]
Putting it all together: the wave equation

Case of a transversal (=shear) plane wave

• Using a similar reasoning, one can show that:

\[
\frac{\partial^2 v}{\partial x^2} - \frac{1}{V_s^2} \frac{\partial^2 v}{\partial t^2} = 0 \quad \text{with} \quad V_s = \sqrt{\frac{\mu}{\rho}}
\]

• \( v \) is the displacement in the transverse direction
Why waves?

- Elastic material ⇒ linear relation between stress and strain
- If stress is applied to an elastic body:
  - The body deforms
  - A **restoring force** appears because the body is elastic
  - Fundamental equation of dynamics ⇒ the applied force plus the restoring force must equal mass x acceleration
- One can show that:
  - Under compressional stress:
    \[
    \frac{\partial^2 u}{\partial x^2} - \frac{1}{V_p^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad \text{with} \quad V_p = \sqrt{\frac{K + \frac{4}{3} \mu}{\rho}}
    \]
  - Under shear stress:
    \[
    \frac{\partial^2 v}{\partial x^2} - \frac{1}{V_s^2} \frac{\partial^2 v}{\partial t^2} = 0 \quad \text{with} \quad V_s = \sqrt{\frac{\mu}{\rho}}
    \]

\( v \) = displacement in the transverse direction
\( u \) = displacement in the longitudinal direction
\( V_p \) and \( V_s \) = propagation velocities
\( K \) = bulk modulus
\( \rho \) = density of the body
\( \mu \) = shear modulus
Why waves?

The solution to the wave equation is:

\[
\frac{\partial^2 u}{\partial x^2} - \frac{1}{V^2} \frac{\partial^2 u}{\partial t^2} = 0
\]

is called the wave equation

The solution to the wave equation is:

\[
u = A \sin 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right)\]

\textit{sine} ⇒ \textbf{it’s a wave!} = oscillation, with:

- \( u \) = displacement
- \( x \) = distance along x-axis, \( t \) = time
- \( A \) = amplitude
- \( (x/\lambda + t/T) \) = phase
- \( T \) = period of the wave
- \( \lambda \) = wavelength of the wave

Velocity = distance traveled in 1 second, therefore: \textit{velocity} = \textit{frequency} \times \textit{wavelength}

\[
V = f \times \lambda \quad \Leftrightarrow \quad V = \frac{\lambda}{T}
\]
Wave equations, so what?

\[ \frac{\partial^2 u}{\partial x^2} - \frac{1}{V_p^2} \frac{\partial^2 u}{\partial t^2} = 0 \quad \text{with} \quad V_p = \sqrt{\frac{K + \frac{4}{3} \mu}{\rho}} \]

**Compressional waves:** particles move along y axis

\[ \frac{\partial^2 v}{\partial x^2} - \frac{1}{V_S^2} \frac{\partial^2 v}{\partial t^2} = 0 \quad \text{with} \quad V_S = \sqrt{\frac{\mu}{\rho}} \]

**Shear waves:** particles move along z axis
Wave equations, so what?

\[ V_P = \sqrt{\frac{K + \frac{4}{3} \mu}{\rho}} \quad V_S = \sqrt{\frac{\mu}{\rho}} \]

- **Bulk modulus** \( K \) = measure of the isotropic stress needed to change the volume of a material (sometimes called "incompressibility")
- **Shear modulus** \( \mu \) = measure of the shear stress needed to change the shape of a material (sometimes called "rigidity")
- Compression:
  - Change of volume and shape
  - Velocity depends on \( K \) and \( \mu \)
- Shear:
  - No change in shape => velocity depends only on \( \mu \)
  - Shear stress needed to change the shape of a fluid = zero => \( V_S = 0 \) in fluids!
Wave equations, so what?

• For a perfect elastic solid:
  \[ \nu = \frac{1}{4} \quad \text{and} \quad K = \frac{5}{3} \mu \]
  \[ \Rightarrow \frac{V_P}{V_S} = \sqrt{K + \frac{4}{3} \mu} \quad \Rightarrow \frac{V_P}{V_S} = \sqrt{\frac{5}{3} \mu + \frac{4}{3} \mu} \]
  \[ \Rightarrow \frac{V_P}{V_S} = \sqrt{3} \]

• Therefore:
  - \( V_P > V_S \)
  - Compressional waves travel faster than shear waves (i.e. arrive first at the seismic station)
  - Compressional waves = Primary waves = P-waves
  - Shear waves = Secondary waves = S-waves
More on seismic waves

- Velocity of seismic waves depend on the material density
- WARNING: $K$ and $\mu$ also depend on density (they increase rapidly with density) ⇒ relationship between velocity and density is not obvious from the equations above…
- Birch’s law (1964): $V = a\rho + b$
- Nafe-Drake curve:
  - Velocity/density relation is not so simple
  - Igneous and metamorphic rocks > sedimentary rocks
More on seismic waves

- **Wavefront**: location of the front of the wave at a given time
  - If medium is homogeneous => velocity constant = circular pattern (spherical in 3D)
  - If medium not homogeneous => complex pattern

- **Raypaths**:
  - Perpendicular to the wavefronts
  - Indicate the direction of propagation of the wave
  - P-waves: particle motion along the ray path
  - S-wave: particle motion transverse to the ray path (SV or SH)
More on seismic waves

