ECE 301
HW2

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#1

ⓐ
① No, the system depends on future
② Yes, does dependent on past

ⓑ
① $x_1(t) = x(t - t_0)$

$y_1(t) = \sin(3t)\, x_1(t)$
$= \sin(3t)\, x(t - t_0)$

$y(t - t_0) = \sin(3(t - t_0))\, x(t - t_0)$

$y_1(t) \neq y(t - t_0)$ TV.

② if $u(t) = 0 \Rightarrow y(t) = t$ For $t \in (-\infty, \infty)$

if $u(t-2) = 0 \Rightarrow y(t-2) \neq t$
thus TV.

ⓒ ① $x_1(t)$, $y_1(t) = x_1^2(t-1)$; $x_2(t)$, $y_2(t) = x_2^2(t-1)$

$x_3 = \alpha x_1(t) + \beta x_2(t)$

$y_3(t) = x_3^2(t-1)$
$= (\alpha x_1(t-1) + \beta x_2(t-1))^2$
$= \alpha^2 x_1^2(t-1) + \beta^2 x_2^2(t-1) + 2\alpha\beta\, x_1(t-1)\, x_2(t-1)$
$\neq \alpha y_1(t) + \beta y_2(t)$ Not linear

#2

$$y = \left(\tfrac{1}{3}\right)^{n+3} u[n+3] * u[n-3]$$

(a)

$$= \sum_{k=-\infty}^{\infty} \left(\tfrac{1}{3}\right)^{k+3} u[k+3]\, u[n-k+3]$$

let $m = k+3$.

$$= \sum_{m=-\infty}^{\infty} \left(\tfrac{1}{3}\right)^{m} u[m]\, u[n-m] = \sum_{m=0}^{n} \left(\tfrac{1}{3}\right)^{m} = \frac{1-\frac{1}{3}^{n+1}}{1-\frac{1}{3}}$$

$$= \frac{3}{2}\left(1-\tfrac{1}{3}^{n+1}\right)u[n]$$

(b) $y[n] = u[n] * h[n]$

$$= \left(\tfrac{1}{3}\right)^{n} u[n-2] * \left(\tfrac{1}{3}\right)^{n} u[n+2]$$

$$= \left(\tfrac{1}{3}\right)^{n} u[n] * \delta[n-2] * \delta[n+2] * u[n] * \left(\tfrac{1}{3}\right)^{n}$$

$$= \sum_{k=0}^{n} \left(\tfrac{1}{3}\right)^{k}\left(\tfrac{1}{3}\right)^{n-k}$$

$$= (1+n)\left(\tfrac{1}{3}\right)^{n}$$

as $n \to \infty$

(c) $x[n] = 1 \quad 1 \quad 1$

$h[n] = 1 \quad -1 \quad 1$

⇒ refer to Prof Michael-Zoltowski notes

$x[n]$

1 −1 1

$y[n] =$ 1 −1 1

$h[n]$ 1 −1 1

1 −1 1

$y[n] = 1 \quad 0 \quad 1 \quad 0 \quad 1$

If $t \in (1,4)$ $\quad y(t) = \int_{t-1}^{3} e^{2(t-2)} d\tau - \int_{3}^{6} e^{2(t-2)} d\tau$

$t \in (4,7]$ $\quad y(t) = \int_{t-1}^{6} e^{-(t-2)} d\tau$

$t \in (7, \infty)$ $\quad y(t) = 0$

(c) $X(t) = u(t)$ $\quad h(t) = \delta(t-3)$

$\boxed{y(t) = u(t-3)}$ or $\boxed{y(t) = 1 \quad t = 3}$

#4 I $y(t) = \delta(t-2) + \delta(2-t)$

(a) Not Memoryless; $y(t)$ dependents on $\delta(t-2)$ and $\delta(2-t)$, dependents on future and past.

(b) define. $u_1(t) = u(t-2)$

$y_1(t) = u_1(t-2) + u_1(-t-2)$

$y_1(t) = u(t-2-2) + u(-t-2-2)$

$y(t-\tau) = u(t-2-2) + u(2-t+2) \neq y_1(t)$

Thus not time invariance

(c) define $\delta_1(t)$ s.t $y_1(t) = \delta_1(t-2) + \delta_1(-t+2)$

$y_2(t) = \delta_2(t-2) + \delta_2(-t+2)$

#5 (a) $s[n] = \sum_{k=-\infty}^{\infty} h[k]$

(b) $h[n] = s[n] - s[n-1]$ (c) $h(t) = \frac{ds(s)}{d(t)} = (-3e^{-3t} + 4e^{-2t})u(t)$

(d) $y_p(t) = ke^{(-1+2j)t}$

$$(-1+2j)ke^{(-1+2j)t} + 4ke^{(-1+2j)t} = e^{(-1+2j)t}$$

$$(-1+2j)k + 4k = 1 \qquad (-k+2jk) + 4k = 1 \qquad k = \frac{1}{2j+3}$$

$$3k + 2jk = 1$$

$$k(3+2j) = 1$$

$$y_p(t) = \left(\frac{1}{2j+3}\right)e^{(-1+2j)t} \quad t>0 \qquad y_h(t) = Ae^{st}$$

$$Ase^{st} + 4Ae^{st} = 0 \Rightarrow Ae^{st}(s+4) = 0 \qquad s = -4$$

$$y(t) = Ae^{-4t} + \left(\frac{1}{2j+3}\right)e^{+(-1+2j)t}$$

$$Ae^{-4(0)} + \left(\frac{1}{3+2j}\right)e^{(-1+2j)\cdot 0} = 1$$

$$A + \frac{1}{3+2j} = 1 \qquad A = \frac{3+2j-1}{3+2j} = \frac{2+2j}{3+2j}$$

$$y(t) = \left(\frac{2+2j}{3+2j}\right)\left[\underbrace{e^{-4t}}_{y_h} + \underbrace{\frac{e^{(-1+2j)t}}{2+2j}}_{y_p}\right]u(t) \quad t>0$$

Important

Need to show $t>0$

(e)