Precision

- Time series = successive estimates of site position + formal errors
- First order analysis:
  - Fit a straight line using a least square adjustment and compute a standard deviation
  - Slope
  - Associated uncertainty (decreases as $1/\sqrt{\#\text{samples}}$)
  - Compute the weighted rms of the scatter to the weighted mean (or to a model) = "long term repeatability"

$$ wrms = \sqrt{\frac{N}{N-1} \sum_{i=1}^{N} \left( y_i - (a + b t_i) \right)^2} \sum_{i=1}^{N} \sigma_i^2 $$

- $y_i$ and $\sigma_i$ = position and associated formal error from the inversion
- $N$ = number of data points
- Comparison of repeatability and standard deviation:
  - Standard deviation is very low and decreases as $1/\sqrt{\#\text{samples}}$
  - Repeatability is a more realistic estimate of the true error but assumes that each estimate is independent = no temporal correlation
Noise models

- 150 km baseline across the San Jacinto active fault, 2.5 years of continuous GPS observations
- Linear trend $\Rightarrow$ fault slip rate: $16.9 \pm 0.6$ mm/yr
- Comparison with "simulated campaigns":
  - Difference between continuous up to 10 mm/yr…
  - 10 mm/yr $\gg$ uncertainties estimates
  - Long period fluctuations in the continuous time series
- What noise model?
Velocity uncertainties

- Uncertainty as a function of time:
  - Continuous GPS measurements in blue
  - Campaign measurements in red

- Conclusion:
  - 2 years at least to reach 1mm/yr precision
  - 4 time longer for campaign measurements

This does not account for temporal correlation between errors.
Noise models

- Langbein et al. (1993), Langbein and Johnson (JGR 1997)
- California (Parkfield, Long Valley, Pearblossom areas):
  - 2-color electronic distance meter measurements
  - Baseline length: 5-10 km
  - 15 years of observations
- Spectral analysis of time series
  - White noise + colored noise, spectral index=2 (random walk)
  - Monument noise? Amplitude of rwn correlated with:
    - Bedrock type
    - Type of monument
Noise models

- Correlated noise "hidden" in time series
- Synthetic time series:

\[ x(t) = x_0 + rt + ao(t) + bk \beta(t) \]

- \( x_0 \) = x-axis intersect
- \( r \) = velocity (constant)
- \( a \) and \( b_k \) = magnitude of the white and colored noise, respectively
- \( \alpha \) and \( \beta \) = uncorrelated random variables

Synthetic time series, 1 year of daily positions: \( b/a = \text{rwn/wn} \) relative amplitude

Rate uncertainty as a function of sampling frequency for 5 years of measurements:
⇒ Little gain with continuous measurements!
⇒ Emphasis on monuments to reduce rwn
Noise models

- Regional analysis: Zhang et al. (1997), GAMIT, double differences
- Global analysis: Mao et al. (1997), GIPSY, point positioning
- Spectral indices for GPS time series range from 0.74 to 1.02 (Mao et al., 1997)
- Dependence in latitude: tropical stations have larger white noise ⇒ Troposphere?
Noise models, a case study
Noise models, a case study

Estimation of the parameters of a model combining white + colored noise:

⇒ Colored noise = flicker
⇒ Amplitude ~ 5 mm
⇒ 8 mm at Chatel and Modane: site environment? (masks)
⇒ White noise alone underestimate uncertainties by a factor of 2
Noise models, a case study

![Graph showing noise models with time series length and sigma rate]

Empirical noise model combining white, flicker, and random walk noise (Mao et al., 1999):

$$\sigma_r \approx \left( \frac{12 \sigma_w^2 + a \sigma_f^2 + \sigma_{rw}^2}{gT^3 + gbT^2 + T} \right)^{1/2}$$

- $T =$ time span
- $g =$ # meas. per year
- $\sigma =$ magnitude of noise
- $a, b =$ empirical constants

⇒ 1 mm/yr uncertainty can be reached in 1 year in the best case, in 10 years in the worse…

Mean values, estimated from the analysis of the REGAL time series

Maximum amplitude of white and flicker noise of the REGAL time series, plus 2 mm/yr of random walk noise (Langbein and Johnson, 1997).

Minimum amplitude of white and flicker noise of the REGAL time series.
Noise sources

Bock et al., 2000

- Kinematic analysis of GPS baselines (50 m, 14 km, 37 km)
- 1 sec. sampling rate
- Epoch-by-epoch processing, no time-correlation introduced
- No flicker noise on very short baseline => tropospheric origin?
Snow on antenna

- Effect of snow on GPS antenna radome: Jaldehag et al., 1996
- Elevation angle dependency strong when snow on radome: local refraction effects
Loading effects

- Geophysical processes loading the earth's crust: oceanic water, continental water, atmospheric air masses
- Radial deformation of elastic earth:
  \[
  u(\varphi, \lambda) = \frac{3}{\rho_e} \sum_{n=0}^{\infty} \frac{h'_n}{2n+1} q_n(\varphi, \lambda)
  \]
  - \(\rho_e\) = mean Earth density
  - \(h'_n\) = load Love number (function of assumed rheology)
  - \(n\) = degree of spherical harmonic series
  - \(q(\lambda,\varphi)\) = surface load

