On the Message Complexity of Wirless Networks with Interference Avoidance

Aly El Gamal
ECE Department and Coordinated Science Laboratory
University of Illinois at Urbana-Champaign

I. Introduction

In the past decade, there has been a significant growth in the usage of wireless networks, and in particular, cellular networks, because of the increased data demands. This has been the driver of recent research for new ways of managing interference in wireless networks. Due to the superposition and broadcast properties of the wireless medium, interfering signals pose a significant limitation to the rate of communication of users in a wireless network. Hence, it is of interest to understand the fundamental limits of communication in interference networks.

In this survey, we review results on lower and upper bounds for the minimum number of communication rounds needed to simulate the following functions in a wireless network. The first is the broadcast problem where it is needed for a source to deliver its message to all remaining nodes in a multi-hop network. The second is the message passing model where each node can deliver a possibly different message to all its neighbours in a single-hop network. The third is the synchronization problem where all nodes need to agree on a global round numbering scheme in an asynchronous activation model.

We only consider interference-avoidance-based schemes. A transmitter can successfully deliver its message to a receiver if and only if no other transmitter connected to the same receiver is active, thereby, ignoring the effect of channel noise as well as the possibility of managing interference through clever coding schemes. We also assume the absence of any collision detection strategy where transmitters can detect other neighbouring active transmitters. We think that the reason behind these common simplifications in the reviewed results is the lack of a systematic abstraction of existing interference management techniques into network combinatorial problems, and it would be interesting for future work to consider the reviewed problems in light of more sophisticated communication theoretic ideas like multiple-access encoding strategies as well as recent advances in wireless infrastructure like the presence of backhaul links that allow for cooperative transmission.

A. Document Organization

In Section II, we review results on the broadcast problem. The case where all nodes are aware of the network topology is discussed in Section II-A and the case where each node is aware only of its own label as well as its neighbours' is discussed in Section II-B. We then review results on single round simulations of the message passing model in Section III. In Section IV, we discuss the wireless synchronization problem, and conclude the survey in Section V by discussing three open problems, each is related to one of the discussed network functions.

II. The Broadcast Problem

In this section, we consider a multi-hop network with n nodes, one of them is a source that transmits its message in the first step. The goal is to find the number of rounds required for all nodes to learn the source's message. Let D be the maximum distance of a node from the source, then D is an obvious lower bound. In the following, we consider two models: In the first, the network topology is *globally* known to all nodes, and in the second, each node is aware only of its own label as well the list of labels belonging to its neighbours.

A. Global Knowledge of Network Topology

We first discuss the case where the network topology is known to all nodes. It is natural to consider centralized algorithms in this setting, since the global knowledge of network topology allows each node to correctly simulate its role in a centralized algorithm. Also, there is no need for randomized algorithms as collisions can always be avoided as a result of the global topology awareness. In [1] and [2], polynomial time deterministic centralized algorithms are proposed for the broadcast problem. The algorithm proposed in [1] requires $O(D+\log^5 n)$ rounds to terminate successfully while the algorithm in [2] requires $O(D\log^2 n)$ rounds. As we consider the case where D is small, we will pay more attention to the result in [2] and summarize the proposed algorithm therein. In [3], it is shown that there is a family of radius-2 graphs for which any schedule for the broadcast problem requires $\Omega(\log^2 n)$ rounds, and hence, proving the optimality of the polynomial-time algorithm in [2] for radius-2 graphs.

The lower bound in [3] is proved using the following probabilistic argument. Let V_i be the set of nodes at distance i from the source, where $i \in \{1, 2\}$. The proof follows by considering a random assignment of connections between nodes in V_1 and nodes in V_2. It is then shown that the probability that any transmission schedule consisting of $\frac{\log^2 n}{100}$ rounds successfully relays the source's message to all nodes in V_2 is bounded away from unity, and hence, there exists a radius-2 graph for which $\frac{\log^2 n}{100}$ rounds of communication cannot suffice to complete broadcast.

Finding a centralized deterministic algorithm that exploits the global network knowledge to achieve the optimal message complexity is not a challenging task. We could do an exhaustive search over all possible sets of active transmitters in each round and choose the scenario with the minimum number of rounds. It is then natural to consider the computational complexity of such algorithms. In [2], a polynomial-time algorithm is shown to complete the broadcast with $O(D\log^2 n)$ rounds, and hence, we know that it achieves optimal message complexity in radius-2 networks.

