ECE301

HW 2

Due on Tuesday Sep. 27th

Please provide steps to explain your answer

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Office Hour change: MSEE180 (9:30am - 10:30am Monday and Wednesday)

Question 1

a.) Consider the continuous-time systems with input x(t) and output y(t)
1.) $y = x(t) - x(t+1)$
2.) $y(t) = \begin{cases} t & |x(t)| \leq 1 \\ 0 & |x(t)| > 1 \end{cases}$
For each of the above systems, is it causal? Justify your answer.

b.) Consider the continuous-time systems with input x(t) and output y(t)
1.) $y(t) = sin(3t)x(t)$
2.) $y(t) = \begin{cases} t & |x(t)| \leq 1 \\ 0 & |x(t)| > 1 \end{cases}$
For each of the above systems, is it time-invariant or time-variant? Justify your answer.

c.) Consider the continuous-time systems with input x(t) and output y(t)
1.) $y(t) = (x(t-1))^2$
2.) $y(t) = O(x(t))$, where O(.) is odd function
For each of the above systems, is it linear? Justify your answer.

d.) Consider the continuous-time systems with input x(t) and output y(t)
1.) $y[n] = x[n-1] + x[n-2]$
Is this system stable? Justify your answer.

Question 2

Consider the discrete-time system with input x[n], impulse response h[n], and output y[n]. Find y[n] by using the formula below

$$y[n] = x[n] * h[n]$$

$$\begin{array}{ll} a.)\ x[n] = (\frac{1}{3})^{n+3}u[n+3]; & h[n] = u[n-3] \\ b.)\ x[n] = (\frac{1}{3})^{n}u[n] * \delta[n-2]; & h[n] = (\frac{1}{3})^{n}u[n] * \delta[n+2] \\ c.)\ x[n] = u[n] - u[n-3]; & h[n] = e^{j\pi n}(u[n] - u[n-3]) \end{array}$$

Question 3

Consider the continuous-time system with input x(t), impulse response h(t), and output y(t). Find y(t) by using the formula below

$$y(t) = x(t) * h(t)$$

$$\begin{array}{ll} a.)\ x(t) = e^{-\alpha t}u(t); & h(t) = e^{-\beta t}u(t) \\ b.)\ x(t) = u(t) - 2u(t-3) + u(t-6); & h(t) = e^{2t}u(1-t) \\ c.)\ x(t) = u(t); & h(t) = \delta(t-3) \end{array}$$

Question 4

I: Consider the continuous-time system with input x(t), impulse response h(t)

$$h(t) = \delta(t-2) + \delta(2-t); \quad x(t) = u(t)$$

a.) Define the expression y(t)
b.) Is this system memoryless? Justify your answer
c.) Is this system time-variant or time-invariant? Justify your answer
d.) Is this system linear? Justify your answer

II: Prove the followings
a.) $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$
b.) $[x_1(t) * x_2(t)] * x_3(t) = x_1(t) * [x_2(t) * x_3(t)]$

Question 5

a.) Define unit step response of an LTI system (discrete-time)
b.) Express h[n] by using unit step response
c.) Determine h(t) of an LTI system with unit step response

$$s(t) = (e^{-3t} - 2e^{-2t} + 1)u(t)$$

d.) Linear constant-coefficient differential equations

$$y'(t) + 4y(t) = x(t), \quad x(t) = e^{(-1+2j)t}u(t), \quad y(0) = 1$$

what is y(t).

Note: Due to the time limit in class for Olympic II, I made part (c) in this section very easy. For review purpose on exam, i.e., I would find some problems such that first find the h(t) by using the unit step response, then use calculated h(t) and given x(t) (input) to find y(t)(output). Discrete time is also fair game.