- Atmospheric load = ground pressure field (measurements+interpolations, e.g. NCEP)
- Oceanic load = ocean tide models (+sat. altimetry)
- Continental water (groundwater+snow) = global models for soil moisture and snow
Continental water loading

- Water cycle + climatological model
- Load = model storage output = snow, ground water, soil water
- 1°x1° grid
- Load dominated by annual signal
- Vertical displacement range caused by total stored water/snow (max-min, 1994 to 1998)
- On average: 9-15 mm over continents
- Max: monsoon and tropical continental areas
- Horizontal: 5 mm max.

VanDam et al., 2001
Continental water loading

- Monthly averages of vertical component
- Atmospheric loading removed
- Annual signal, 2-3 cm
- Hydrological loading does not explain fully GPS height residuals
- Goal: validate models
- Amplitude varies from year to year

VanDam et al., 2001
Continental water loading

- Effect on secular trends for vertical displacement
- Small effect in most regions (<0.5 mm/yr)
- 20 years => < 0.3 mm/yr at all sites

TRENDS 1996-1998

VanDam et al., 2001
Precision and accuracy

Comparison of the position of site Grasse (right panel: NE, left panel: Up) obtained using 2 different geodetic techniques (GPS, SLR) and different processing strategies.

The scatter of a series of measurements made using independent techniques is an indicator of the **accuracy** of the position estimate.

Accuracy = external control

The scatter of a series of measurements made using the same technique is an indicator of the **precision** of the position estimate.

Precision = internal control

Daily positions (NE) for SJDV over a 6 month time period.

**SJDV PLANIMETRIE**
A short baseline

- 1.3 km long baseline continuously observed during 10 days

- Processing of GPS phase data (on L1) with research software

- Repeatability, horizontal components:
  - 24 hr sessions: < 1 mm
  - 15 min sessions: ~ 5 mm
Influence of baseline length

- Permanents GPS sites (IGS network), 1 to 2 years of continuous measurements

- Repeatability (horizontal components):
  - Short baseline (36 km) = 2.0 mm
  - Medium baseline (160 km) = 2.3 mm
  - Long baseline (870 km) = 7.3 mm
  - Very long baseline (2300 km) = 10.0 mm
Influence of session duration

- Landslide in the French Alps (La Clapière, 50x10⁶ m³)
- Reference site outside of the landslide + 3 sites on the landslide ⇒ baselines ~ 1 km
- Continuous observations during 6 days
- Processing of the phase data (L1), using 24hr, 12hr, 6hr, 1hr sessions
  ⇒ Shorter sessions are affected by a high-frequency noise
  ⇒ HF noise is correlated with PDOP variations and multipath (enhanced by topo + snow).

Variations in meters

![Graph of length variations](image)

![Graph of height variations](image)
Influence of session duration

- Three baselines observed continuously during 30 days
- Length = 30, 60 and 260 km
- Sophisticated processing of the phase data (LC)
  - 1, 6, and 24 hr sessions
  - Research software (GAMIT)
  - Precise IGS IGS, estimation of tropospheric parameters, etc…

![Graph showing influence of session duration on repeatability]
Influence of the processing strategy

- 260 km long baseline observed continuously during 160 days
- Processing of the phase GPS data (LC) using 24 hour sessions with:
  - A commercial software (GPPS), broadcast orbits, no tropospheric estimation, etc.
  - A research software (GAMIT), IGS precise orbits, tropospheric estimation, etc.

Result:
- GPPS: \( \text{wrms} = 6 \text{ cm} \)
- GAMIT: \( \text{wrms} = 3 \text{ mm} \)
- But mean length differ by 0.6 mm only!
The quest for millimeter precision... The recipe

- Receivers:
  - Record phase and pseudorange data
  - Dual frequency

- Antennas:
  - Design that minimizes multipath
  - Calibrated + phase diagram known

- Measurements:
  - Long sessions (24 hours), repeated 2-3 times (=> power!)
  - Or continuous recording at permanent sites
  - Sampling rate 30 seconds, elevation cut-off 10°

- Sites: stable, secure, and perennial

- Reliable field operators!

- Post-processing of phase data:
  - Ionosphere-free combination LC
  - Double differences (eliminate clocks) => need for at least 2 stations
  - Models:
    - Antenna phase center variations
    - Tropospheric zenith delays (+ horizontal gradients)
    - Solid-Earth tides, ocean loading (+ atmospheric and hydrological loading...)
    - Orbit perturbations: solar radiation pressure, yaw
  - A priori tables:
    - Earth orientation parameters for accurate conversions between inertial and Earth-fixed frames
    - Lunar and solar ephemerides (tidal effects)
    - Precise GPS orbits (from IGS)
    - Accurate terrestrial reference frame (ITRF)

⇒ Research software (GAMIT, BERNESE, GIPSY, etc.)