The algorithm in [2] relies on an efficient procedure for relaying the source's message. Assuming that all nodes at distance d from the source (members of V_d) have received the message, it is desired to find a polynomial-time subroutine to deliver the message to all nodes at distance $d+1$ from the source (members of V_{d+1}) in a minimum number of rounds. This subroutine is then used iteratively to broadcast the source's message in a multi-hop network. We now explain briefly the key idea in the design of this subroutine in [2], which the authors call the *spokesman election* algorithm. A subset $S \subseteq V_d$ is efficiently identified such that activating the transmitters in S guarantees successful reception at more than $p|V_{d+1}|$ receivers in V_{d+1}, where p is a constant such that $0 < p < 1$. For every $S \subseteq V_d$, define $R_S \subseteq V_{d+1}$ as the set of receivers that successfully receive the source's message if the set S of transmitters are activated. The spokesman election algorithm consists of two phases. First, the average of the size of R_S is computed over all sets S of size k, where $1 \leq k \leq |V_d|$. More precisely, the following is evaluated,

$$w_k = \frac{1}{\binom{V_d}{k}} \sum_{S \subseteq V_d : |S| = k} |R_S|, 1 \leq k \leq V_d \tag{1}$$

The first phase is concluded by choosing $k^* = \arg\max_k w_k$ as the size of the set of active transmitters. We then proceed with the second phase to determine the k members of the set. The members of the active transmitter set S^* are chosen sequentially. In step i, $1 \leq i \leq k$, the value of w_k is re-evaluated for each candidate as an average of $|R_S|$ over all sets $S \subseteq V_d : |S| = k$ that contain the $i-1$ previously chosen members and the candidate considered for the i^{th} position in S. The candidate with the highest such average is selected. The key to the efficiency of the spokesman election algorithm is the easy computability of the w_k values in (1).

B. Local Knowledge of Network Topology

The broadcast problem was considered in [4], [5], and [6] for networks where each node can only be aware of its own label as well as the labels of its neighbors. In [4], a randomized transmission protocol is used to achieve broadcast with probability at least $1-\epsilon$, in a number of rounds that is $O\left(\left(D + \log\frac{n}{\epsilon}\right)\log n\right)$. The proposed randomized algorithm uses neither the processor labels nor the fact that each node is aware of its labels, and hence, making it suitable to use in a dynamic wireless network where fading conditions can lead to changes in the topology. While the combinatorial optimization problem of choosing the set of active transmitters may be difficult even for a centralized algorithm that has access to the network topology, the simple yet useful idea behind the randomized algorithm is that making each node transmit with a certain probability, in a way that is oblivious to what decisions other nodes are taking, will eventually lead to completion of broadcast with high probability in a sufficiently large number of rounds, the probabilistic analysis are elementary and not included in this survey.

In [4], the authors claimed to show that any deterministic broadcast schedule using only local knowledge of the network topology requires $\Omega(n)$ rounds to complete for certain types of graphs. The goal of proving this bound was to show that there is an exponential gap between determinism and randomization for the setting of local network topology. The graphs used to prove this lower bound are radius-2 graphs with only one node at distance 2 from the source, and this node is connected to only a subset of the nodes that are at distance 1 from the source. In [5], the authors claim to strengthen this lower bound by showing that $(n-1)$ rounds are required by any deterministic broadcast schedule for the same type of graphs that is used in the proof of the lower bound in [4]. However, the authors in [6] proved both of the above bounds to be incorrect by constructing a logarithmic broadcast schedule for the same type of graphs. In the following paragraph, we call this type of graphs BGI graphs, following the initials of the authors in [4].

