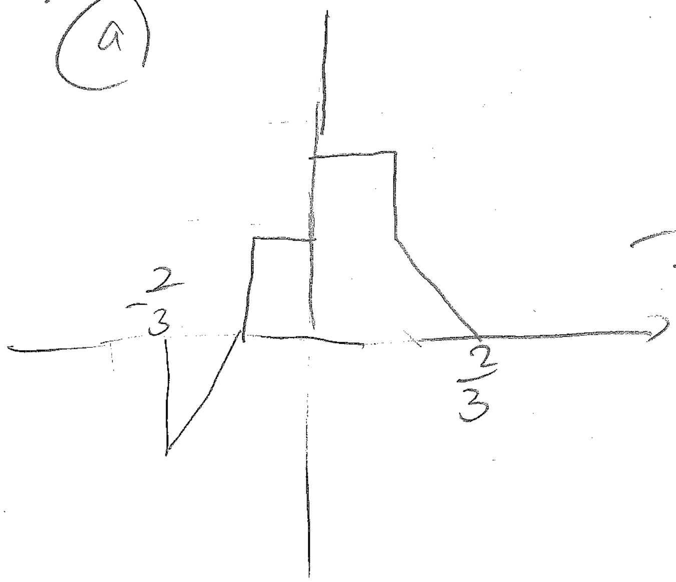
HW1

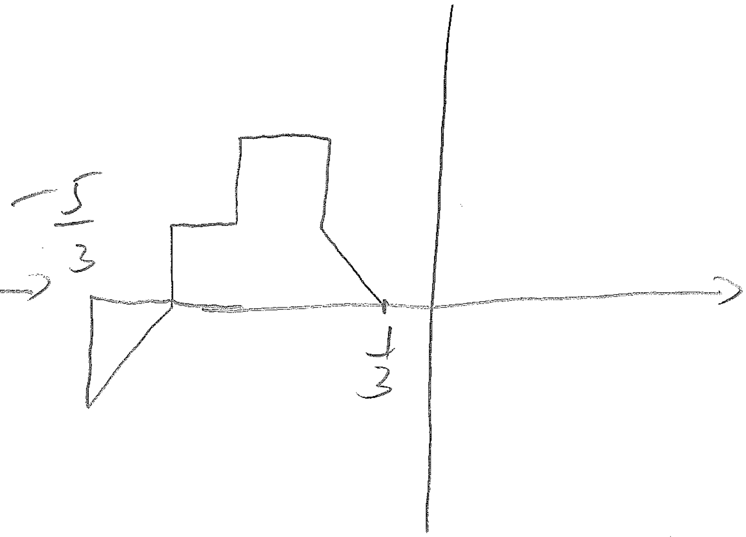
#1

a

X(3t)

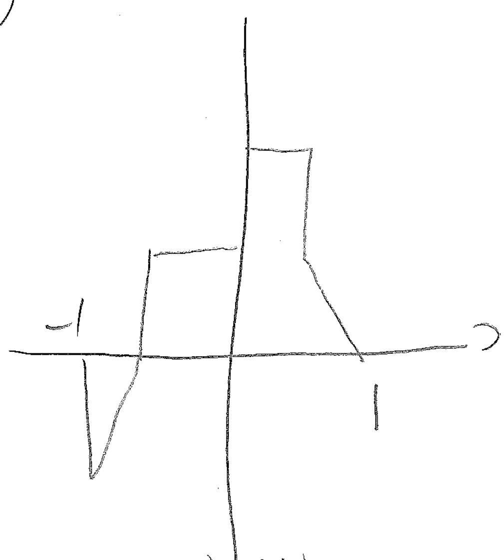


X(3t+1)



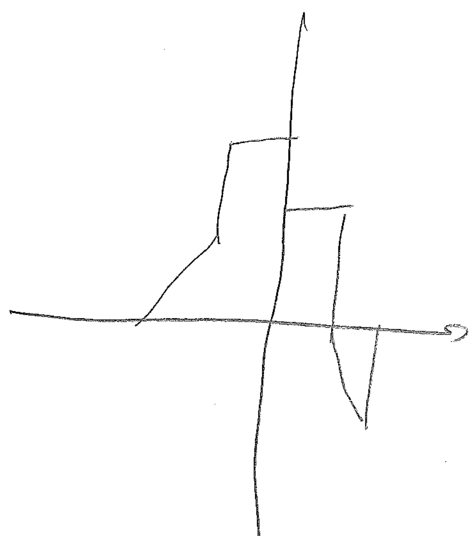
b

X(2t)

