

# Lecture - Wednesday Oct. 7

## Time Scaling

Continuous time

$$x(at) = \sum_{k=-\infty}^{\infty} a_k e^{jk(\omega_0 t)}$$

Fourier coefficient remain the same

change in fundamental frequency

Discrete-time

$$x_m[n] = \begin{cases} x[n/m] & \text{if } n \text{ is multiple of } m \\ 0 & \text{otherwise} \end{cases}$$

$$\xrightarrow{\text{FS}} \frac{1}{m} a_k$$

F.100 jahrsbegrenzung - lautet

## Periodic Convolution

$$z[n] = \sum_{r=0}^{N-1} x[r] y[n-r] \xrightarrow{FS} N a_k b_k$$

Hilfsp

$$= \sum_{n=0}^{N-1} \sum_{k=0}^{\infty} a_k e^{j k \omega_0 n} \sum_{r=0}^{N-1} b_r e^{j k \omega_0 (n-r)}$$

## Multiplication

$$x[n] y[n] \xrightarrow{FS} \sum_{\ell=0}^{N-1} a_\ell b_{k-\ell}$$

## Running Sum

$$\sum_{k=-\infty}^n x[k] \xleftrightarrow{\text{FS}} \left( \frac{1}{1 - e^{j\omega k (2\pi/N)}} \right) a_k$$

~~Bliss goes J. West  
K = -∞ to ∞~~

## First Difference

$$x[n] - x[n-1] \xleftrightarrow{\text{FS}} (1 - e^{-j\omega k (2\pi/N)}) a_k$$

why?

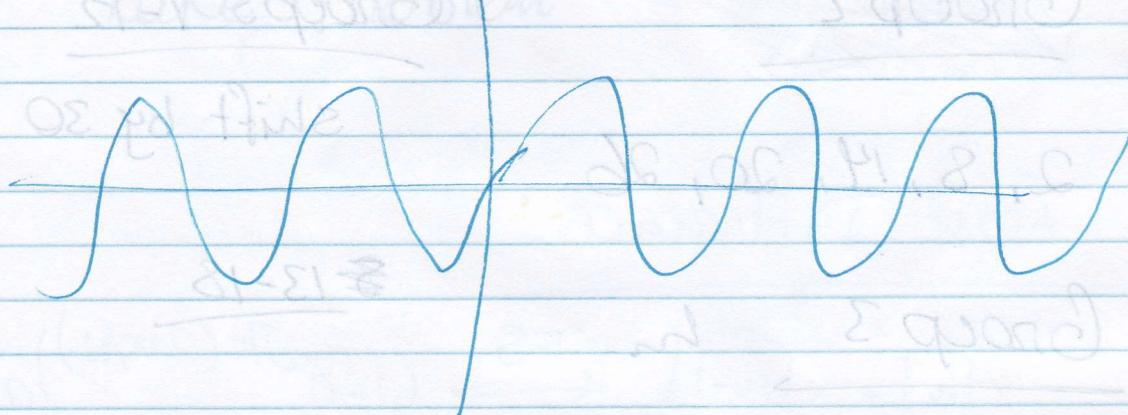
## Parseval's Relation

$$\frac{1}{N} \sum_{n=1}^N |x[n]|^2 = \sum_{k=1}^N |a_k|^2$$

## Review Lecture (Exam 1)

$\alpha$ ,  $P_{\text{avg}}$ ,  $E_{\text{avg}}$  and  $P_{\text{dc}}$

$$y(t) = \sin(\omega_0 t)$$



$$E_{\text{avg}} = \infty$$

$$P_{\text{avg}} = ?$$

First identify whether value  
is zero, finite or infinity

If finite, remember that you are  
~~not~~ considering the magnitude  
square

- \* Signal Scaling and Shifting
- \* Fundamental Period/frequency
  - Definition
  - Sum of two periodic signals
- \* Properties of Fourier series coefficients
- \* Linearity and time invariance
  - Characterized by impulse response