The idea behind the algorithm proposed in [6] for broadcast in BGI graphs, is that the source can also listen to the wireless communication between nodes at distance 1 and those at distance 2. The source uses this feedback to identify one node at distance 1 that is connected to the node at distance 2 in logarithmic time, it then informs the identified node to relay its message in a total of two more rounds. The identification of one node with degree 2 is carried through a binary search procedure, where each step takes place in two rounds. In the first round of each step, the source broadcasts the labels of a candidate set of nodes at distance 1, this set will consist of consecutive labels and its size equals half the number of possible nodes with degree 2. In the second round, each of the nodes in the candidate set will broadcast its own label only if it is connected to the node at distance 2 from the source (note that each node is aware of the list of its own neighbours). If the source detected no transmission then all elements of the current candidate set are known to have degree 1 and eliminated from future candidacy. If the source detected a transmission from only one node, then it identifies this node as having degree 2. If the source detects collision then it proceeds the next step of the binary search by only selecting the first half of the current candidate set as the next candidate set. The algorithm can be easily seen to terminate in logarithmic time. We finally note here that assuming that the source cannot distinguish between the case where there is more than one active transmitter and the case where there are no active transmitters, will only lead to increase the number of rounds needed for each step of the binary search to three instead of two.

The authors in [6] then proceed to use the binary search procedure with auxiliary theorems from [7] and [8] to construct a broadcast algorithm that requires $O\left(n^{\frac{4}{5}}\log n\right)$ for general radius-2 graphs, and hence, showing that broadcast is possible on these graphs in sublinear time. The first auxiliary result is from [7], and shows the existence of a protocol for bipartite graphs that delivers a message known to all nodes in the left partite set to all nodes in the right partite set under the assumption that the degree of any node in the right partite set is bounded by a fixed constant, and the algorithm completes in time logarithmic in the number of nodes and linear in the right degree bound. The second auxiliary result is from [8], and shows the existence of a protocol for exchanging messages in a graph where all node degrees are bounded by a fixed constant and each node is transmitting a possibly unique message, and the algorithm finishes in time that is logarithmic in the number of nodes and quadratic in the degree bound.

The algorithm in [6] uses the above mentioned auxiliary results with the binary search procedure as follows. We label the set of nodes at distance one and two from the source as V_1 and V_2, respectively. A binary search is first used by the source to identify the node in V_1 with maximum number of neighbours in V_2, and then this node is informed to relay the source's message in a dedicated communication round. This process is iteratively used until all the nodes in V_1 that have not been identified, have no more than d_1 neighbours in V_2, where d_1 is a pre-specified constant. The first auxiliary result of [7] is then used in a bipartite graph formed by nodes in V_1 and nodes in V_2 that have not yet received the source's message. The result implies the existence of an efficient protocol that delivers the source's message to all nodes in V_2 whose degree is less than a pre-specified constant d_2. The list of labels for nodes in V_2 who received the source's message in this last step is delivered to all nodes in V_1 using the protocol whose existence is proved in the second auxiliary result of [8]. A binary search procedure is finally used to iteratively identify each node in V_1 with uninformed neighbours in V_2. Following the analysis, and by careful choice of the constants d_1 and d_2, it is shown in [6] that the algorithm completes in $O(n^{\frac{4}{5}} \log n)$ rounds.

In order to prove that an exponential gap exists between deterministic and randomized algorithms for networks with local topology knowledge, it is shown in [6] that for any deterministic broadcast schedule, there exists a radius-2 graph for which the schedule requires $\Omega(n^{\frac{1}{4}})$ rounds. The exponential gap conclusion follows from this result and the logarithmic time randomized algorithm of [4]. In the lower bound proof, a radius-2 graph is constructed for each fixed schedule, with the property that the number of nodes in V_1 is strictly greater than the number of nodes in V_2, and each node in V_1 has exactly one neighbour in V_2. It is intuitive to think of these graphs as worst case constructions as the relaying of the source's message by a node in V_1 cannot result in delivering it to more than one node in V_2.

It is worth noting that the $\Omega(n)$ lower bound of [4] holds if we restrict the choice of the broadcast algorithm to oblivious algorithms, where the transmit sets must be fixed in advance, and also for the case where the links from the source to nodes in V_1 are directed, and hence, the source cannot listen to transmissions of nodes in V_1. This last assumption can be justified in practice for cases where the source node uses higher transmit power than the relay nodes of V_1, for example in a heterogeneous cellular downlink scenario where the source is a macrocell basestation and each of the nodes in V_1 is a microcell basestation.

