#1 $X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$

(a) $X_1(e^{j\omega}) = X(e^{j\omega})\,X(e^{j\omega})$

(b) $X_2(e^{j\omega}) = e^{j\omega}X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega})$

(c) $y[n] = x[n] * h[n] = e^{-j\omega n_0}X(e^{j\omega})$

(d) $X_3(e^{j\omega}) = \frac{1}{1-e^{-j\omega}}X(e^{j\omega}) + \pi X(e^{j0})\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)$

#2 $x[n] = \cos\omega_0 n = \frac{1}{2}e^{j\omega_0 n} + \frac{1}{2}e^{-j\omega_0 n}$

(a) $X(e^{j\omega_0}) = \sum_{l=-\infty}^{\infty}\pi\,\delta(\omega-\omega_0-2\pi l) +$

$\sum_{l=-\infty}^{\infty}\pi\,\delta(\omega+\omega_0-2\pi l)$

(b) $Y(e^{j\omega_0}) = \frac{1}{j}\sum_{l=-\infty}^{\infty}\pi\,\delta(\omega-\omega_0-2\pi l) - \frac{1}{j}\sum_{l=-\infty}^{\infty}\pi\,\delta(\omega+\omega_0-2\pi l)$

(c) $2X(e^{j\omega_0}) + 3Y(e^{j\omega_0})$

#3 $X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}$ $\quad Y(e^{j\omega}) = \frac{1}{(1-ae^{-j\omega})^2} + \frac{1}{1-ae^{-j\omega}}$

$$H(e^{j\omega}) = \frac{\frac{1}{(1-ae^{-j\omega})^2} + \frac{1}{1-ae^{-j\omega}}}{\frac{1}{1-ae^{-j\omega}}} = \frac{\frac{1-ae^{-j\omega}+1}{(1-ae^{-j\omega})^2}}{\frac{1}{1-ae^{-j\omega}}}$$

$$H(e^{jw}) = \frac{2 - ae^{-jw}}{1 - ae^{-jw}}$$

$$= \frac{1}{1 - ae^{-jw}} + \frac{1 - ae^{-jw}}{1 - ae^{-jw}} = \frac{1}{1 - ae^{-jw}} + 1$$

$$h[n] \Rightarrow a^n u[n] + \delta[n]$$

#4 (a) $X_1(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$

$$= 2\sum_{n=1}^{\infty} \left(\tfrac{1}{2} e^{-jw}\right)^n = \frac{e^{-jw}}{1 - \frac{1}{2}e^{-jw}}$$

(b) $X_2(e^{jw}) = \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{|n|} \cos\left(\tfrac{\pi}{8}(n-1)\right) e^{-jwn}$

$$= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n \cos\left(\tfrac{\pi}{8}(n-1)\right) e^{-jwn} + \sum_{n=-\infty}^{-1} \left(\tfrac{1}{2}\right)^{-n} \cos\left(\tfrac{\pi}{8}(n-1)\right) e^{-jwn}$$

$$\Rightarrow \tfrac{1}{2} e^{-jw} \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n-1} \cos\left(\tfrac{\pi}{8}(n-1)\right) e^{-jw(n-1)} + 2e^{-jw} \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{m+1} \cos\left(\tfrac{\pi}{8}(m+1)\right) e^{jw(m+1)}$$

$$= \tfrac{1}{2} e^{-jw} \sum_{n=1}^{\infty} \left(\tfrac{1}{2}\right)^n \cos\left(\tfrac{\pi}{8} n\right) e^{-jwn} + 2e^{-jw} \sum_{m=1}^{\infty} \left(\tfrac{1}{2}\right)^m \cos\left(\tfrac{\pi}{8} m\right) e^{jwm}$$

$$\Rightarrow \tfrac{1}{4} e^{-jw} \sum_{m=1}^{\infty} \left(\tfrac{1}{2}\right)^m \left(e^{j\frac{\pi}{8}n} + e^{-j\frac{\pi}{8}n}\right) e^{-jwn} + e^{-jw} \sum_{m=1}^{\infty} \left(\tfrac{1}{2}\right)^m \left(e^{j\frac{\pi}{8}m} + e^{-j\frac{\pi}{8}m}\right) e^{jwm}$$

$$\Rightarrow \tfrac{1}{2} e^{-jw} \left[\frac{e^{j(w-\frac{\pi}{8})}}{1 - \frac{1}{2}e^{j(w-\frac{\pi}{8})}} + \frac{e^{j(w+\frac{\pi}{8})}}{1 - \frac{1}{2}e^{j(w+\frac{\pi}{8})}} + \frac{e^{j(w+\frac{\pi}{8})}}{1 - \frac{1}{2}e^{j(w+\frac{\pi}{8})}} + \frac{e^{j(w-\frac{\pi}{8})}}{1 - \frac{1}{2}e^{j(w-\frac{\pi}{8})}} \right]$$

#5 (a) $Y(e^{jw}) - \frac{1}{2} e^{-jw} Y(e^{jw}) = X(e^{jw})$

$$H(e^{jw}) = \frac{1}{1 - \frac{1}{2} e^{-jw}}$$

(b) $h[n] = \left(\frac{1}{2}\right)^n u[n]$

(c) $Y(e^{jw}) = X(e^{jw}) H(e^{jw})$

$$= \frac{-2}{1 - \frac{1}{3} e^{-jw}} + \frac{3}{1 - \frac{1}{2} e^{-jw}} \Rightarrow$$

$$y[n] = -2\left(\frac{1}{3}\right)^n u[n] + 3\left(\frac{1}{2}\right)^n u[n]$$

#6 example 5.9 in book 379