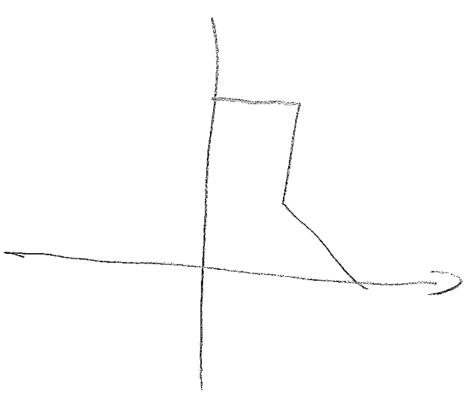


c

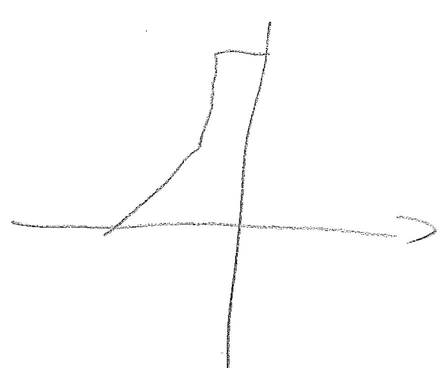
X(-2t)



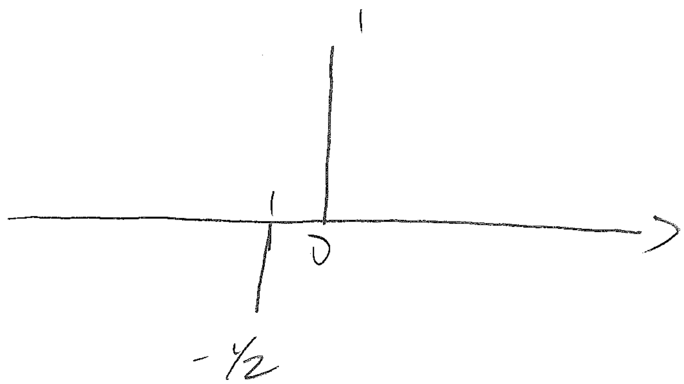
x(2t)u(t)



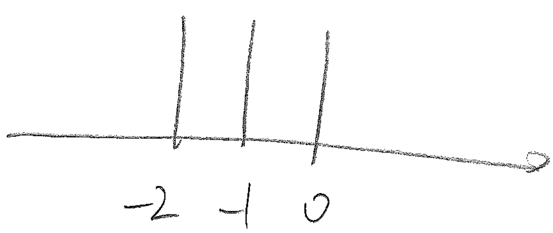
x(-2t)u(-t)



(d) $x[4n+1]$



(e) $x[(n+1)^2]$



#2 (a) $T_0 = \frac{2\pi}{2\pi} = 1$, Fundamental freq. $F_0 = \frac{1}{T_0}$

$x(t) = x(t+1)$ for $\forall t$, periodic.

(b) $x(t) = [\cos(3t - \frac{2\pi}{3})]^2$

$= [\frac{1}{2}e^{j(3t-\frac{2\pi}{3})} + \frac{1}{2}e^{-j(3t-\frac{2\pi}{3})}]^2$

$= \frac{1}{4}[e^{j(6t-\frac{4}{3}\pi)} + e^{-j(6t-\frac{4\pi}{3})} + 2]$

since $e^{j(6t-\frac{4}{3}\pi)} + e^{-j(6t-\frac{4\pi}{3})}$

$T_0 = \frac{2\pi}{6} = \frac{\pi}{3}$ $\quad F_0 = \frac{1}{T_0}$

$x(t) = x(t+\frac{\pi}{3})$ for $\forall t$, periodic

(c) $x[n] = \cos(\frac{\pi}{7}n)$

$N = \frac{2\pi m}{\omega_0} = \frac{2\pi}{\frac{\pi}{7}}m = 14m$, $\quad m = 1$ smallest.

