HW4

$\boxed{\#1}$ Refer to HW3, Q2's Input

$\boxed{\#2}$ (a) $X(j\omega) = \frac{1}{1+j\omega}$ $\quad Y(j\omega) = \frac{6}{(1+j\omega)(4+j\omega)}$

(b) $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{6}{4+j\omega}$ $\quad h(t) = 6e^{-4t}\underline{u(t)}$

must have

(c) $\int_{-\infty}^{\infty} |h(t)|\,dt = \int_0^{\infty} 6e^{-4t}\,dt \Rightarrow < \infty$.

$\boxed{\#3}$ $H(e^{j\omega}) = \sum_{n=0}^{9} a^n e^{-j\omega n}$ $\quad$ define $\alpha = a^n e^{-j\omega n}$

(a) $\begin{cases} N & \alpha = 1 \\ \frac{1-\alpha^N}{1-\alpha} & \alpha \neq 1 \end{cases}$ where $N = 10$

$H(e^{j\omega}) = \frac{1-(ae^{-j\omega})^N}{1-ae^{-j\omega}}$

(b) $S[n] = \frac{1}{2j}\left[e^{j\frac{\pi}{3}n} h[n] - e^{-j\frac{\pi}{3}n} h[n]\right]$

$S(e^{j\omega}) = \frac{1}{2j}\left(\left(H(e^{j(\omega-\frac{\pi}{3})}\right) - H\left(e^{j(\omega+\frac{\pi}{3})}\right)\right)$

\#4

#4 (a) $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \frac{1}{2+j\omega}$

(b) $Y(j\omega) = H(j\omega)\, X(j\omega) = \frac{5}{6+j\omega 5+(j\omega)^2}$

$Y(j\omega) = \frac{A}{3+j\omega} + \frac{B}{2+j\omega} \qquad A=-5 \quad B=5$

$y(t) = -5e^{-3t}u(t) + 5e^{-2t}u(t)$

#5 $H(e^{j\omega})\big|_{\omega=0} = \frac{32}{25}$ $\cancel{H(e^{j\omega})\big|_{\omega=\pi} = 1.28}$

$H(e^{j\omega})\big|_{\omega=\pi} = 1.28$

$y[n] = \frac{32}{25}(3)\, e^{j0n} + \frac{32}{25}(-1)^n.$