ECE301

HW 5

Due on Thursday Nov. 17th

Please provide steps to explain your answer

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Office Hour change: MSEE180 (9:30am - 10:30am Monday and Wednesday)

Question 1

a.) Define that the fix notation $X_1(jw)$ and $X_2(jw)$ are Fourier transform of $x_1(t)$ and $x_2(t)$ respectively.
Prove Fourier transform of x_1 multiple x_2 is the convolution of $X_1(jw) * X_2(jw)$ that is

$$\mathcal{F}\{x_1(t)x_2(t)\} = X_1(jw) * X_2(jw)$$

b.) Given a frequency response $H(e^{jw})$, where $w \in (-\pi, \pi)$,

$$H\left(e^{jw}\right) = \frac{1 + e^{-2jw}}{1 + \frac{9}{16}e^{-2jw}}$$

Determine the overall output $y[n]$, when the input is:

$$x[n] = 2cos\left(\frac{\pi}{2}n\right) + cos(\pi n) + 4sin(\pi n)$$

Question 2

Consider an LTI system whose response to the input

$$x(t) = [e^{-t} + e^{-3t}]u(t)$$

is

$$y(t) = [2e^{-2t} - 2e^{-4t}]u(t).$$

(a). Find the frequency response of this system,
(b). Determine the system's impulse response.

Question 3

Use the properties of the Fourier Transform to prove the following statement

Let $y(t) = x(t) * h(t)$. Prove: If $|H(jw)| = 1$ for all w, then

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Question 4

Consider the signal

$$x(t) = e^{-t}\left[u(t) - u(t-1)\right]$$

Determine the Fourier transform of each of the signals shown in figure.1

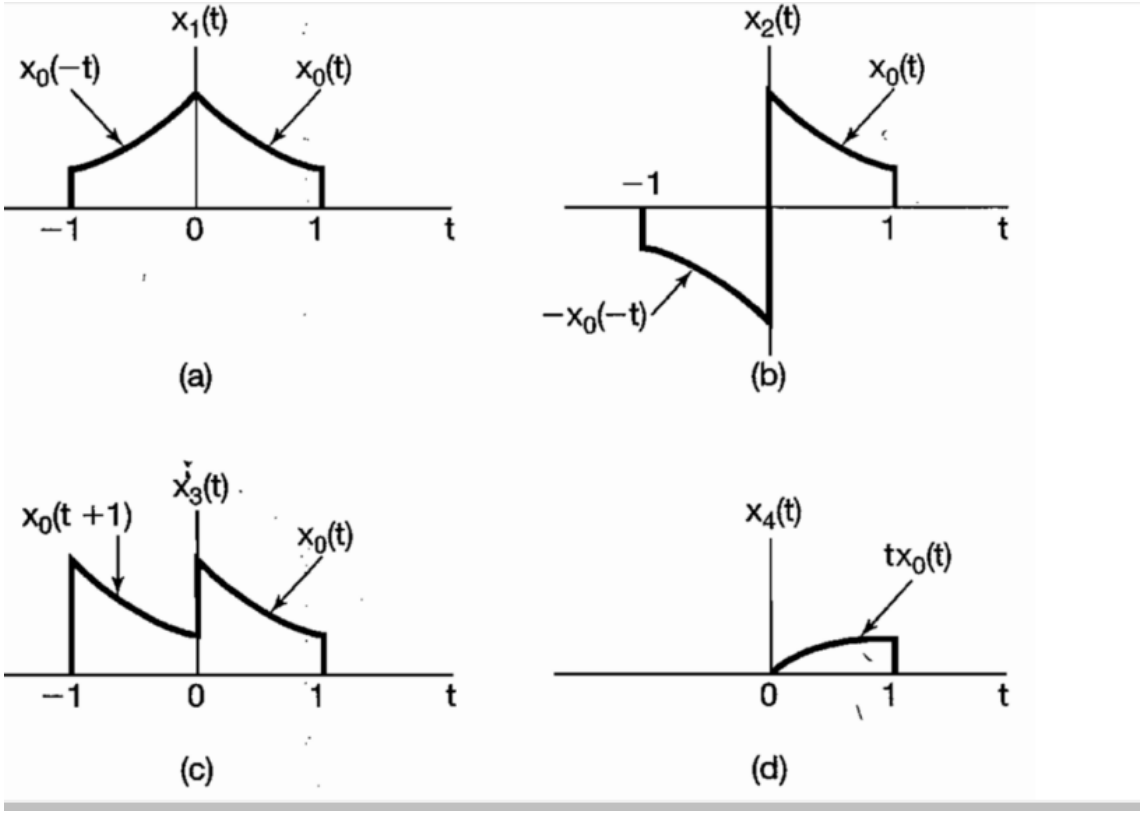

Figure 1: Shape

Question 5

Consider LTI system that is related by the differential equation

$$y''(t) + 5y'(t) + 6y(t) = 2x(t)$$

(a) Find the impulse response of this system
(b) What is the Fourier transform of the output $Y(jw)$, if $x(t) = e^{-2t}u(t)$