We finally conclude this section by highlighting the fact that broadcast can be completed in time independent of the number of nodes n if messages can be split into bits that are transmitted in separate communication rounds. In this case, all nodes at distance i from the source can relay the one-bit message in one round by being all active if the bit value is 1 and all inactive if the bit value is 0. A receiver decodes a value of 1 if and only if it detects active transmission, even if there is collision. This algorithm completes in $(D+L)$ rounds , where L is the length of the message in bits. However, in practice, this is usually inefficient because of the high overhead induced in this case by the number of control bits possibly needed for combating noise and achieving synchronization.

III. Simulating the Message Passing Model

Unlike the broadcast problem model where it is desired that each node delivers the same message to all nodes of the network, regardless of the network topology, the message passing model requires that each node delivers a possibly different message to each of its neighbours, and no relaying of messages is required. Since the message passing model is suitable for describing wireline networks, algorithms and converse analysis for many network functions like consensus and shared memory systems already exist in the literature based on the message passing model. It is therefore useful to understand the minimum number of communication rounds needed for a wireless network to simulate a single round of the message passing model. For brevity, we refer to this problem in the following discussion as the single round simulation (SRS) problem.

Let Δ_i, and Δ_o, denote the maximum in-degree, and out-degree of any node in the network, respectively. In [3], a randomized algorithm achieves SRS with probability $1-\epsilon$, in $O\left(\Delta_i \Delta_o \log \frac{n}{\epsilon}\right)$ rounds. The algorithm executes in Δ_o phases, each phase consists of multiple rounds. In each phase, each transmitter will target one receiver and keeps trying to transmit its message with probability $\frac{1}{\Delta_i}$ in each round until it succeeds. The proof of the probabilistic completion of the algorithm in $O\left(\Delta_i \Delta_o \log \frac{n}{\epsilon}\right)$ rounds uses elementary probabilistic tools.

We now discuss the design of deterministic distributed algorithms for the SRS problem. It is easy to see that a trivial algorithm that dedicates each round to the transmission of a single message completes the simulation in $O(n\Delta_o)$ rounds. In [3], an algorithm that achieves SRS in $O(\Delta_o \Delta_i^2 \log^2 n)$ rounds is designed based on the following key lemma.

Lemma 1: Let $1 \leq x_1, \ldots, x_s \leq n$ be s distinct integers. Then for every $1 \leq i \leq s$ there exists a prime $p \leq s \log n$ such that $x_i \neq x_j (\text{mod } p)$ for every $j \neq i$.

The algorithm executes in Δ_o phases. Each transmitter targets a single receiver in each phase. The idea is that it is guaranteed that there is a communication round for each transmitter-receiver pair, in which neither the receiver node nor any of its neighbours other than the specified transmitter is transmitting a message. Let $\{p_1, \ldots, p_s\}$ be the prime numbers in the range $\{2, 3, \ldots, (\Delta_i + 1) \log n\}$, then each phase consists of s subphases, and the i^{th} subphase consists of p_i rounds. In each subphase, the k^{th} transmitter transmits its message in the round number $(k \text{ mod } p_i)$. It follows from Lemma 1 that for each transmitter k, there is a prime number p_i among $\{p_1, \ldots, p_s\}$

such that in the round numbered (k mod p_i) of the i^{th} subphase, transmitter k will be able to successfully deliver its message to its target receiver. The complexity analysis is straightforward since there are Δ_o phases, in each phase there are at most $O(\Delta_i \log n)$ subphases, and in each subphase, there are at most $O(\Delta_i \log n)$ rounds. It follows that $O(\Delta_o \Delta_i^2 \log^2 n)$ rounds suffice for a deterministic distributed algorithm to achieve SRS for every graph.

IV. The Synchronization Problem

Many network functions like that achieved by the algorithm relying on Lemma 1 above assume that all participating nodes agree on a global round numbering. In [9], this problem is studied for complete graphs and a general model that assumes the availability of F orthogonal frequency slots. In order to model possible interference from other wireless devices as well as protocols run by the same participating nodes, the model in [9] assumes an adversary that can disrupt up to t frequencies in each communication round. The adversary is also responsible for activating any of the participating devices, which are assumed to be initially inactive. It is important to notice that the adversary in the model is not intended to reflect the actions of a real adversary but is rather used to model a worst-case scenario for the realization of random phenomena. Once a node is activated, it tries to commit to a round numbering after a minimum number of communication rounds. Once a node commits, it cannot change the round numbering scheme, meaning that the round number has to be incremented in each round.