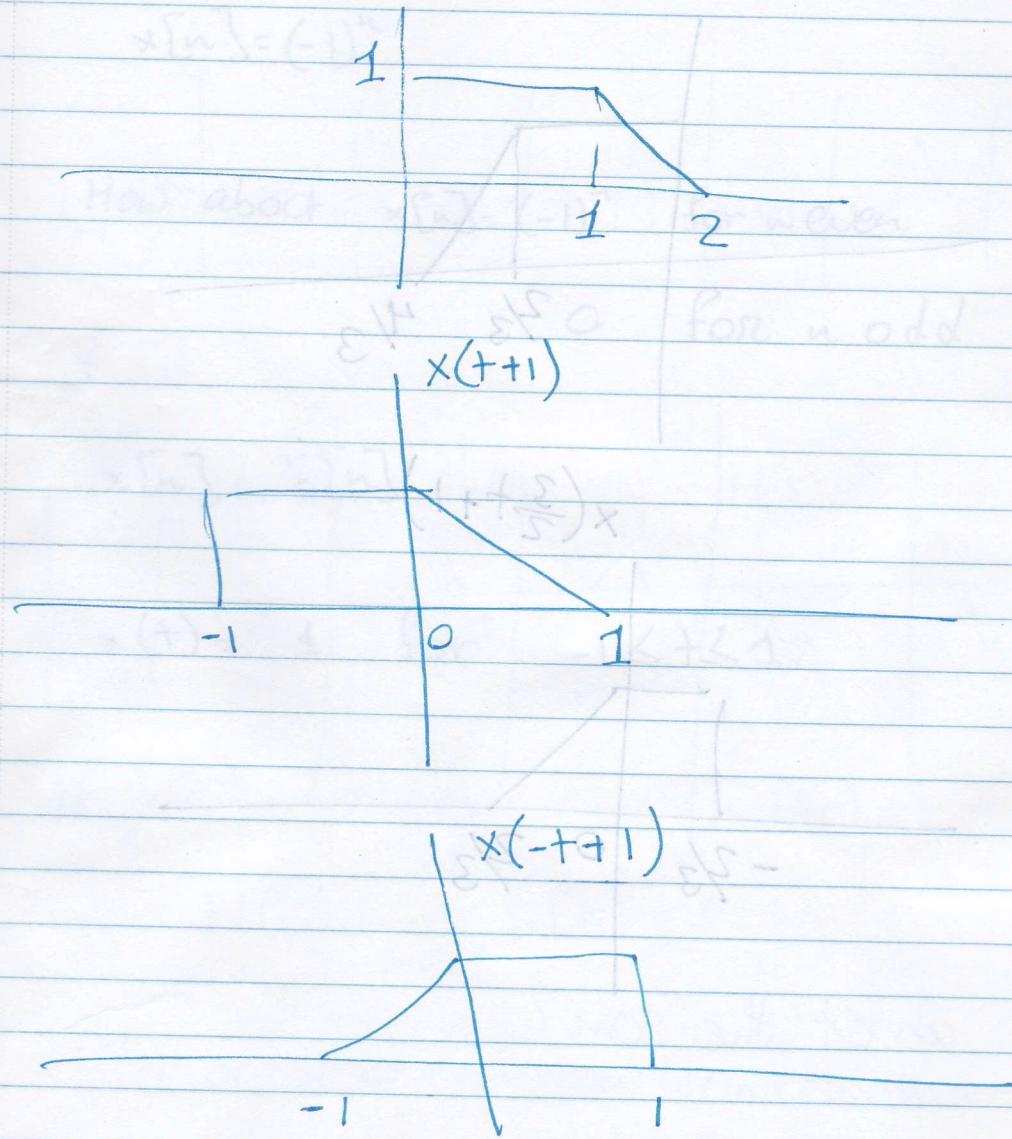
Properties of systems translate  
to properties of the impulse  
response

For example

When is a system memoryless?

when is an LTI system memoryless?

