

ECE301

HW 7

DUE ON THURSDAY DEC. 8TH

Please provide steps to explain
your answer

T.A.: Jing Guo

email: guo349@purdue.edu

**Office Hour change: MSEE180
(9:30am - 10:30am Monday and
Wednesday)**

Question 1

A signal $x(t)$ with Fourier transform $X(j\omega)$ undergoes impulse-train sampling to generate

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

where $T = 10^{-4}s$. For each of the following sets of constraints on $x(t)$ and/or $X(j\omega)$, does the sampling theorem guarantee that $x(t)$ can be recovered exactly from $x_p(t)$?

- $X(j\omega) = 0$ for $|\omega| > 4000\pi$
- $X(j\omega) = 0$ for $|\omega| > 14000\pi$
- $\text{Re}\{X(j\omega)\} = 0$ for $|\omega| > 4000\pi$
- $x(t)$ real and $X(j\omega) = 0$ for $\omega > 4500\pi$
- $X(j\omega) * X(j\omega) = 0$ for $|\omega| > 14000\pi$
- $|X(j\omega)| = 0$ for $|\omega| > 4500\pi$

Question 2

a.) The signal $x_a(t) = u(t + 2) - u(t - 2)$ is sampled every $T_s = 0.5s$ to form $x[n] = x_a(nT_s)$. Determine a closed-form expression for the DTFT $X(e^{j\omega})$ of the $x[n]$. Check table 5.2 in the textbook

b.) The signal $x_a(t) = t\{u(t + 2) - u(t - 2)\}$ is sampled every $T_s = 0.5s$ to form $x[n] = x_a(nT_s)$. Determine a closed-form expression for the DTFT $X(e^{j\omega})$ of the $x[n]$.

Question 3

Consider the input signal $x_0(t)$ below

$$x_0(t) = e^{-j25t} + e^{-j15t} + e^{-j5t} + 1 + e^{j5t} + e^{j15t} + e^{j25t}$$

This signal is first input to an analog filter with impulse response (hints check table 4.2)

$$h_{LP}(t) = \frac{\sin(16t)}{\pi t} + \frac{\sin(20t)}{\pi t} + \frac{\sin(30t)}{\pi t}$$

to form $x(t) = x_0(t) * h_{LP}(t)$. $x(t)$ is sampled as a rate of $w_s = 60$ to form $x[n]$. The DT signal $x[n]$ is then went through DT LTI system with impulse response

$$h[n] = \frac{\sin(\frac{\pi}{3}n)}{\pi n} + \frac{\sin(\frac{2\pi}{3}n)}{\pi n}$$

- Plot both Fourier Transform of $h_{LP}(t)$ and the DTFT of $h[n]$.
- Write your expression for the output $y[n]$

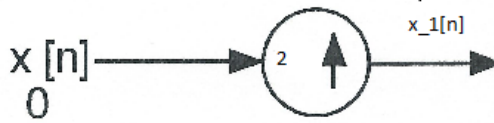


Figure 1

Question 4

Consider the continuous-time signal $x_a(t)$ below.

$$x_a(t) = T_s \left\{ \frac{\sin(15t)}{\pi t} + \frac{\sin(20t)}{\pi t} + \frac{\sin(30t)}{\pi t} \right\}$$

- a.) A discrete-time signal is created by sampling $x_a(t)$ according to $x[n] = x_a(nT_s)$ for $T_s = \frac{2\pi}{60}$. Plot the DTFT of $x[n]$, over $-\pi < w < \pi$
- b.) Repeat part (a) for $T_s = \frac{2\pi}{50}$. Sketch the DTFT of $x[n]$, over $-\pi < w < \pi$.
 hints: check table 5.2 from text book

Question 5

- a.) Consider the continuous time signal $x(t)$ below. A discrete time signal is created by sampling $x(t)$ according to $x[n] = x(nT_s)$ with $T_s = \frac{\pi}{10}$. Plot the magnitude of the DTFT of $x[n]$, over $-\pi < w < \pi$

$$x(t) = T_s \left(\frac{\sin(10t)}{\pi t} + \frac{\sin(5t)}{\pi t} \right)$$

- b.) The discrete-time signal, $x_1[n]$, is created by running $x[n]$ from part(a) through the DT system see Fig.1

$$h_1[n] = 3 \frac{\sin\left(\frac{\pi n}{3}\right)}{\pi n}$$

hints: Upsampling