The authors in [9] mention that it is not known whether the wireless synchronization problem can be solved deterministically, and restrict their study to probabilistic algorithms. A probabilistic algorithm is called regular if for each round of communication, all nodes use the same probability distribution for frequency selection and the same probability of transmission on the selected frequency. It is shown that any probabilistic algorithm that solves the synchronization problem with probability $1 - \frac{1}{n}$, requires at least $\Omega\left(\frac{Ft}{F-t} \log n\right)$ rounds, and if it is regular then it requires at least $\Omega\left(\frac{\log^2 n}{(F-t)\log\log n} + \frac{Ft}{F-t}\log n\right)$ rounds. The first terms follows from the number of rounds needed for a node to broadcast alone in any frequency, which is a necessary condition for achieving synchronization. The $\Omega\left(\frac{\log^2 n}{(F-t)\log\log n}\right)$ bound on the number of rounds needed for a successful broadcast was derived in [10] for a strongly related problem called the *Wake-up* problem. In the Wake-up problem, it is required to find the minimum number of communication rounds needed for a node to activate all active remaining nodes through a successful broadcast. The second term in the $\Omega\left(\frac{\log^2 n}{(F-t)\log\log n} + \frac{Ft}{F-t}\log n\right)$ bound follows a worst case scenario for the adversary interference, and disappears when the number of disrupted frequencies $t = 0$.

A regular probabilistic algorithm is introduced in [9] and achieves synchronization with probability $1-\frac{1}{n}$, in $O\left(\frac{F}{F-t}\log^2 n + \frac{Ft}{F-t}\log n\right)$ rounds. The basic procedure behind the algorithm employs a random frequency hopping protocol at each node to contend for a *leadership* position with all remaining nodes. Once a node is a leader, it broadcasts the global round number. We note that the leader cannot be fixed in advance as the activation times of nodes is not known, and a node has to be able to synchronize in the minimum number of rounds once it is activated, regardless of the activation times of other nodes. We also note from the lower and upper bound expressions that the gap between the bounds seems to be related to the way the broadcast problem is handled in the proposed algorithm rather than the interference avoidance problem.

V. Concluding Remarks

We discussed the simulation of three network functions in wireless networks, we now conclude this discussion by an open problem for each. First, the broadcast problem, we have reviewed results for two settings: the network topology is known to all nodes in one case, and in the other each node is only aware of its own label as well as a list of labels corresponding to its neighbours. The inefficiency due to the local knowledge assumption in the second case can be seen from the results. For example, any deterministic algorithm requires at least $\Omega\left(n^{\frac{1}{4}}\right)$ rounds while with global network knowledge there is a deterministic algorithm the completes broadcast in $O(D\log^2 n)$ rounds. The problem we propose is the following: Is there a loss in the asymptotic message complexity if we restrict the choice of algorithms for the broadcast problem with local network topology to these that lead the source to learn the network topology?. Learning the topology can be done through means of feedback as in [6]. Moreover, if we consider a generalized broadcast problem in a connected graph where we have n phases, and in each phase a different node acts a source, then our question becomes more interesting as it may be possible for the nodes to jointly learn the network topology in an initialization phase, through a procedure akin to the one in [11].

We then discussed the simulation of the message passing model that allows each node to deliver a possibly different message to each of its neighbours. The results reviewed from [3] present lower and upper bounds on the message complexity of this problem as functions of the maximum in-degrees and out-degrees over all participating nodes. We note that this may not be a good performance criterion for lower bounds and may lead to the design of inefficient algorithms for cases where there are only few nodes with maximum degrees and the remaining node degrees are much smaller. It is therefore desired to study the message complexity of this problem as a function of more appropriate properties of the graph adjacency matrix.

We finally reviewed results on the wireless synchronization problem, where it is desired to find the minimum number of communication rounds for nodes in an asynchronous activation model to agree on a global round numbering scheme. We reviewed bounds on probabilistic solutions to the problem. It is mentioned in [9] that it is not known whether solving this problem deterministically is possible. The authors suggest that the multi-selector function of [12] may be useful to find a deterministic solution. The multi-selector is a combinatorial function that generates a sequence of channel assignments for participating nodes such that every sufficiently large subset of nodes is partitioned onto distinct channels by at least one of these assignments. In [13], an optimal deterministic algorithm has been found for the wireless synchronization problem for the case where the difference in activation times between different nodes is bounded.