thus $N = 14$ $\quad F_0 = \frac{1}{N}$

$\boxed{x[n] = x[n+14] \text{ periodic } \forall n \in \mathbb{Z}}$

(d) $x[n] = 2\cos(\frac{\pi}{5}n) + \sin(\frac{\pi n}{3}) - 2\cos(\frac{\pi}{15} + \frac{\pi}{6})$

$N_1 = \frac{2\pi}{\frac{\pi}{5}}m$ $\quad N_2 = \frac{2\pi}{\frac{\pi}{3}}m$ $\quad N_3 = \frac{2\pi}{\frac{\pi}{15}}$ $\quad \boxed{x[n] = x[n+30] \text{ periodic } \forall n}$

$= 10m_1$ $\quad = 6m_2$ $\quad = 30m_3$

$m = 3$ $\quad m_2 = 5$ $\quad m_3 = 1$

thus $N = 30$ $\quad F = \frac{1}{N_0}$

fn := function.

#3 a) For even fn $f(x) = f(-x)$; For odd fn $f(-x) = -f(x)$

1) $E(x(-t)) = \frac{1}{2}[x(-t) + x(-(-t))]$
$= \frac{1}{2}[x(-t) + x(t)] = E(x(t))$

2) $O(x(-t)) = \frac{1}{2}[x(-t) - x(-(-t))]$
$= -\frac{1}{2}[x(t) - x(-(t))] = -O(x(t))$

b) x is odd, y is even.

1. define $f(t) = x(t)\,y(t)$
$f(-t) = x(-t)\,y(-t) = -x(t)\,y(t) = -f(t)$
thus odd fn

2. define $g(t) = y(t)y(t)$
$g(-t) = y(-t)y(-t) = y(t)\,y(t) = g(t)$
$\therefore$ even fn