## Shifting and scaling



$$\times \left(\frac{3}{2} + \right)$$

To proportion of the jumps

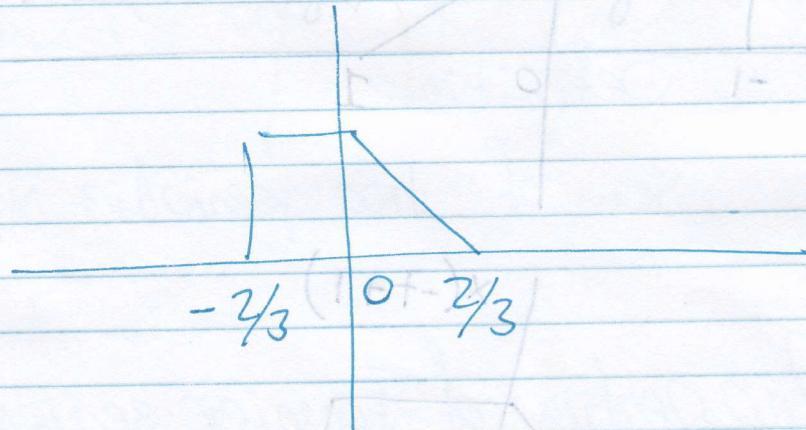
response

For example

When we have a random walk

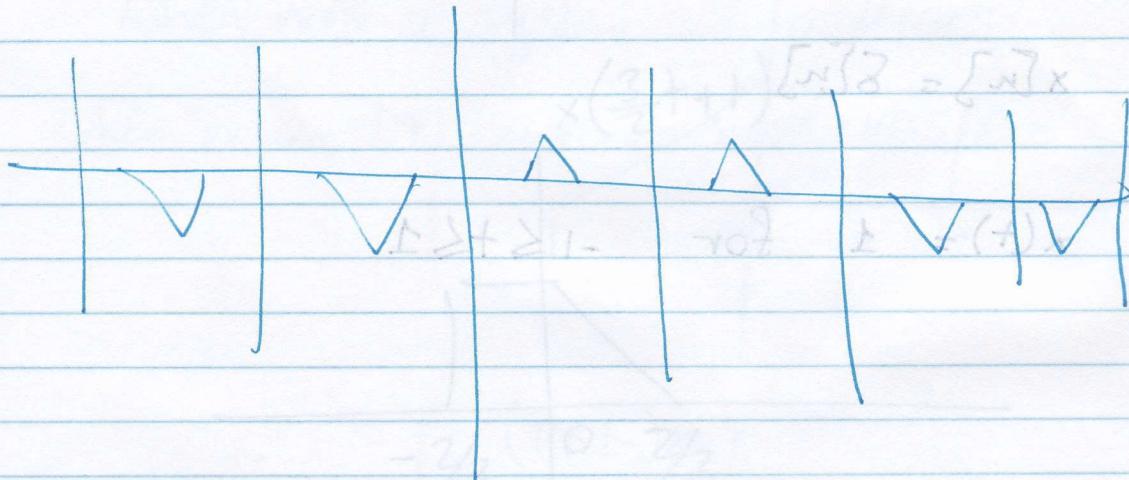
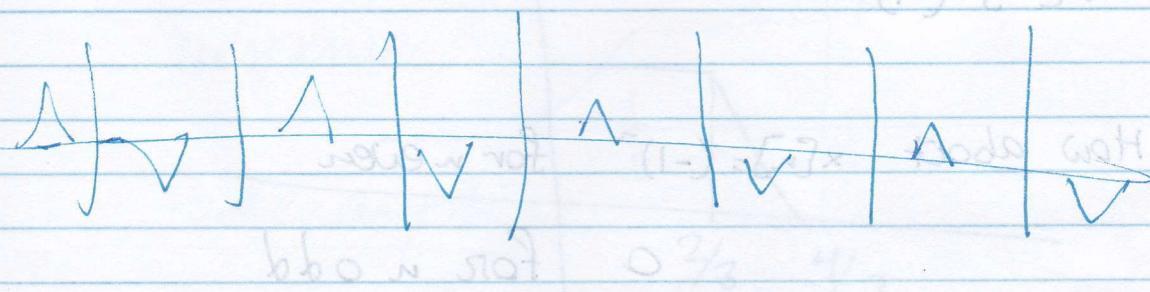
when it is an LTI

$$\times \left(\frac{3}{2} + 1\right)$$



Fundamental Period

$$T(1) = \sqrt{\pi}$$



How about the sum?

