

* Go over time transformation

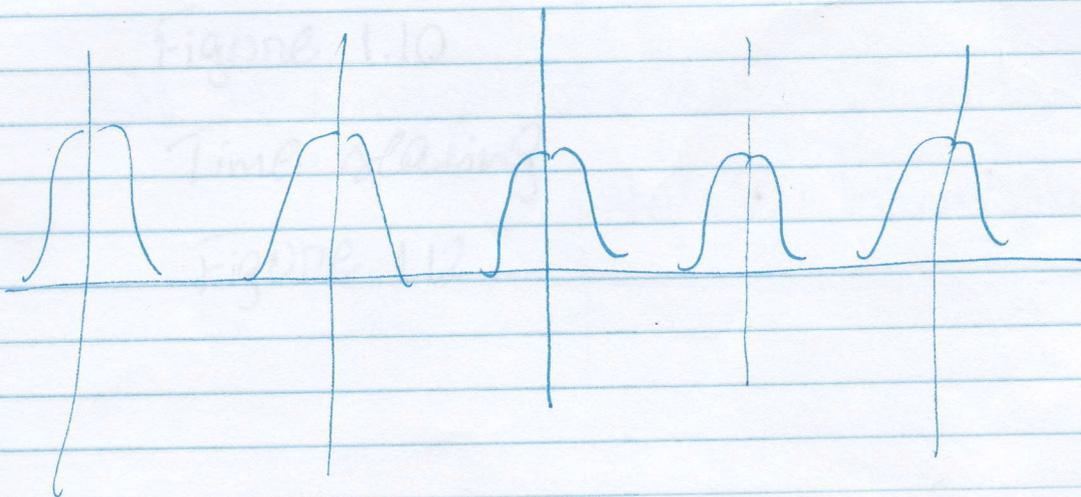
Second Lecture

* Periodic Signals

$$- x(t) = x(t+T)$$

- Natural response for systems in which energy is conserved
- Ideal LC circuits without resistive energy dissipation
- Ideal mechanical systems without frictional losses

Figure 1.14



$$x(t) = x(t+mT)$$

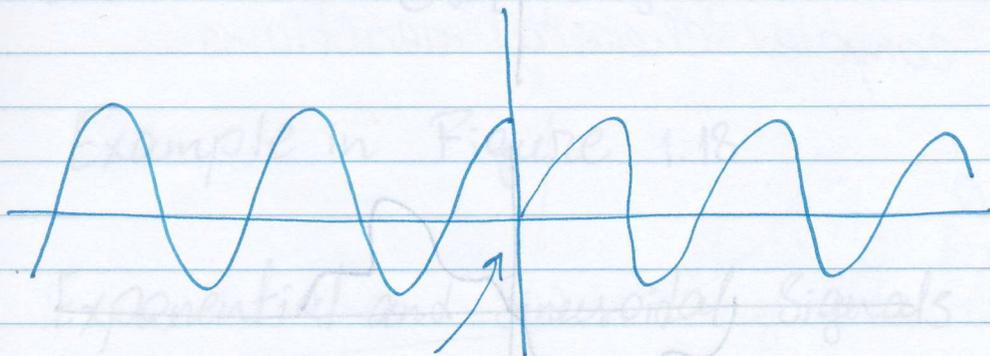
periodic with periods $2T, 3T, 4T, \dots$

T_0 is the fundamental period

Example 1.4

$$x(t) = \begin{cases} \cos(t) & \text{if } t < 0 \\ \sin(t) & \text{if } t \geq 0 \end{cases}$$

$$\cos(t+2\pi) = \cos(t) \quad \sin(t+2\pi) = \sin(t)$$



Discontinuity at $t=0$ does
not recur

* Go over time transformation

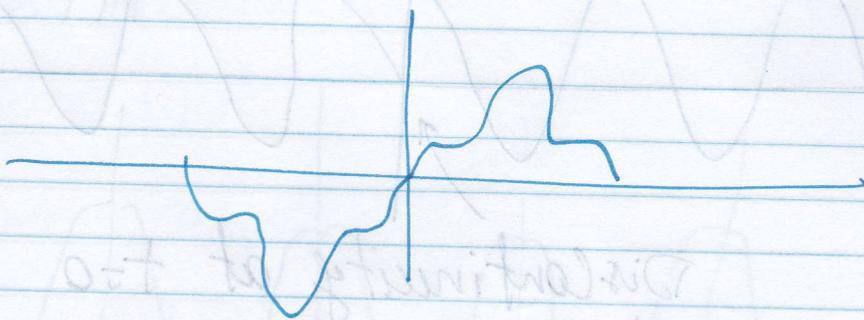
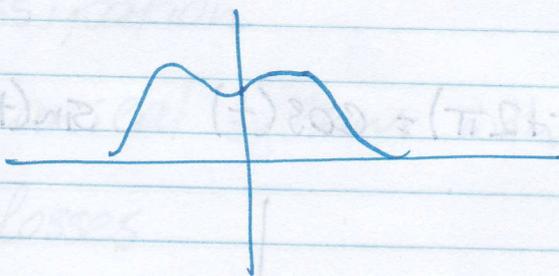
Even and Odd Signals