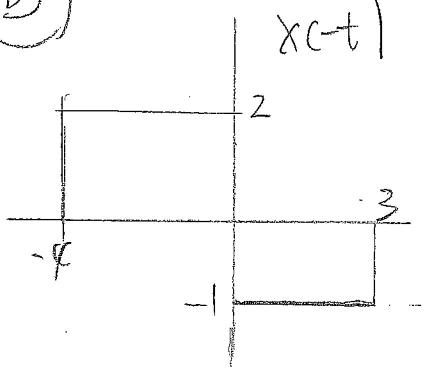
#4

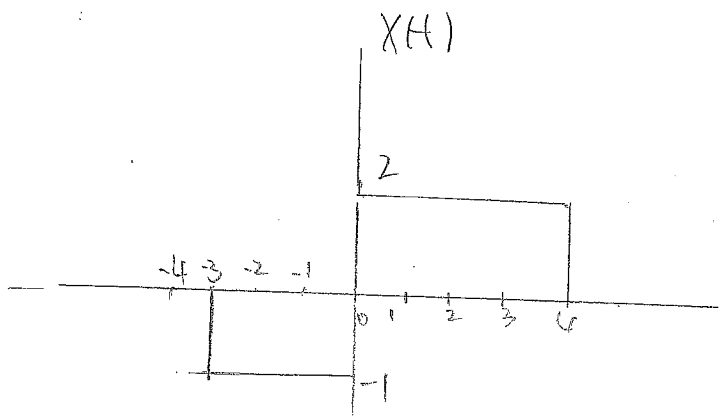
(a) defin $y(t) = x(t)\,x(-t)$

$y(-t) = x(-t)\,x(t) = -x(t)(-x(t)) = x(t)\,x(t) = y(t)$

thus odd fn

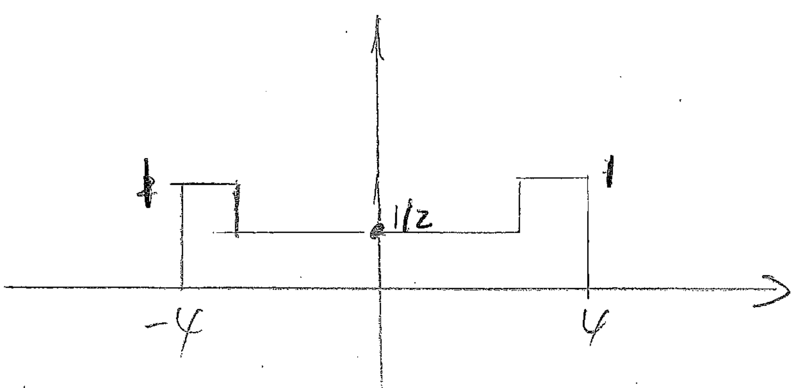
(b)

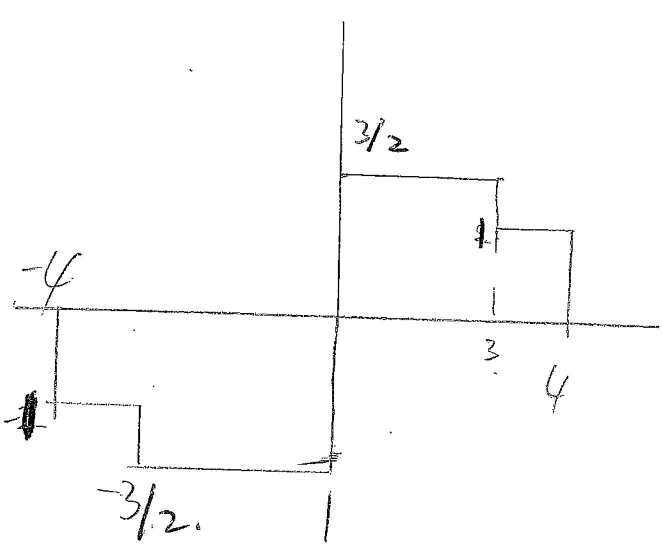




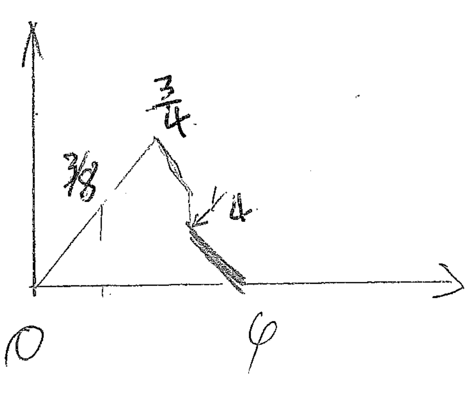
2) $E(t) = \frac{1}{2}(x(t) + x(-t))$

① $O(t) = \frac{1}{2}(x(t) - x(-t))$





(c)



$\frac{3}{4} \times 2$

$\frac{1}{4}$

#5

(a) $x(t) = e^{j(2t+\frac{\pi}{6})}$

$|x(t)| = 1$. thus.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} dt = \lim_{T\to\infty} = 1$$

(b) $x(t) = \sin(t)$

$$E_\infty = \int_{-\infty}^{\infty} \sin^2 t\, dt = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \sin^2(t) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} \frac{1-\cos(2t)}{2} dt = 1/2$$

(c) $x_1[n] = (1/3)^n u[n]$ $E_\infty = \sum_{n=-\infty}^{\infty} \left|\left(\frac{1}{3}\right)^n u[n]\right|^2 \Rightarrow \sum_{n=0}^{\infty} (1/3) = 9/8$

$P_\infty = 0$ b/c $E_\infty < \infty$

(d) $x[n] = e^{j(\frac{2\pi}{3}n + \frac{\pi}{8})}$ $|x[n]|^2 = 1$

$$E_\infty = \sum_{n=-\infty}^{\infty} 1 = \infty$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N} 1 = 1$$

(d) $x[n] = j\sin\frac{\pi}{4}n$ $x[n] = |j\sin\frac{\pi}{4}|^2 = (\sin\frac{\pi}{4})^2$ $E_\infty = \infty$

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N} 1-\cos^2\frac{\pi}{4} \Rightarrow \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}\frac{1-\cos(\frac{\pi}{2}n)}{2} = \frac{1}{2}$$