- The amplitude of seismic waves changes as they travel because of:
  - Geometry: as the wave propagates away from the source, the wavefront occupies a larger area ⇒ amplitude has to decrease to conserve energy
  - Attenuation:
    - If the rocks are not fully elastic, some amount of energy is lost during propagation (e.g., by frictional heating) ⇒ amplitude decreases.
    - Example: if a rock contains fluids, seismic waves are attenuated (oil, water, partial melting)

- The velocity of seismic waves is (usually) not the same in all directions = seismic anisotropy
  - Physical properties of minerals are different depending on their symmetry
  - If minerals are aligned (e.g. ductile flow in lower crust or mantle),
  - Example: oceanic ridges, anisotropy in the upper mantle perpendicular to the ridge direction
More on seismic waves

- P and S waves propagate inside a body = body waves
- If we add a particular boundary condition = free-surface (Earth’s surface) ⇒ surface waves:
  - Analogous to water waves and travel along the Earth's surface.
  - Travel more slowly than body waves
  - Low frequency, they are more likely than body waves to stimulate resonance in buildings, and are therefore the most destructive type of seismic wave.
- Two types of surface waves:
  - Rayleigh waves (ground roll, can be seen during eq as ground moves slowly up and down)
  - Love waves
Surface waves

- Love waves are dispersive = velocity depends on frequency
- Rayleigh waves not dispersive in homogeneous medium but: as frequency decrease they penetrate deeper => travel faster => dispersion
- As a result of dispersion:
  - Wave packets propagate at group velocity
  - Individual waves propagate as phase velocity
  - Seismic pulse is stretched out as it propagates away from the source
Surface waves

- In Earth (typically), phase velocity increases with wavelength ⇒ longer wavelengths propagate faster ⇒ long wavelengths arrive first (at large distances from seismic source)
Surface waves

- Surface wave dispersion can be used to probe the Earth
- Longer periods penetrate deeper => probe deeper levels:
  - 35 s => crust
  - 100 s => upper mantle
- (Compare to body wave tomography)
## More on seismic waves

<table>
<thead>
<tr>
<th>Type</th>
<th>Particle motion</th>
<th>Typical velocity</th>
<th>Other characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P</strong></td>
<td>Alternating compressions (&quot;pushes&quot;) and dilations (&quot;pulls&quot;) which are directed in the same direction as the wave is propagating (along the raypath); and therefore, perpendicular to the wavefront</td>
<td>( V_p \sim 5 - 7 \text{ km/s in typical Earth's crust; } ) ( \gg 8 \text{ km/s in Earth's mantle and core; } 1.5 \text{ km/s in water; } 0.3 \text{ km/s in air} )</td>
<td>P motion travels fastest in materials, so the P-wave is the first-arriving energy on a seismogram. Generally smaller and higher frequency than the S and Surface-waves. P waves in a liquid or gas are pressure waves, including sound waves.</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>Alternating transverse motions (perpendicular to the direction of propagation); commonly polarized such that particle motion is in vertical or horizontal planes</td>
<td>( V_s \sim 3 - 4 \text{ km/s in typical Earth's crust; } ) ( \gg 4.5 \text{ km/s in Earth's mantle; } \sim 2.5-3.0 \text{ km/s in (solid) inner core} )</td>
<td>S-waves do not travel through fluids, so do not exist in Earth's outer core (inferred to be primarily liquid iron) or in air or water or molten rock (magma). S waves travel slower than P waves in a solid and, therefore, arrive after the P wave.</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>Transverse horizontal motion, perpendicular to the direction of propagation and generally parallel to the Earth’s surface</td>
<td>( V_L \sim 2.0 - 4.5 \text{ km/s in the Earth depending on frequency of the propagating wave} )</td>
<td>Love waves exist because of the Earth’s surface. They are largest at the surface and decrease in amplitude with depth. Love waves are dispersive, that is, the wave velocity is dependent on frequency, with low frequencies normally propagating at higher velocity. Depth of penetration of the Love waves is also dependent on frequency, with lower frequencies penetrating to greater depth.</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>Motion is both in the direction of propagation and perpendicular (in a vertical plane), and “phased” so that the motion is generally elliptical – either prograde or retrograde</td>
<td>( V_R \sim 2.0 - 4.5 \text{ km/s in the Earth depending on frequency of the propagating wave} )</td>
<td>Rayleigh waves are also dispersive and the amplitudes generally decrease with depth in the Earth. Appearance and particle motion are similar to water waves.</td>
</tr>
</tbody>
</table>

http://www.eas.purdue.edu/~braile/edumod/slinky/slinky.htm
What have we learned?

• Solids deform when stress is applied
  – Strain is reversible in elastic materials
  – Assuming small deformations, Hooke’s law gives the relation between stress and strain in an elastic body
• Elastic materials are characterized by:
  – Young modulus $E$
  – Poisson’s ratio $\nu$
  – Shear modulus $\mu$
• When stress is applied to a piece of rock, a restoring force appears because of the elasticity of the body:
  – The wave equation is derived from the fact that the applied force plus the restoring force equal mass $\times$ acceleration.
  – The solution to the wave equation gives displacement as a function of position and time: displacement is oscillatory.
• There are:
  – Compressional and shear waves = body waves
  – Love and Rayleigh waves = surface waves
  – Compressional waves travel faster than shear waves
  – Propagation velocity is a function of density and elastic properties