Even signal: identical to its
time-reversed counterpart

$$x(-t) = x(t)$$

Odd Signal

$$x(-t) = -x(t)$$



An odd signal must necessarily be 0 at $t=0$

$$\text{Ev}\{x(t)\} = \frac{1}{2} [x(t) + x(-t)]$$

$$\text{Od}\{x(t)\} = \frac{1}{2} [x(t) - x(-t)]$$

$$x(t) = \text{Ev}\{x(t)\} + \text{Od}\{x(t)\}$$

Check that $\text{Ev}\{x(t)\}$ is even

Check that $\text{Od}\{x(t)\}$ is odd

Example in Figure 1.18

Exponential and Sinusoidal Signals

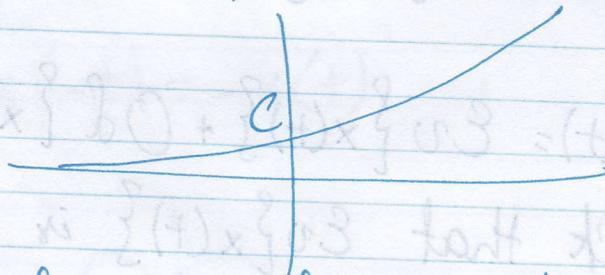
Basic building blocks

- Continuous-time

$$x(t) = Ce^{at}$$

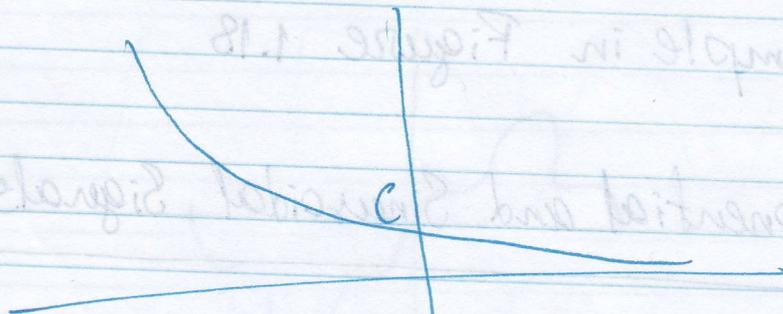
C and a are in general complex
if C and a are real, then $x(t)$
is called a real exponential

$$a > 0$$



chain reactions in atomic explosions
complex chemical reactions

$$a < 0$$



Radio active decay

damped mechanical systems

Using

one of

* Homework Problems 2.61 *
and 2.62

for $a = 0$, $x(t)$ is constant

If a is purely imaginary

E.g. $x(t) = e^{j\omega_0 t}$

periodic signal

$$e^{j\omega_0 t} = e^{j\omega_0 (t+T)}$$

$$= e^{j\omega_0 t} e^{j\omega_0 T}$$

$$e^{j\omega_0 T} = 1$$

If $\omega_0 = 0$ then $x(t) = 1$ ← Periodic for any value of T

If $\omega_0 \neq 0$ then the fundamental period

$$T_0 = \frac{2\pi}{|\omega_0|}$$

Homework

$e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$ have the same
fundamental period

Any one-to-one mapping of a periodic
signal results
in a periodic
signal

Sinusoidal Signal

$$x(t) = A \cos(\omega_0 t + \phi)$$

t has seconds as unit

ϕ radians

ω_0 radians per second

$$\omega_0 = 2\pi f_0$$

f_0 cycles per second

Hertz (Hz)

T_0 is the fundamental period

Eq. (1.24)

$$T_0 = \frac{2\pi}{|\omega_0|} = \frac{1}{|f_0|}$$

Using

Euler's relation

$$A \cos(\omega_0 t + \phi) = \frac{A}{2} e^{j\phi} e^{j\omega_0 t} + \frac{A}{2} e^{-j\phi} e^{-j\omega_0 t}$$

Sinusoidal and Complex Exponential

Signals: $E_\infty = \infty$ $P_\infty < \infty$

$$\int_0^{T_0} |e^{j\omega_0 t}|^2 dt$$

$$= \int_0^{T_0} 1 dt = T_0$$

$$P_\infty = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |e^{j\omega_0 t}|^2 dt = 1$$

Homework Problem 1.3

Homework

A harmonically related set of complex exponentials is a set of periodic exponentials with fundamental frequencies that are all multiples of a single positive frequency ω_0

$$\omega_0 = \frac{2\pi}{T_0}$$

$$e^{j\omega_0 T_0} = 1$$

$$\omega = k \omega_0$$

$$k = 0, \pm 1, \pm 2, \dots$$

$$\phi_k(t) = e^{jk\omega_0 t}$$

Fundamental frequency $|k|\omega_0$

Fundamental period $\frac{2\pi}{|k|\omega_0} = \frac{T_0}{|k|}$

Third Lecture

* Consistent with the use of harmonics
in music

Tones resulting from variations in acoustic pressure at frequencies that are integer multiples of a fundamental frequency.

Superposition of harmonically related periodic exponentials.

General Complex-Exponential Signals

$$C e^{at}$$

$$C = |C| e^{j\theta} \quad \text{polar form}$$

$$a = \alpha + j\omega_0 t \quad \text{rectangular form}$$

$$C e^{at} = |C| e^{j\theta} e^{(r+j\omega_0)t}$$

$$|C| e^{rt} e^{j(\omega_0 t + \theta)} \quad (*)$$

Using Euler's relation

$$C e^{at} = |C| e^{rt} \cos(\omega_0 t + \theta) \\ + j |C| e^{rt} \sin(\omega_0 t + \theta)$$

For $r < 0$, the real and imaginary parts are sinusoidal

For $r > 0$, sinusoidal multiplied by a growing exponential

For $r < 0$, sinusoidal multiplied by a decaying exponential

Figure 1.23