REFERENCES

[1] I. Gaber, and Y. Mansour "Broadcast in radio networks," *In Proc.* 6^{th} *Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, pp. 577-585, 1995.

[2] I. Chlamtac, and O. Weinstein "The wave expansion approach to broadcasting in multihop radio networks," *IEEE Transactions on Communications*, vol. 33, pp. 1240-1246, 1985.

[3] N. Alon, A. Bar-Noy, N. Linial and D. Peleg "On the complexity of radio communication," *In Proc. Symposium on Theory of Computing*, 1989.

[4] R. Bar-Yehuda, O. Goldreich, and A. Itai "On the time complexity of broadcast in radio networks: an exponential gap between determinism and randomization," *Journal of Computer and System Sciences*, vol. 45, pp. 104-126, 1992.

[5] F. K. Hwang "The time complexity of deterministic broadcast radio networks," *Discrete Applied Mathematics*, vol. 60, pp. 219-222, 1995.

[6] D. Kowalski, and A. Pelc "Deterministic Broadcasting Time in Radio Networks of Unknown Topology," *In Proc.* 43^{rd} *Annual IEEE Symposium on Foundations of Computer Science (FOCS)*, 2002.

[7] A. E. F. Clementi, A. Monti, and R. Silvestri "Selective families, superimposed code, and broadcasting on unknown radio networks," *In Proc.* 12^{th} *Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, pp. 709-819,, 2001.

[8] G. De Marco, and A. Pelc "Faster broadcasting in unknown radio networks," *Information Processing letters* vol. 79, pp. 53-56, 2001.

[9] S. Dolev, S. Gilbert, R. Guerraoui, F. Kuhn and C. Newport "The wireless synchronization problem," *In Proc.* 28^{th} *Annual ACM Symposium on Principles of Distributed Computing (PODC)*, Calgary, 2009.

[10] T. Jurdzinski and G. Stachowiak "Probabilistic algorithms for the wakeup problem in single-hop radio networks," *In Proc. International Symposium on Algorithms and Computation*, 2002.

[11] J. Yang, S. Draper, and R. Nowak "Learning the interference graph of a wireless network", *available at http://arxiv.org/abs/1208.0562*, Aug. 2012.

[12] S. Gilbert, R. Guerraoui, D. Kowalski, and C. Newport "Interference-resilient information exchange," *In Proc. IEEE Conference on Computer Communications*, 2009.

[13] L. Barenboim, S. Dolev and R. Ostrovsky "Deterministic and Energy-Optimal wireless synchronization," *In Proc. Distributed Computing,* 25^{th} *International Symposium (DISC)*, Rome, 2011.

[14] L. Gaieniec, A. Pelc and D. Peleg "The wakeup problem in synchronous broadcast systems," *SIAM Journal of Discrete Mathematics*, vol. 14, no. 2, pp. 207-222, 2001.

[15] B. Chlebus and D. Kowalski "A better wake-up in radio networks," *In Proc. International Symposium on Principles of Distributed Computing (PODC)*, 2004.

[16] M. Bradonnjic, E. Kohler and R. Ostrovsky "Near-Optimal use for wireless network synchronization," *In Proc.* 5^{th} *International Workshop on Algorithmic Aspects of Wireless Sensor Networks*, 2009.

[17] J. Polastre, J. Hill and D. Culler "Versatile low power media access for wireless sensor networks," *In Proc.*2^{nd} *International Conference on Embedded Networked Sensor Systems*, Baltimore, 2004.

[18] S. Palchaudhuri and D. Johnson "Birthday paradox for energy conservation in sensor networks," *In Proc.* 5^{th} *Symposium of Operating Ssystems Design and Implementation*, 2002.

[19] T. Moscibroda, P. Von Rickenbach, R. Wattenhofer "Analyzing the Energy-Latency trade-off during the deployment of sensor networks," *In Proc.* 25^{th} *IEEE International Conference on Computer Communications (INFOCOMM)*, 2006.