#1 ⓑ $H(e^{jw}) = \dfrac{1 + e^{-2jw}}{1 + \frac{9}{16} e^{-2jw}}$ $\qquad w = \frac{\pi}{2}, -\frac{\pi}{2}, \pi, -\pi$

(part)ⓐ at last page

$H(e^{jw})\Big|_{w=\pi} = 1.28$

$y[n] = \frac{32}{25}(-1)^n$

#2 $X(jw) = \dfrac{1}{1+jw} + \dfrac{1}{3+jw} = \dfrac{2(2+jw)}{(1+jw)(3+jw)}$

$Y(jw) = \dfrac{2}{2+jw} - \dfrac{2}{4+jw} = \dfrac{2}{(2+jw)(4+jw)}$ $\qquad \dfrac{Y}{X} = \dfrac{\frac{6}{(2+jw)(4+jw)}}{\frac{2(2+jw)}{(1+jw)(3}}$

$H(jw) = \dfrac{Y(jw)}{X(jw)} = \dfrac{2(3+jw)(jw+1)}{(4+jw)(2+jw)^2} = \dfrac{A}{4+jw} + \dfrac{B}{2+jw} + \dfrac{C}{(2+jw)^2}$

$A = 1.5 \quad B = 0.5 \quad C = -1$

$h(t) = 1.5e^{-4t}u(t) + \frac{1}{2}e^{-2t}u(t) \cancel{+} + e^{-2t}u(t)$

#3 $\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(jw)|^2 dw \Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(jw)X(jw)|^2 dw$

$\Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(jw)|^2 |X(jw)|^2 dw$

$\Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(jw)|^2 dw = \int_{-\infty}^{\infty} |x(t)|^2 dt$

#4

(a) $X_1(t) = X_0(t) + X_0(-t)$ $\qquad X_0 = \frac{1-e^{-(1+jw)}}{1+jw}$

$$X_1(jw) = X_0(jw) + X_0(-jw) = \frac{2-2e^{-1}\cos w - 2we^{-1}\sin w}{1+w^2}$$

(b) $X_2(t) = X_0(t) - X(-t)$

$$X_2(jw) = X_0(jw) - X(-jw) = j\left[\frac{-2w+2e^{-1}\sin(w)+2we^{-1}\cos w}{1+w^2}\right]$$

(c) $X_3(t) = X_0(t) + X_0(-t+1)$

$$X_3(jw) = X_0(jw) + e^{-jw}X_0(-jw) = \frac{1+e^{-jw}-e^{-1}(1+e^{jw})}{1+jw}$$

(d) $$X_4(jw) = j\frac{d}{dw}X(jw) = \frac{1-2e^{-1}e^{-jw}-jwe^{-1}e^{-jw}}{(1+jw)^2}$$

#5 $(-w^2+5jw+6)Y(jw) = 2X(jw)$

(a) $$H = \frac{2}{-w^2+5jw+6} \Rightarrow \frac{2}{(jw+2)(jw+3)}$$

$$\frac{A}{jw+2} + \frac{B}{jw+3} = \frac{A(jw+3)+B(jw+2)}{(jw+2)(jw+3)}$$

$A+B=0$ $\quad -A=+B$ $\quad A=2$

$3A+2B=2$ $\quad 3A-2A=2$ $\quad B=-2$

$$\frac{2}{jw+2} - \frac{2}{jw+3} = H(jw)$$

$$h(t) = 2(e^{-2t}-e^{-3t})u(t)$$

(b) answer in the box, for practice

$X(j\omega) = \frac{1}{(2+j\omega)}$ Check $y(t)$ shown below

$Y(j\omega) = 2\left(\frac{1}{(2+j\omega)^2} - \frac{1}{(j\omega+3)(j\omega+2)}\right)$

$y(t) = 2te^{-2t}u(t) + 2e^{-3t}u(t) + 2e^{-2t}u(t)$

$\frac{A}{j\omega+3} + \frac{B}{j\omega+2}$

$A + B = 0$

$2A + 3B = 1$

$2A - 3A = 1$

$A = -1 \quad B = 1$

$\frac{+1}{3+j\omega} - \frac{1}{j\omega+2}$

(#1)

$\Leftarrow \begin{cases} \frac{1}{2\pi} F^{-1}(X_1(j\omega) * X_2(j\omega)) = x_1 x_2 \text{ ①} \\ F\{x_1 x_2\} = X_1(j\omega) * X_2(j\omega) \text{ ②} \end{cases}$

You can interchange.

use ①

$\left(\frac{1}{2\pi}\right)^2 \int_{\mathbb{R}} \int_{\mathbb{R}} X_1(j\alpha) X_2(j(\omega-\alpha))\, d\alpha \; e^{-j\omega t}\, dt$

$s = \omega - \alpha$

$= \left(\frac{1}{2\pi}\right)^2 \int_{\mathbb{R}} \int_{\mathbb{R}} X_1(j\alpha) X_2(js)\, e^{j(s+\alpha)}\, ds\, d\alpha$

$= \left(\frac{1}{2\pi}\right)^2 \int_{\mathbb{R}} X_1(j\alpha) e^{j\alpha} d\alpha \int_{\mathbb{R}} X_2(js) e^{js}\, ds$

$\Rightarrow x_1(t)\, x_2(t)$

holds