## Lecture - Wed. Oct. 7 (cont.)

Example 3.14

Fourier series and LTI systems

LTI systems

If  $x(t) = e^{st}$  then  $y(t) = H(s)e^{st}$ ,

where  $H(s) = \int_{-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$

If  $x[n] = z^n$  then  $y[n] = H(z)z^n$

$$H(z) = \sum_{k=-\infty}^{\infty} h[k] z^{-k}$$

$H(s)$  or  $H(z)$  system functions

When  $s = j\omega$ ,  $H(j\omega)$  is the frequency response of the system,

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

Discrete-time  
 $Z = e^{j\omega}$

$$H(e^{j\omega}) = \sum h[n] e^{-jn\omega}$$

$$x(t) = \sum a_k e^{jk\omega_0 t}$$

$$y(t) = \sum a_k H(jk\omega_0) e^{jk\omega_0 t}$$

$$x(t) \xleftrightarrow{FS} a_k$$

$$y(t) \xleftrightarrow{FS} a_k H(jk\omega_0)$$

### Example 3.16

$$x(t) = \sum_{k=-3}^3 a_k e^{jk2\pi t}$$

$$a_0 = 1 \quad a_1 = a_{-1} = \frac{1}{4} \quad a_2 = a_{-2} = \frac{1}{2}$$

$$a_3 = a_{-3} = \frac{1}{3}$$

Note that  $y(t)$  is real

why?

### Example 3.17

$$x[n] = \cos\left(\frac{2\pi n}{N}\right)$$

$$h[n] = \alpha^n u[n], -1 < \alpha < 1$$

$$x[n] = \frac{1}{2} e^{j(2\pi/N)n} + \frac{1}{2} e^{-j(2\pi/N)n}$$

$$H(e^{j\omega}) = \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n} = \sum_{n=0}^{\infty} (\alpha e^{-j\omega})^n$$

\* Problem 1.54 \*

$$H(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega}}$$

$$y[n] = \frac{1}{2} H(e^{j2\pi/N}) e^{j(2\pi/N)n}$$

$$+ \frac{1}{2} H(e^{-j2\pi/N}) e^{-j(2\pi/N)n}$$

$$= \frac{1}{2} \left( \frac{1}{1 - \alpha e^{-j2\pi/N}} \right) e^{j(2\pi/N)n}$$

$$+ \frac{1}{2} \left( \frac{1}{1 - \alpha e^{j2\pi/N}} \right) e^{-j(2\pi/N)n}$$

If we write

$$\frac{1}{1 - \alpha e^{-j2\pi/N}} = r e^{j\theta} \quad \text{otherwise}$$

then

$$y[n] = r \cos \left( \frac{2\pi}{N} n + \theta \right)$$

for example if  $N=4$

$$\frac{1}{1 - \alpha e^{-j2\pi/4}} = \frac{1}{1 + \alpha j} \stackrel{?}{=} \frac{1}{\sqrt{1 + \alpha^2}} e^{j(-\tan^{-1}(\alpha))}$$

$$y[n] = \frac{1}{\sqrt{1+\alpha^2}} \cos \left( \frac{\pi n}{2} - \tan^{-1}(\alpha) \right)$$

wed. 14  
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## The continuous-time Fourier Transform

$$\left( \theta + n \frac{\pi}{N} \right) \Rightarrow \theta = \text{large}$$

Half-dimensions

$$S_o + 1 \quad T_o + 1 \quad H(T_o)$$